

Name: \_\_\_\_\_  
AP Calculus: Area Approximations

Date: \_\_\_\_\_  
Ms. Loughran

1. Suppose a car is moving with increasing velocity and suppose we are given the velocity every second, as in the table below.

Time(sec)	0	1	2	3	4	5
Velocity(ft/sec)	20	30	38	44	48	50

Assuming that the car's velocity is always increasing, give upper and lower estimates for the distance the car traveled in the 5 seconds.

2. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below:

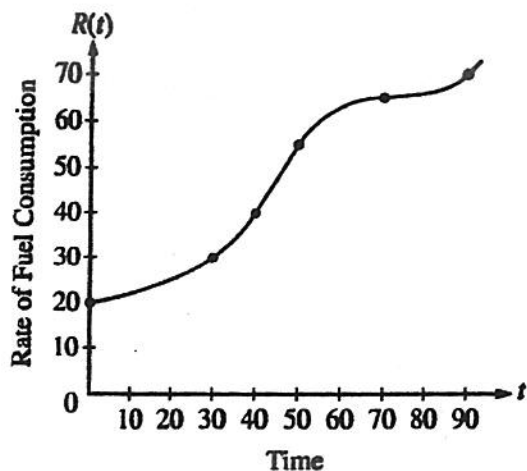
Time spent running (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

- (a) Assuming that Roger's speed is always decreasing, give upper and lower estimates for the distance Roger ran during the first half hour.  
(b) Give upper and lower estimates for the distance Roger ran in total.  
(c) How often would Jeff have needed to measure Roger's pace in order to find lower and upper estimates within 0.1 mile of the actual distance that he ran?

3. The table below shows the values of  $R(t)$ , a differentiable function of  $t$ .

$t$	0	1	2	3	4	5	6	7	8
$R(t)$	4.6	5.4	6.1	6.5	6.8	6.3	6.0	5.5	4.8

- (a) Use a midpoint Riemann sum with four equal subintervals of equal length to approximate  $\int_0^8 R(t) dt$ .
- (b) Use a trapezoidal approximation with four equal subintervals of equal length to approximate  $\int_0^8 R(t) dt$ .



$t$ (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

4. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above. Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain.
5. The temperature, in degrees Celsius, of the water in a pond is a differentiable function  $W$  of time  $t$ . The table below shows the water temperature as recorded every 3 days over a 15-day period. Approximate  $\int_0^{15} W(t) dt$  by using the trapezoidal approximation with subintervals of length  $\Delta t = 3$ .

$t$ (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

6. A metal wire of length 8 centimeters is heated at one end. The table below gives selected values of the temperature  $T(x)$ , in degrees Celsius, of the wire  $x$  cm from the heated end. The function  $T$  is decreasing and twice differentiable.

Estimate  $\int_0^8 T(x)dx$  using four subintervals and the (a) trapezoidal rule (b) a left hand Riemann sum.

Dist. $x$ (cm)	0	1	5	6	8
Temp. $T(x)$ ( $^{\circ}\text{C}$ )	100	93	70	62	55