ROSLYN HIGH SCHOOL CALCULUS AB S. Conrad, S. Weiss

CHAIN RULE CHARTS

1. The following table of values was computed for a pair of functions f and g and their derivatives f' and g'. Complete the table for the two composite functions h and k and their derivatives h' and k', if $h(x) = f[g(x)]$ and $k(x) = g[f(x)]$.

x	f(x)	f'(x)	g(x)	g'(x)	h(x)	h'(x)	k(x)	k'(x)
0	1	5	2	-5				
1	3	-2	0	1				
2	0	2	3	1				
3	2	4	1	-6				

2. Use the table from question 1 to find the DERIVATIVE, at x = 2, of
   a) $f(x) + g(x)$
   b) $[f(x)][g(x)]$
   c) $f(x)/g(x)$
   d) $f[g(x)]$

3. The table at the right records the values of f, g, f', and g' at x = 1 and x = 2. Find the value of each of the following:

x	f(x)	g(x)	f'(x)	g'(x)
1	2	2	4	5
2	5	7	9	3

   a) $(f + g)'(2)$
   b) $(\frac{f}{g})'(1)$
   c) $\frac{d}{dx}f[g(x)]$ at x = 1
   d) $(gof)'(1)$
   e) $[f^2(x)]'$ at x = 2
   f) $[f(x^2)]'$ at x = 1
   g) $(fof)'(1)$

4. Determine the derivative g'(x) in terms of f' if
   a) $g(x) = f(x^2)$
   b) $g(x) = f(\sin^2 x) + f(\cos^2 x)$
   c) $g(x) = f[f(x)]$
   d) $g(x) = f(f[f(x)])$

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CHAIN RULE PROBLEMS

For problems 1-6, let f and g be differentiable at $x = 2$, $f(2) = 3$, $f'(2) = 1$, $g(2) = 7$, and $g'(2) = -3$. Find the value of each indicated derivative at $x = 2$.

1. $\frac{d}{dx}[f(x)g(x)]$

2. $\frac{d}{dx}[f^2(x)g(x)]$

3. $\frac{d}{dx}[f^3(x)]$

4. $\frac{d}{dx}[3f(x) - g^3(x)]$

5. $\frac{d}{dx}\left[\frac{2g(x) - f(x)}{f^2(x)}\right]$

6. $\frac{d}{dx}[f(x)/3g(x)]$

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7. The population of Sunbelt City increases at the rate of 10 000 people per day. If the area of the city grows to keep the ratio of 1 square mile per 1000 people, how fast is the area increasing per day?

8. If $h(x) = f[u(x)]$, $u(2) = 3$, $u'(2) = 7$, and $f'(3) = -2$, find $h'(2)$.

9. Let $y = [u^2(t) + 1]^3$. Find dy/dt at $t = 3$ if $u(3) = -2$ and $u'(3) = -1$.

10. If $f(x) = \sin x$, $g(x) = x^2$, and $h(x) = g[f(x)]$, find an equation of the line tangent to $y = h(x)$ at $x = \frac{1}{4}\pi$.

11. Given the curve defined by $x = \frac{t}{1+t}$, $y = 2t^2$ for $t \geq 0$. Find the slope of the tangent to the curve at the point $(\frac{1}{3}, \frac{1}{2})$.

12. If $f(x) = \ln x$, $g(x) = x^2 - 4$, and $h(x) = f[g(x)]$, find $h'(7)$.

*13. Determine the constant c for which the straight line joining $(0,3)$ to $(5,-2)$ is tangent to $y = \frac{c}{x+1}$.