2018 [Remember - notes in brackets are my explanations for you.]

1 (a) $\int_0^{300} r(t)\,dt = 270$ 270 people enter line during $0 < t < 300$.

(b) $P(t)$ = # of people in line at time t.

[We integrate rate, $r(t) - 0.7$, to get $P(t)$.]

$r(t)$: rate get in line

0.7: rate exit line

$$\int_0^{300} (r(t) - 0.7)\,dt = P(300) - P(0)$$

$$\int_0^{300} (r(t) - 0.7)\,dt \quad P(300) - 20$$

$$\underbrace{\int_0^{300} (r(t) - 0.7)\,dt + 20}_{80} = P(300)$$

$$80 = P(300)$$

[Use calculator]

so 80 ppl in line at t = 300 seconds.

(c) people exit line at rate of 0.7 ppl/s

Need $0.7x = 80$, where x = time

$$x = \frac{80}{0.7}$$

$$300 + \frac{80}{0.7} = 414.286 \text{ seconds}$$

300: [time to get 80 people in line]

$\frac{80}{0.7}$: [time for 80 people to exit]

(d) looking for abs min for $P(t)$ so use candidate test

$$\underbrace{\int_0^t r(x)\,dx}_{} - 0.7t = P(t) - P(0)$$

[integrate rate at which enter line] [subtract how many people exit line]

so $$\int_0^t r(x)\,dx - 0.7t + P(0) = P(t)$$

t	$P(t)$
0	20
33.0132....	3.803
166.574....	158.070
300	80

times at which $r(t) - 0.7 = 0$

↓

[find in calculator]

To the nearest whole #, there are 4 ppl in line at $t = 33.013$ seconds, making this the time the # of ppl reaches a minimum

(2) (a) $v'(t) = a(t)$

$$a(3) = -2.118$$

The acc of the particle at $t = 3$ is -2.118.

(b) $$\int_0^3 v(t)\,dt = x(3) - x(0)$$

$$\int_0^3 v(t)\,dt = x(3) - (-5)$$

$$\underbrace{\int_0^3 v(t)\,dt - 5}_{\text{[use calculator]}} = x(3)$$

$$-1.760 = x(3)$$

The position of the particle at $t = 3$ is -1.760.

(c) $$\int_0^{3.5} v(t)\,dt = 2.844$$

means the displacement of the particle from $t = 0$ to $t = 3.5$ is 2.844

$$\int_0^{3.5} |v(t)|\,dt = 3.737$$

means the total distance travelled by the particle from $t = 0$ to $t = 3.5$ is 3.737

(d) need $v(t) = x_2'(t)$

$v(t) = 2t - 1$

[use graph of $v(t)$ to see where intersection with line $2t-1$ would occur]

$t = 1.571$

The two particles have same velocity when $t = 1.571$.

(3) (a) know that $g = f'$

$$\int_{-5}^{1} g(x)\,dx = f(1) - f(-5)$$

[use area to find value of this integral]

$$\frac{-3(3+4)}{2} + 0 + \frac{1(2)}{2} = 3 - f(-5)$$

$$-\left(\frac{-3(7)}{2} + \frac{2}{2} - 3\right) = f(-5)$$

$$\frac{21}{2} - \frac{2}{2} + \frac{6}{2} = f(-5)$$

$$\frac{25}{2} = f(-5)$$

[do not need to simplify - could leave here]

(b) $\int_{1}^{6} g(x)\,dx$

$$= \int_{1}^{3} g(x)\,dx + \int_{3}^{6} g(x)\,dx$$

[split because we use given eq for $g(x)$ for $3 < x < 6$ but area for $1 < x < 3$]

$$= 2(2) + 2\int_{3}^{6} (x-4)^2\,dx$$

$$= 4 + 2 \cdot \left.\frac{(x-4)^3}{3}\right]_3^6$$

$$= 4 + \frac{2}{3}\left((6-4)^3 - (3-4)^3\right) = 10$$

(c) $g = f'$

Since g is ⊕ and increasing on $0 < x < 1$ and $4 < x < 6$ then f is both increasing & concave up

(d) f would have a point of inflection at $x = 4$ because g changes from decreasing to increasing.

4) (a) $H'(6) \approx \dfrac{H(7) - H(5)}{7-5} = \dfrac{11-6}{2} = \dfrac{5}{2}$

$H'(6)$ is the rate that the height of the tree is changing in meters per year when $t = 6$ years.

(b) $H(t)$ is differentiable on $3 < t < 5$ so $H(t)$ is continuous on $3 < t < 5$ so MVT applies.

$$\frac{H(5) - H(3)}{5-3} = \frac{6-2}{2} = 2$$

MVT guarantees that $H'(?) = 2$ in the interval $3 < t < 5$.

(c) $$\underbrace{\frac{\int_2^{10} H(t)\,dt}{10-2}}$$ = avg height of tree in interval $2 \le t \le 10$.

$$\frac{1}{8}\int_2^{10} H(t)\,dt \approx \underbrace{\frac{1}{8}\left(\frac{(1.5+2) + 2(2+6) + 2(6+11) + 3(11+15)}{2}\right)}_{\frac{263}{32}}$$

$\boxed{\text{but does not need to be simplified to a single fraction}}$

(d) $G(x) = \frac{100x}{1+x}$ [know $\frac{dx}{dt} = 0.03, G(x) = 50$
need $G'(x)$]

$$G'(x)\frac{dx}{dt} = \frac{(1+x)100\frac{dx}{dt} - \frac{dx}{dt}(100x)}{(1+x)^2}$$

<u>to find x:</u>

$$50 = \frac{100x}{1+x}$$

$$50(1+x) = 100x$$

$$50 + 50x = 100x$$

$$50 = 50x$$

$$1 = x$$

$$G'(x) \cdot 0.03 = \frac{(1+1)100(0.03) - 0.03(100 \cdot 1)}{(1+1)^2}$$

$$G'(x) = \underbrace{\frac{200(0.03) - 0.03(100)}{0.03(4)}}$$

$$= \frac{3}{4}$$ [but do not need to simplify]

When the tree is 50 meters tall the height of the tree is changing at a rate of $\frac{3}{4}$ meters per year.

[make sure to write sentence & use units even if you are not using the simplified fraction]

(5) (a) $\dfrac{f(\pi)-f(0)}{\pi-0} = \dfrac{e^{\pi}\cos\pi - e^{0}\cos 0}{\pi}$

$= \dfrac{e^{\pi}(-1) - 1\cdot 1}{\pi} = \dfrac{-e^{\pi}-1}{\pi}$

(b) $f'(x) = e^{x}\cdot -\sin x + e^{x}\cos x$

$f'(\frac{3\pi}{2}) = e^{\frac{3\pi}{2}}\cdot -\sin\frac{3\pi}{2} + e^{3\pi/2}\cos\frac{3\pi}{2}$

$= e^{\frac{3\pi}{2}}\cdot -(-1) + e^{3\pi/2}(0)$

$= e^{\frac{3\pi}{2}}$

(c) need absolute min so use candidate test

x	$f(x)$
0	1
$\pi/4$	$\frac{\sqrt{2}}{2}e^{\pi/4}$
$5\pi/4$	$-\frac{\sqrt{2}}{2}e^{5\pi/4}$
2π	$e^{2\pi}$

$\longrightarrow$ abs min of $f(x)$ is $-\frac{\sqrt{2}}{2}e^{5\pi/4}$

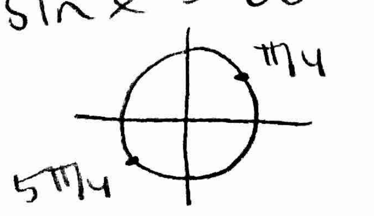
$f'(x) = 0$

$-e^{x}\sin x + e^{x}\cos x = 0$

$-e^{x}(\sin x - \cos x) = 0$

$-e^{x} \neq 0 \qquad \sin x - \cos x = 0$

$\sin x = \cos x$



$x = \pi/4 \text{ \& } 5\pi/4$

(d) $\lim_{x \to \pi/2} \frac{f(x)}{g(x)} \longrightarrow \frac{0}{0}$ ← $f(\frac{\pi}{2})$

↑ $g(\pi/2)$

get from plugging in $\pi/2$ so we can use L'Hopital's Rule

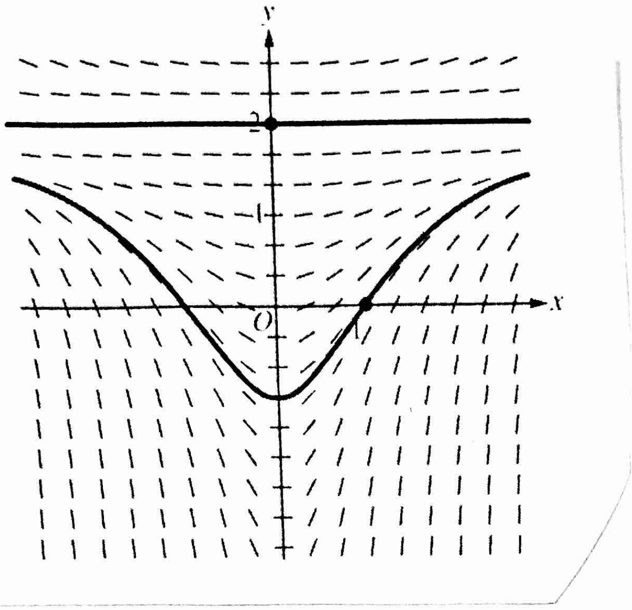
$\lim_{x \to \frac{\pi}{2}} \frac{f'(x)}{g'(x)} = \frac{-e^{\frac{\pi}{2}}}{2}$

← $f'(\frac{\pi}{2})$ from $f'(x)$ found in part (b)

↑ $g'(\frac{\pi}{2})$ from graph

(6)

(a)



(b) [(1, 0) → point
slope → use given $\frac{dy}{dx}$ to find $\left.\frac{dy}{dx}\right|_{(1,0)}$]

$$\left.\frac{dy}{dx}\right|_{(1,0)} = \frac{1}{3}(1)(0-2)^2 = \frac{4}{3}$$

$y = \frac{4}{3}(x-1)$ → eq of tan line at (1, 0)

$$y = \frac{4}{3}(0.7-1)$$

$$y = \frac{4}{3}(-0.3) = \frac{4}{3}\left(-\frac{3}{10}\right) = -\frac{4}{10} \approx f(0.7)$$

(c) $\frac{dy}{dx} = \frac{1}{3}x(y-2)^2$

$$\frac{dy}{(y-2)^2} = \frac{1}{3}x\,dx$$

$$\int \frac{1}{(y-2)^2}\,dy = \frac{1}{3}\int x\,dx$$

$$\int (y-2)^{-2}\,dy = \frac{1}{3}\int x\,dx$$

$$\frac{(y-2)^{-1}}{-1} = \frac{1}{3}\cdot\frac{x^2}{2} + C$$

$$\frac{-1}{y-2} = \frac{x^2}{6} + C$$

$$f(1) = 0$$

$$\frac{-1}{-2} = \frac{1^2}{6} + C$$

$$\frac{1}{2} = \frac{1}{6} + C$$

$$\frac{2}{6} = C$$

$$\frac{-1}{y-2} = \frac{x^2}{6} + \frac{2}{6}$$

$$\frac{-1}{y-2} = \frac{x^2+2}{6}$$

$$y-2 = -\frac{6}{x^2+2}$$

$$y = -\frac{6}{x^2+2} + 2$$