Name:_____ Date:_____

PC: Double Angle Formulas Ms. Loughran

Double Angles

Double-Angle Formulas

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = \cos^2 x - \sin^2 x$
or $2\cos^2 x - 1$
or $1 - 2\sin^2 x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

One application of these formulas is in evaluating trigonometric functions.

A second application of these formulas is in solving trigonometric equations.

Examples:

1. If $\angle x$ is in Quadrant II and $\cos x = -\frac{4}{5}$. Find the value of $\sin 2x$.

2. Prove the identity: $\sin 2x = \dfrac{2\tan x}{1 + \tan^2 x}$

Exercise Set A

1. If A is a positive acute angle and $\cos A = \frac{4}{5}$, what is the value of $\cos 2A$?

   (1) 1 (2) $\frac{7}{25}$ (3) $\frac{9}{25}$ (4) $\frac{24}{25}$

2. If $\sin x = \frac{3}{5}$ and angle x is obtuse, then the value of $\sin 2x$ is

   (1) $\frac{6}{5}$ (2) $-\frac{6}{5}$ (3) $\frac{24}{25}$ (4) $-\frac{24}{25}$

3. If $\sin x = \frac{3}{5}$ and x is an acute angle, what is the numerical value of $\sin 2x$?

4. If $\sin A = \frac{2}{5}$, find the value of $\cos 2A$.

5. If $\sin x = a$ and $\cos x = b$, express $\sin 2x$ in terms of a and b.

6. If x is a positive acute angle and $\cos x = \frac{5}{13}$, find $\sin 2x$.

7. If $\sin x = \frac{5}{6}$, what is the value of $\cos 2x$?

8. If $\tan A = \frac{1}{3}$, find the value of $\tan 2A$.

9. If $\tan \theta = 1$, then the value of $\tan 2\theta$ is

   (1) 1 (3) –2

   (2) 2 (4) undefined

10. $\angle A$ is in Quadrant I and $\sin A = \frac{\sqrt{5}}{3}$. Find, in simplest form, the value of:

    a. $\sin 2A$ b. $\cos 2A$

11. Which is a solution of the equation $\sin x = \cos 2x$?

    (1) 15° (2) 60° (3) 30° (4) 45°

12. Which is a solution of the equation $\sin^2 2x + \sin 2x - 2 = 0$?

    (1) 30° (2) 45° (3) 60° (4) 90°

In 13–18, find all values of θ in the interval $0 \le \theta < 2\pi$ that satisfy the given equation.

13. $\cos 2\theta = -\sin \theta$

14. $\cos 2\theta = \cos \theta$

15. $\cos 2\theta + 2 = \sin \theta$

16. $\sin 2\theta - \cos \theta = 0$

17. $\cos 2\theta + \cos \theta + 1 = 0$

18. $2\cos^3 \theta + \cos \theta \cos 2\theta = 0$

In 19–22, find all values of θ in the interval $0° \le \theta < 360°$ that satisfy the given equation. As necessary, express values to the nearest degree.

19. $3\cos 2\theta + \sin \theta - 2 = 0$

20. $3\cos 2\theta - \sin \theta - 2 = 0$

21. $3\cos 2\theta + 5\sin \theta - 2 = 0$

22. $5\sin \theta + 2\cos 2\theta - 3 = 0$

In 23–31, for all values of the angle for which the given expression is defined, choose an equivalent expression.

23. $\sin 2\theta \sec \theta$ is equivalent to

    (1) $\tan \theta$ (2) $\tan 2\theta$ (3) $2\cos \theta$ (4) $2\sin \theta$

24. $\frac{1}{2} \sec x \sin 2x$ is equivalent to

    (1) $\sin x$ (2) $-\sin x$ (3) $\cos x$ (4) $-\cos x$

25. $2\sin^2 A + \cos 2A$ is equivalent to

    (1) 1 (2) 2 (3) $\sin^2 A$ (4) $-\sin^2 A$

26. $\frac{\sin 2A}{\sin^2 A}$ is equivalent to

    (1) 1 (2) 2 (3) $2\tan A$ (4) $2\cot A$

27. $\frac{\sin 2A}{2\sin A}$ is equivalent to

    (1) 1 (3) $\frac{\sin A}{2}$

    (2) $\cos A$ (4) $\frac{1 - 2\cos^2 A}{2\sin A}$

28. $\frac{\sin 2A}{\cos A} - \sin A$ is equivalent to

    (1) 1 (2) $\cos A$ (3) $\sin A$ (4) $2\sin A$

29. $\frac{2\cos x}{\sin 2x}$ is equivalent to

    (1) $\sin x$ (2) $2\sin x$ (3) $2\csc x$ (4) $\csc x$

30. $\sin 2A + \cos A$ is equivalent to

    (1) $\cos A(2\sin A + 1)$ (3) $2(\sin A + \cos A)$

    (2) $\cos A(\cos A + 1)$ (4) $\cos A(\sin A + 1)$

31. $(\sin x - \cos x)^2$ is equivalent to

    (1) 1 (3) $1 - \sin 2x$

    (2) $-\cos 2x$ (4) $1 - \cos 2x$

32. Which statement is true for all real values of θ?

    (1) $\cos^2 \theta - \sin^2 \theta = 1$

    (2) $\cos \theta + \sin \theta = 1$

    (3) $\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

    (4) $\cos 2\theta = 2\cos \theta$

33. Which equation is an identity?

    (1) $\sin 4x = 4\sin x \cos x$

    (2) $\cos 4x = \cos^4 x - \sin^4 x$

    (3) $\sin^2 4x + \cos^2 4x = 1$

    (4) $\sin^4 x + \cos^4 x = 1$

In 34–69, prove that the statement is an identity for all values of the angle for which the expressions are defined.

34. $\sin 2x = \tan x(2 - 2\sin^2 x)$

35. $2 - \sec^2 x = (\cos 2x)(\sec^2 x)$

36. $\cot x \sin 2x - \cos 2x = \sec^2 x - \tan^2 x$

37. $\sin 2\theta \cot \theta - 2\sin^2 \theta = 2\cos 2\theta$

38. $\cot \theta = \frac{\sin 2\theta}{2\sin^2 \theta}$

39. $\cot\theta = \dfrac{\sin 2\theta}{1-\cos 2\theta}$

40. $\tan 2x \csc x = \dfrac{2\cos x}{\cos 2x}$

41. $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

42. $\tan\theta = \dfrac{1-\cos 2\theta}{\sin 2\theta}$

43. $\dfrac{\cos(90^\circ-\theta)}{\sin 2\theta} = \dfrac{\sec\theta}{2}$

44. $\dfrac{\sin 2\theta}{\tan\theta} = \dfrac{2}{1+\tan^2\theta}$

45. $\dfrac{2\tan x - \sin 2x}{2\sin^2 x} = \tan x$

46. $\dfrac{\cos 2x + 1}{\sec^2 x - \tan^2 x} = 2\cos^2 x$

47. $\dfrac{1}{2}\sin 2A = \dfrac{\tan A}{1+\tan^2 A}$

48. $\dfrac{\sin 2\theta}{1+\cos^2\theta-\sin^2\theta} = \tan\theta$

49. $\cos 2x = \dfrac{1-\tan^2 x}{\sec^2 x}$

50. $\dfrac{\sin 2\theta}{\tan\theta} = \dfrac{2}{1+\tan^2\theta}$

51. $\dfrac{\sin\theta+\sin 2\theta}{1+\cos\theta+\cos 2\theta} = \tan\theta$

52. $\sec x \sin 2x = \dfrac{2-2\cos^2 x}{\sin x}$

53. $\tan\theta + \cot\theta = \dfrac{2}{\sin 2\theta}$

54. $1+\tan^2\theta = \dfrac{2\sin\theta}{\cos\theta\sin 2\theta}$

55. $\dfrac{2\sin^2 A}{\sin 2A} + \cot A = \sec A \csc A$

56. $\dfrac{\cos 2\theta}{\sin\theta} + \sin\theta = \dfrac{\cot\theta}{\sec\theta}$

57. $\dfrac{1}{2}(\cot A - \tan A) = \dfrac{1}{\tan 2A}$

58. $\dfrac{\sin 2\theta}{\sin\theta} - \dfrac{\cos 2\theta}{\cos\theta} = \dfrac{1}{\cos\theta}$

59. $\dfrac{2\sin^2 x}{\sin 2x} + \dfrac{1}{\tan x} = \sec x \csc x$

60. $\dfrac{\cos 2x}{\sin x} + \dfrac{\sin 2x}{\cos x} = \csc x$

61. $2-\tan^2 A = 1 + \dfrac{\cos 2A}{\cos^2 A}$

62. $\dfrac{\cos x}{\sin x} + \dfrac{\sin x}{\cos x} = \dfrac{2}{\sin 2x}$

63. $\dfrac{2\cos x}{\sin 2x} = \dfrac{1}{\sin x\cos^2 x} - \dfrac{\tan^2 x}{\sin x}$

64. $(\cos 2\theta)(1+\tan^2\theta) = 2 - \dfrac{1}{\cos^2\theta}$

65. $\dfrac{\cos 2\theta}{\cos\theta(\cos\theta-\sin\theta)} = 1+\tan\theta$

66. $\tan\theta + \dfrac{2-4\sin^2\theta}{\sin 2\theta} = \cot\theta$

67. $\dfrac{1+\cos 2A}{\sin 2A} = \cot A$

68. $\csc 2x + \cot 2x = \cot x$

69. $\dfrac{1+\cos 2\theta}{1-\cos 2\theta} = \cot^2\theta$