

An ellipse is the locus of all points in a plane such that the sum of the distances from two given points in the plane, called foci, is constant.

The standard form of the equation of an ellipse with center at (h,k), major axis of length 2a units and minor axis of length 2b units, where $c^2 = a^2 - b^2$, is as follows:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$
, when the major axis is parallel to the x-axis,

or

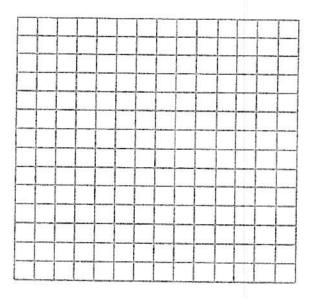
$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$
, when the major axis is parallel to the y-axis.

The foci are located on the major axis with formulas:

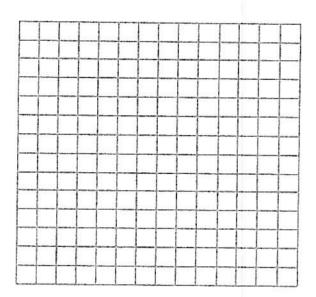
$$(h+c,k)$$
 and $(h-c,k)$ if the major axis is parallel to the x -axis $(h,k+c)$ and $(h,k-c)$ if the major axis is parallel to the y-axis

In all ellipses, $a^2 > b^2$. You can use this information to determine the orientation of the major axis from the values given in the equation. If a^2 is the denominator of the x term, the major axis is parallel to the x-axis. If a^2 is the denominator of the y term, the major axis is parallel to the y-axis. The **vertices** of the ellipse are the endpoints of the major axis. The **covertices** are the endpoints of the minor axis.

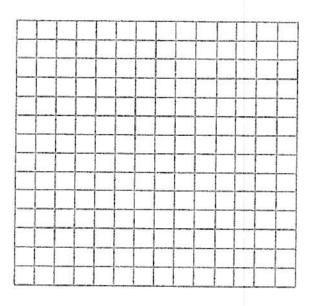
1. Graph
$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$



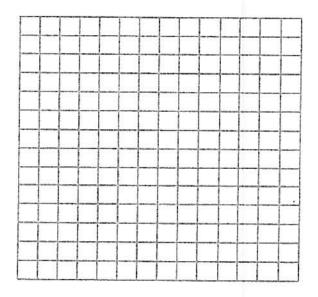
2. Graph
$$\frac{y^2}{9} + \frac{x^2}{4} = 1$$



3. Graph
$$\frac{(x-4)^2}{121} + \frac{(y+5)^2}{64} = 1$$



4. Graph
$$\frac{(y+2)^2}{25} + \frac{(x-3)^2}{16} = 1$$



Find the coordinates of the center, the foci, the vertices and covertices of the ellipse with the equation:

5.
$$4x^2 + y^2 - 8x + 6y + 9 = 0$$

6.
$$4x^2 + 9y^2 - 8x - 54y + 49 = 0$$

7.
$$9x^2 + 4y^2 - 18x + 16y = 11$$

Practice Exercises

For 1-8, find the coordinates of the center, the foci, the vertices and covertices of each ellipse.

1)
$$\frac{x^2}{49} + \frac{y^2}{169} = 1$$

2)
$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

$$3) \ \frac{x^2}{95} + \frac{y^2}{30} = 1$$

4)
$$\frac{x^2}{169} + \frac{y^2}{64} = 1$$

5)
$$\frac{x^2}{64} + \frac{(y-6)^2}{121} = 1$$

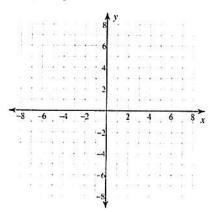
6)
$$\frac{(x+5)^2}{81} + \frac{(y-1)^2}{144} = 1$$

7)
$$\frac{(x-3)^2}{49} + \frac{(y-9)^2}{4} = 1$$

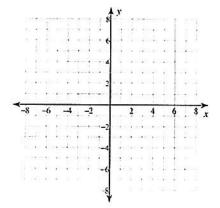
8)
$$\frac{x^2}{64} + \frac{(y-8)^2}{9} = 1$$

Graph each equation.

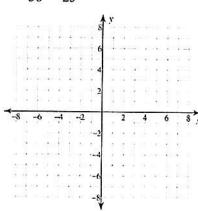
9)
$$\frac{x^2}{4} + \frac{y^2}{9} = 1$$



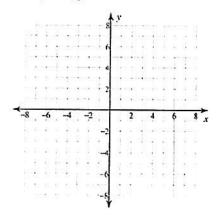
$$10) \ \frac{x^2}{49} + y^2 = 1$$



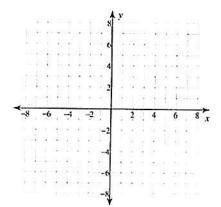
11)
$$\frac{x^2}{36} + \frac{y^2}{25} = 1$$



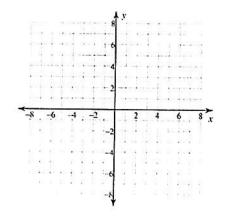
12)
$$\frac{x^2}{9} + \frac{y^2}{49} = 1$$



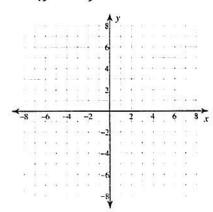
13)
$$\frac{x^2}{49} + \frac{(y-3)^2}{16} = 1$$



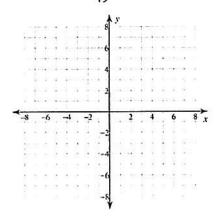
14)
$$\frac{(x-1)^2}{4} + \frac{y^2}{49} = 1$$



15)
$$\frac{x^2}{49} + \frac{(y-1)^2}{9} = 1$$



16)
$$(x+5)^2 + \frac{y^2}{49} = 1$$



For 17-20, find the coordinates of the center, the foci, and the vertices of each ellipse.

17)
$$-16y + 52 = -2x^2 - 8x - y^2$$

18)
$$4y^2 - 338x + 32y = -169x^2 + 443$$

19)
$$\frac{(x+4)^2}{4} + \frac{(y+9)^2}{64} = 1$$

20)
$$126y + 9y^2 - 8x - 131 = -4x^2$$