THE FACTOR THEOREM

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THEOREM: $f(x)$ is a polynomial function of degree ≥ 1 and $x - c$ is a factor of f if and only if $f(c) = 0$ (i.e. c is a zero of f).

EXAMPLE: If $f(x) = 2x - 6$ and 3 is a zero of $f(x)$, then _____ is a factor of $f(x)$.

EXAMPLE: If $x + 1$ is a factor of $f(x)$, then _____ is a zero of $f(x)$.

EXAMPLE: If $f(x) = (x - 3)(x + 4)(2x - 1)$, then the zeros of $f(x)$ are: _____.

EXAMPLE: If $f(4) = 0$, then a factor of f is: _____.

EXAMPLE: If $3x - 1$ is a factor of $f(x)$, then $f(\quad) = 0$.

Note: The complete factorization of a polynomial will include factors with ONLY integral coefficients.

PROBLEMS

1. Show 2 ways that $y - 1$ is a factor of $y^3 - 3y^2 + 3y - 1$.
2. Show 2 ways that $x - 2$ is a factor of $x^5 - 32$.
3. Factors of $x^3 + x^2 - 4x - 4$ are $(x - 2)$, $(x + 2)$ and $(x + 1)$. What are the zeros of the polynomial?
4. Given that the zeros of $x^3 - 6x^2 + 11x - 6$ are 1, 2, and 3. What are the factors of the polynomial? Check by multiplication.
5. If $f(x) = x^3 - x^2 + kx - 12$, find k so that $f(x)$ is exactly divisible by $x + 3$.
6. If $f(x) = 4x^3 - 2x^2 - 3x + k$, find k so that $f(x)$ is exactly divisible by $x - 2$.
7. If $f(x) = ax^5 + ax^4 + 13x^3 - 11x^2 - 10x - a$, find $f(1)$ if $f(-1) = 0$.
8. Show that -3 is a zero of $f(x) = x^3 + 7x^2 + 7x - 15$.
9. Find all zeros of $f(x) = 3x^3 - 4x^2 - 9x + 10$ given that one zero is 1.
10. One root of $x^3 + 8x^2 + 11x - 20 = 0$ is -5. Find the complete set of solutions of this equation.
11. Show that $(x + 1)$ is a factor of $x^3 - 2x^2 + 3 = 0$. Use this information to find the solution set of this equation.
12. One zero of $4x^3 - 11x^2 + 5x + 2$ is $-\frac{1}{4}$. Find the complete factorization of this polynomial and find the remaining two zeros.

MULTIPLE ZEROS/ROOTS

13. a) Show that $3x + 1$ is a multiple factor of $p(x) = 27x^5 - 18x^3 - 8x^2 - x$.
    b) What is the multiplicity of the zero $-\frac{1}{3}$?
    c) Find all solutions of $p(x) = 0$.

14. a) Prove that 1 is a multiple root of $x^5 - 3x^4 + 8x^2 - 9x + 3 = 0$ and find its multiplicity.
    b) Use this information to solve the equation in part a).

15. The zeros of a polynomial $g(x)$ are 3, -2, and 1, where 1 has a multiplicity of two.
    a) What is the degree of $g(x)$?
    b) What are the factors of $g(x)$?
    c) Find $g(x)$ in expanded form.

16. Given that i is a multiple root of $x^4 + 2x^2 + 1 = 0$. Find the complete solution set.

17. If a polynomial equation of degree 5 has a solution set of $\{1\}$, what is the multiplicity of this root?

18. It is given that 1 is a multiple root of $x^5 - 3x^4 - 6x^3 + 26x^2 - 27x + 9 = 0$. Find all roots.

19. Solve $8x^5 - 12x^4 + 38x^3 - 49x^2 + 24x - 4 = 0$ if $\frac{1}{2}$ is a multiple root.

NUMBER OF ROOTS - CONJUGATE PAIRS

20. One zero of a quadratic polynomial function with real coefficients is $3 + 4i$. What is the other zero?

21. If a cubic polynomial function with real coefficients has exactly one real zero, how many non-real zeros does it have?

22. Can a polynomial equation with real coefficients of odd degree have all imaginary roots? Explain.

23. Find the complete solution set of $x^4 - 2x^3 + 3x^2 - 8x - 4 = 0$ if one root is $2i$.

24. Find the complete solution set of $x^4 - 4x^3 + 6x^2 - 4x - 15 = 0$ if one root is $1 - 2i$.

25. Find the roots of $f(x) = x^4 - 3x^2 - 28$.

26. One zero of $f(x) = x^3 - x^2 - 5x + 21$ is $2 + i\sqrt{3}$. Find the remaining zeros.