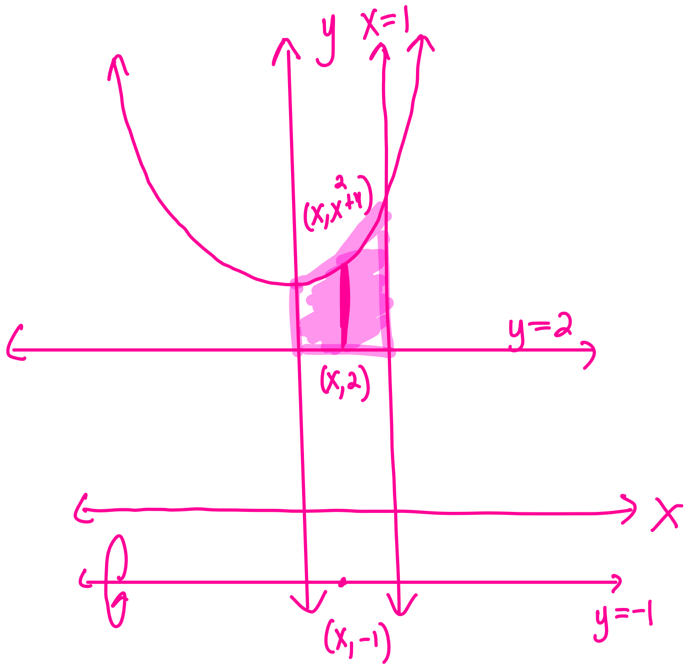
Name: ____ Date: __________

AP Calculus AB: Volume Ms. Loughran

Do Now:

1. Find the volume of the solid that results when the region bounded by $y = x^2 + 4,\ y = 2,\ x = 0,$ and $x = 1$ is revolved about the line $y = -1$.



$R = x^2+4-(-1) = x^2+5$

$r = 2-(-1) = 3$

$$V = \pi\int_0^1 \left((x^2+5)^2 - 3^2\right) dx$$

x^4+10x^2+25-9

$$V = \pi\int_0^1 \left(x^4+10x^2+16\right) dx = \pi\left[\frac{x^5}{5} + \frac{10x^3}{3} + 16x \Big|_0^1\right]$$

$$\pi\left[\frac{1}{5} + \frac{10}{3} + 16 - 0\right]$$

Homework 03/22, 03/25

2003

1. If $y = (x^3 + 1)^2$, then $\frac{dy}{dx} =$ $2(x^3+1) \cdot 3x^2$

(A) $(3x^2)^2$ (B) $2(x^3 + 1)$ (C) $2(3x^2 + 1)$ (D) $3x^2(x^3 + 1)$ (E) $6x^2(x^3 + 1)$

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2. $\int_0^1 e^{-4x}\,dx =$

(A) $\frac{-e^{-4}}{4}$ (B) $-4e^{-4}$ (C) $e^{-4} - 1$ (D) $\frac{1}{4} - \frac{e^{-4}}{4}$ (E) $4 - 4e^{-4}$

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3. For $x \geq 0$, the horizontal line $y = 2$ is an asymptote for the graph of the function f. Which of the following statements must be true?

(A) $f(0) = 2$

(B) $f(x) \neq 2$ for all $x \geq 0$

(C) $f(2)$ is undefined.

(D) $\lim_{x \to 2} f(x) = \infty$

(E) $\lim_{x \to \infty} f(x) = 2$

$6x+4-(6x+9)$

4. If $y = \dfrac{2x+3}{3x+2}$, then $\dfrac{dy}{dx} = \dfrac{(3x+2)(2)-(2x+3)(3)}{(3x+2)^2}$

(A) $\dfrac{12x+13}{(3x+2)^2}$ (B) $\dfrac{12x-13}{(3x+2)^2}$ (C) $\dfrac{5}{(3x+2)^2}$ (D) $\dfrac{-5}{(3x+2)^2}$ (E) $\dfrac{2}{3}$

5. $\int_0^{\frac{\pi}{4}} \sin x\, dx =$

(A) $-\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{2}}{2}$ (C) $-\frac{\sqrt{2}}{2} - 1$ (D) $-\frac{\sqrt{2}}{2} + 1$ (E) $\frac{\sqrt{2}}{2} - 1$

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6. $\lim_{x\to\infty} \frac{x^3 - 2x^2 + 3x - 4}{4x^3 - 3x^2 + 2x - 1} =$

(A) 4 (B) 1 (C) $\frac{1}{4}$ (D) 0 (E) -1

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8. $\int x^2 \cos(x^3)\, dx =$

(A) $-\frac{1}{3}\sin(x^3) + C$

(B) $\frac{1}{3}\sin(x^3) + C$

(C) $-\frac{x^3}{3}\sin(x^3) + C$

(D) $\frac{x^3}{3}\sin(x^3) + C$

(E) $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right) + C$

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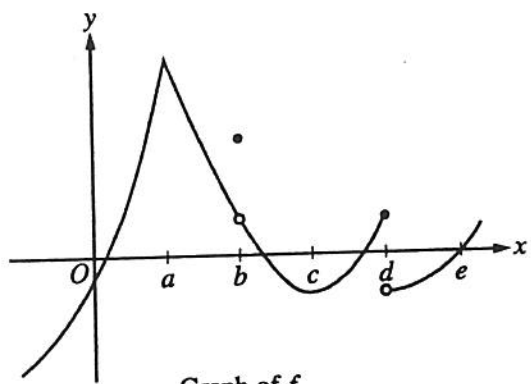
9. If $f(x) = \ln\left(x + 4 + e^{-3x}\right)$, then $f'(0)$ is

(A) $-\frac{2}{5}$ (B) $\frac{1}{5}$ (C) $\frac{1}{4}$ (D) $\frac{2}{5}$ (E) nonexistent

$f'(x) = \frac{1}{x+4+e^{-3x}} \cdot 1-3e^{-3x}$

$f'(0) = \frac{1}{0+4+e^0}(1-3e^0) = \frac{-2}{5}$



Graph of f

13. The graph of a function f is shown above. At which value of x is f continuous, but not differentiable?

(A) a (B) b (C) c (D) d (E) e

14. If $y = x^2 \sin 2x$, then $\frac{dy}{dx} =$

$x^2 2\cos(2x) + 2x\sin 2x$

$2x(x\cos(2x) + \sin 2x)$

(A) $2x \cos 2x$

(B) $4x \cos 2x$

(C) $2x(\sin 2x + \cos 2x)$

(D) $2x(\sin 2x - x\cos 2x)$

(E) $2x(\sin 2x + x\cos 2x)$

15. Let f be the function with derivative given by $f'(x) = x^2 - \frac{2}{x}$. On which of the following intervals is f decreasing?

(A) $(-\infty, -1]$ only

(B) $(-\infty, 0)$

(C) $[-1, 0)$ only

(D) $(0, \sqrt[3]{2}]$

(E) $[\sqrt[3]{2}, \infty)$

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16. If the line tangent to the graph of the function f at the point $(1, 7)$ passes through the point $(-2, -2)$, then $f'(1)$ is

(A) -5 (B) 1 (C) 3 (D) 7 (E) undefined

$m = \frac{7-(-2)}{1-(-2)} = \frac{9}{3} = 3$

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19. A curve has slope $2x + 3$ at each point (x, y) on the curve. Which of the following is an equation for this curve if it passes through the point $(1, 2)$?

(A) $y = 5x - 3$

(B) $y = x^2 + 1$

(C) $y = x^2 + 3x$

(D) $y = x^2 + 3x - 2$

(E) $y = 2x^2 + 3x - 3$

$x+2 = 4x-7$ @ $x=3$

$5=5$

$$f(x) = \begin{cases} x + 2 & \text{if } x \le 3 \\ 4x - 7 & \text{if } x > 3 \end{cases}$$

20. Let f be the function given above. Which of the following statements are true about f ?

I. $\lim_{x \to 3} f(x)$ exists. ✓

II. f is continuous at $x = 3$. ✓

III. f is differentiable at $x = 3$.

$$f'(x) = \begin{cases} 1 & x \le 3 \\ 4 & x > 3 \end{cases}$$

$1 \neq 4$

(A) None

(B) I only

(C) II only

(D) I and II only

(E) I, II, and III

Section I
Part A

24. Let f be the function defined by $f(x) = 4x^3 - 5x + 3$. Which of the following is an equation of the line tangent to the graph of f at the point where $x = -1$?

(A) $y = 7x - 3$
(B) $y = 7x + 7$
(C) $y = 7x + 11$
(D) $y = -5x - 1$
(E) $y = -5x - 5$

$f(-1) = 4(-1)^3 - 5(-1) + 3 = -4 + 5 + 3 = 4$
$f'(x) = 12x^2 - 5$
$f'(-1) = 12(-1)^2 - 5 = 7$

$y - 4 = 7(x+1)$
$y = 7x + 11$

25. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = 2t^3 - 21t^2 + 72t - 53$. At what time t is the particle at rest?

(A) $t = 1$ only
(B) $t = 3$ only
(C) $t = \frac{7}{2}$ only
(D) $t = 3$ and $t = \frac{7}{2}$
(E) $t = 3$ and $t = 4$

$v(t) = 6t^2 - 42t + 72$
$v(t) = 6(t^2 - 7t + 12)$
$v(t) = 6(t-3)(t-4)$
$0 = 6(t-3)(t-4)$
$t = 3, 4$

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26. What is the slope of the line tangent to the curve $3y^2 - 2x^2 = 6 - 2xy$ at the point $(3, 2)$?

(A) 0 (B) $\frac{4}{9}$ (C) $\frac{7}{9}$ (D) $\frac{6}{7}$ (E) $\frac{5}{3}$

$(3,2)$

$6y\frac{dy}{dx} - 4x = -2x\frac{dy}{dx} - 2y$

$6(2)\frac{dy}{dx} - 4(3) = -2(3)\frac{dy}{dx} - 2(2)$

$12\frac{dy}{dx} - 12 = -6\,dy/dx - 4$

$18\,dy/dx = 8$

$dy/dx = \frac{8}{18} = \frac{4}{9}$

27. Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?

(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4 (E) 13

$x^3 + x = 2$

$x^3 + x - 2 = 0$

$$\begin{array}{r|rrrr} 1 & 1 & 0 & 1 & -2 \\ & & 1 & 1 & 2 \\ \hline & 1 & 1 & 2 & 0 \end{array}$$

$x^2 + x + 2 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4(1)(2)}}{2(1)}$ ← imaginary

$g'(2) = \frac{1}{f'(1)} = \frac{1}{4}$

$f'(x) = 3x^2 + 1$

$f'(1) = 3(1)^2 + 1 = 4$

$g: (2,1)$

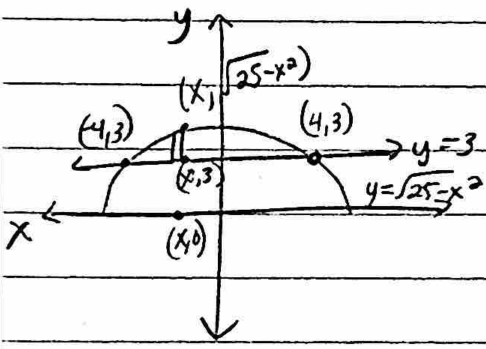
$f(1, 2)$

Homework 03-26

(9) $y=\sqrt{25-x^2}$, $y=3$

$\downarrow$

$x^2+y^2=25$



$R=\sqrt{25-x^2}-0=\sqrt{25-x^2}$

$r=3-0=3$

$$V=\pi\int_{-4}^{4}\left(\left(\sqrt{25-x^2}\right)^2-(3)^2\right)dx$$

$$V=\pi\int_{-4}^{4}(25-x^2-9)\,dx=\pi\int_{-4}^{4}16-x^2=\pi\cdot\left[16x-x^3/3\right]_{-4}^{4}$$

$-64/3$

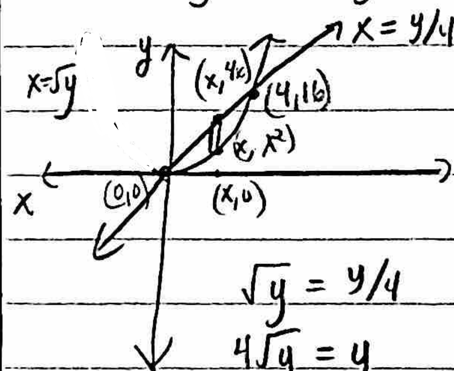
$$\pi\left[16(4)-4^3/3-\left(16(-4)-(-4)^3/3\right)\right]$$

$$\pi\left(64-64/3+64-64/3\right)$$

$$\pi\left(128-128/3\right)=\left(256\pi/3\right)$$

$\to x^2=y$ $\qquad 4x=y$

(13) $x=\sqrt{y}$ $\qquad x=y/4$



$R=4x-0=4x$

$r=x^2-0=x^2$

$\sqrt{y}=y/4$

$4\sqrt{y}=y$

$16y=y^2$

$y^2-16y=0$

$y(y-16)=0$

$y=0$ | $y=16$

$x=0$ $\quad$ $x=4$

$$V=\pi\int_{0}^{4}\left((4x)^2-(x^2)^2\right)dx$$

$$V=\pi\int_{0}^{4}(16x^2-x^4)\,dx=\pi\cdot\left[\frac{16x^3}{3}-\frac{x^5}{5}\right]_{0}^{4}$$

$$=\pi\left[\frac{16(4)^3}{3}-\frac{(4)^5}{5}-0\right]=\pi\left[\frac{1024}{3}-\frac{1024}{5}\right]$$

$$\pi\left[\frac{5120}{15}-\frac{3072}{15}\right]=\frac{2048\pi}{15}$$

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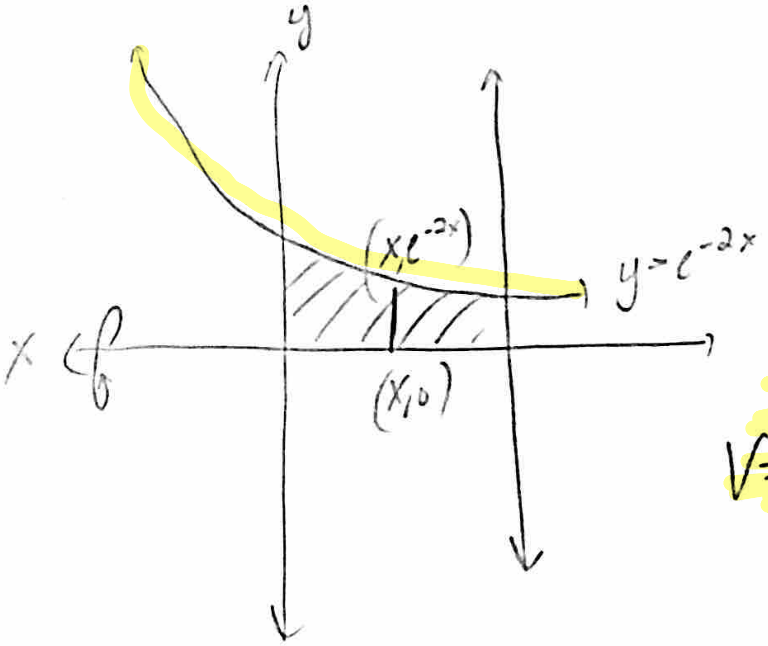
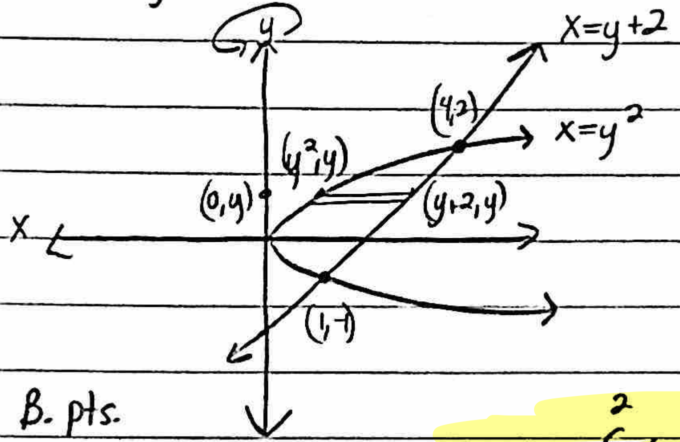


$r = e^{-2x}$

$$V = \pi \int_0^1 \left(e^{-2x}\right)^2 \, dx$$

$$\pi \int_0^1 e^{-4x} \, dx = \pi \left[-\frac{1}{4} e^{-4x} \Big|_0^1 \right] = \pi \left[-\frac{1}{4} e^{-4} - \left(-\frac{1}{4} e^{0} \right) \right] = \left(-\frac{1}{4e^4} + \frac{1}{4} \right) \pi$$

(21) $x = y^2 , \ x = y+2$



$R = y+2-0 = y+2$

$r = y^2 - 0 = y^2$

B. pts.

$y^2 = y+2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$

$x = 4 \quad x = 1$

$$V = \pi \int_{-1}^{2} \left[(y+2)^2 - (y^2)^2 \right] dy$$

$$= \pi \int_{-1}^{2} \left[y^2 + 4y + 4 - y^4 \right] dy$$

$$V = \pi \left[\frac{y^3}{3} + 2y^2 + 4y - \frac{y^5}{5} \Big|_{-1}^{2} \right]$$

$$V = \pi \left(\frac{(2)^3}{3} + 2(2)^2 + 4(2) - \frac{(2)^5}{5} - \left(\frac{(-1)^3}{3} + 2(-1)^2 + 4(-1) - \frac{(-1)^5}{5} \right) \right)$$

$$V = \pi \left(8/3 + 8 + 8 - 32/5 - \left(-1/3 + 2 - 4 + 1/5 \right) \right)$$

$9/3$

$$V = \pi \left(8/3 + 16 - 32/5 + 1/3 - 2 + 4 - 1/5 \right)$$

$$V = \pi \left(18 + 3 - 33/5 \right)$$

$$V = \pi \left(21 - 33/5 \right) = \pi \left(\frac{105}{5} - \frac{33}{5} \right)$$

$$= \frac{72\pi}{5}$$