AP Calculus AB

THEOREM (*The Fundamental Theorem of Calculus, Part 2*). *If f is continuous on an interval I, then f has an antiderivative on I. In particular, if a is any point in I, then the function F defined by*

$$F(x) = \int_a^x f(t)\,dt$$

is an antiderivative of f on I; that is, $F'(x) = f(x)$ for each x in I, or in an alternative notation

$$\frac{d}{dx}\left[\int_a^x f(t)\,dt\right] = f(x)$$

If a definite integral has a variable upper limit of integration and a continuous integrand, then the derivative of the integral with respect to its upper limit is equal to the integrand evaluated at the upper limit.

$$\frac{d}{dx}\left[\int_a^{g(x)} f(t)\,dt\right] = f(g(x))g'(x)$$

In words, *to differentiate an integral with a constant lower limit and a function as the upper limit, substitute the upper limit into the integrand, and multiply by the derivative of the upper limit.*

Name: ___________________________________

AP Calculus - Using the Fundamental Theorem of Calculus

1. If $F(x)=\int_1^{x^4}\sqrt{1+u^3}\,du$, find $F'\left(\sqrt[3]{2}\right)$.

2. Find $\int_0^1 g''(t)dt$ where $g(t)=\int_1^t\sqrt{x^2+1}\,dx$.

3. If $f(x)=e^{\int_2^x \frac{tdt}{1+t^4}}$ find $f'(2)$.

4. If $f(t)=\int_{t^2}^4\frac{dx}{x^8+1}$ find $\int_{-1}^0 f''(t)dt$.

5. If f' is continuous and f passes through (1, 3) and (3, 1), find $\int_1^3 f'(x)dx$.

6. Evaluate $\int_0^1 \frac{d}{dx}\left(\sqrt{1+x^3}\right)dx$

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(a) Given $5x^3 + 40 = \int_c^x f(t)\,dt$.

(i) Find $f(x)$.

(ii) Find the value of c.

(b) If $F(x) = \int_x^3 \sqrt{1 + t^{16}}\,dt$, find $F'(x)$.