

Name: \_\_\_\_\_

Date: \_\_\_\_\_

AP Calculus AB: Additional Algebraic Techniques for Anti-differentiation

Do Now:

Evaluate each of the following.

$$1. \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2}$$
$$\lim_{x \rightarrow 0} \frac{e^x - 1}{2x}$$
$$\lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$$

$$2. \lim_{x \rightarrow \frac{\pi}{3}} \frac{\cos x - \frac{1}{2}}{x - \frac{\pi}{3}}$$
$$\lim_{x \rightarrow \frac{\pi}{3}} \frac{-\sin x}{1} = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$3. \lim_{x \rightarrow 0} \frac{x}{e^x} = \frac{0}{1} = 0$$

$$4. \lim_{x \rightarrow \infty} \frac{e^x}{x^2}$$
$$\lim_{x \rightarrow \infty} \frac{e^x}{2x}$$
$$\lim_{x \rightarrow \infty} \frac{e^x}{2} = \infty$$

$$5. \lim_{x \rightarrow \pi} \frac{1 + \overset{0}{\cos x}}{\overset{0}{\sin 2x}}$$

$$\lim_{x \rightarrow \pi} \frac{-\sin x}{2 \cos 2x} = \frac{0}{2} = 0$$

### Additional Algebraic Techniques for Anti-differentiation

$$1. \int \frac{x^2}{x+3} dx$$

$$x+3 \overline{\begin{array}{r} x-3 \\ x^2 \\ -x^2+3x \\ \hline -3x \\ +3x+9 \\ \hline 9 \end{array}} \quad x-3 + \frac{9}{x+3}$$

$$\int \left( x-3 + \frac{9}{x+3} \right) dx \quad \begin{array}{l} u = x+3 \\ du = dx \end{array}$$

$$\int (x-3) dx + 9 \int \frac{1}{x+3} dx$$

$$\frac{x^2}{2} - 3x + 9 \ln|x+3| + C$$

$$2. \int_{-2}^1 \frac{x^2}{x+3} dx$$

definite integral

$$\left. \left( \frac{x^2}{2} - 3x + 9 \ln|x+3| \right) \right|_{-2}^1$$

$$\frac{1}{2} - 3 + 9 \ln 4 - \left( 2 - 6 + 9 \ln 1 \right)$$

$$\frac{1}{2} - 3 + 9 \ln 4 - 8$$

$$\frac{1}{2} - 11 + 9 \ln 4$$

$$\frac{-21}{2} + 9 \ln 4$$

## Completing the square

$$\begin{aligned}
 &2x^2 - 12x + 26 \\
 &2(x^2 - 6x + 9 - 9 + 13) \\
 &2(x-3)^2 + 2(4) \\
 &2(x-3)^2 + 8 \\
 &2[(x-3)^2 + 4]
 \end{aligned}$$

completing the square

3.  $\int \frac{1}{2x^2 - 12x + 26} dx$

$$\int \frac{1}{2[(x-3)^2 + 4]} dx$$

$$\frac{1}{2} \int \frac{1}{(x-3)^2 + 4} dx$$

$$\frac{1}{2} \int \frac{1}{4\left(\frac{(x-3)^2}{4} + 1\right)} dx$$

$$\frac{1}{4} \cdot \frac{1}{2} \int \frac{1}{\left(\frac{x-3}{2}\right)^2 + 1} dx$$

$$\frac{1}{8} \int \frac{1}{\left(\frac{x-3}{2}\right)^2 + 1} dx$$

$$\begin{aligned}
 u &= \frac{x-3}{2} = \frac{1}{2}(x-3) \\
 du &= \frac{1}{2} dx \\
 2du &= dx
 \end{aligned}$$

$$\frac{1}{8} \cdot 2 \int \frac{1}{u^2 + 1} du = \frac{1}{4} \tan^{-1} u + C = \frac{1}{4} \tan^{-1} \left( \frac{x-3}{2} \right) + C$$

4.  $\int \frac{dx}{\sqrt{21 - 4x - x^2}}$

$$\begin{aligned}
 &21 - 4x - x^2 \\
 &-x^2 - 4x + 21 \\
 &-(x^2 + 4x + 4 - 4 - 21) \\
 &-(x+2)^2 - (-25) \\
 &-(x+2)^2 + 25 \\
 &25 - (x+2)^2
 \end{aligned}$$

$[\arcsin(u)]' = \frac{1}{\sqrt{1-u^2}}$

$$\int \frac{dx}{\sqrt{25 - (x+2)^2}}$$

$$\int \frac{dx}{\sqrt{25\left(1 - \frac{(x+2)^2}{25}\right)}}$$

$$\int \frac{dx}{\sqrt{25} \sqrt{1 - \frac{(x+2)^2}{25}}}$$

$$\frac{1}{5} \int \frac{dx}{\sqrt{1 - \left(\frac{x+2}{5}\right)^2}}$$

$$\begin{aligned}
 u &= \frac{1}{5}(x+2) \\
 du &= \frac{1}{5} dx \\
 5du &= dx
 \end{aligned}$$

$$\frac{1}{5} \cdot 5 \int \frac{du}{\sqrt{1-u^2}}$$

$$\begin{aligned}
 &1 \cdot \arcsin(u) + C \\
 &\arcsin\left(\frac{x+2}{5}\right) + C
 \end{aligned}$$

## Practice

1.  $\int \frac{x^2}{x^2+4} dx$

5.  $\int \frac{dx}{x^2+2x+2}$

2.  $\int \frac{1}{x^2+6x+14} dx$

6.  $\int \frac{1}{\sqrt{-2x^2-8x-6}} dx$

3.  $\int_0^1 \frac{x^3}{2x^2+1} dx$

7.  $\int_3^4 \frac{dx}{x^2-6x+10}$

4.  $\int \frac{x+6}{x-3} dx$

# Homework 02-16

Name: Key  
 AP Calculus AB: Evaluating Definite Integrals of Piecewise Functions

Date: \_\_\_\_\_  
 Ms. Loughran

1. Evaluate  $\int_0^8 h(x) dx$  if  $h(x) = \begin{cases} 4 & , x < 4 \\ x & , x \geq 4 \end{cases}$

$$\int_0^4 4 dx + \int_4^8 x dx$$

$$4x \Big|_0^4 + \frac{x^2}{2} \Big|_4^8 = 16 - 0 + (32 - 8) = 40$$

2. Evaluate  $\int_0^4 g(x) dx$  if  $g(x) = \begin{cases} 2x-5 & , x < 2 \\ x^2 & , x \geq 2 \end{cases}$

$$\int_0^2 (2x-5) dx + \int_2^4 x^2 dx$$

$$x^2 - 5x \Big|_0^2 + \frac{x^3}{3} \Big|_2^4 = -6 + \left( \frac{64}{3} - \frac{8}{3} \right) = \frac{38}{3}$$

3. Evaluate  $\int_{-3}^0 f(x) dx$  if  $f(x) = \begin{cases} -x^3 & , x < -2 \\ -3x+2 & , -2 \leq x < \pi \\ \sin x & , x \geq \pi \end{cases}$

$$\int_{-3}^{-2} -x^3 dx + \int_{-2}^{\pi} (-3x+2) dx = -\frac{x^4}{4} \Big|_{-3}^{-2} + \left( -\frac{3x^2}{2} + 2x \right) \Big|_{-2}^{\pi}$$

3b)  $\int_{-3}^{-2} -x^3 dx + \int_{-2}^{\pi} (-3x+2) dx + \int_{\pi}^5 \sin x dx = \frac{65}{4} + \frac{-3\pi^2}{2} + 2\pi + 10 - \cos 5 - 1$   
 $= \frac{101}{4} - \frac{3\pi^2}{2} + 2\pi - \cos 5$

$$\frac{65}{4} + 10 - (-6 - 4) = \frac{105}{4}$$

4. Evaluate  $\int_{-2}^5 f(x) dx$  if  $f(x) = \begin{cases} -2x^2 & , x < 0 \\ 5x & , 0 \leq x < 2 \\ \frac{1}{x} & , x \geq 2 \end{cases}$

$$\int_{-2}^0 -2x^2 dx + \int_0^2 5x dx + \int_2^5 \frac{1}{x} dx = -\frac{2x^3}{3} \Big|_{-2}^0 + \frac{5x^2}{2} \Big|_0^2 + \ln|x| \Big|_2^5 = \frac{-16}{3} + 10 + \ln 5 - \ln 2$$

$$\frac{14}{3} + \ln \frac{5}{2}$$

5. Evaluate  $\int_{-3}^1 h(x) dx$  if  $h(x) = \begin{cases} 0 & , x \leq 0 \\ 5e^{-5x} & , x > 0 \end{cases}$

$$\int_{-3}^0 0 dx + \int_0^1 5e^{-5x} dx = 5 \cdot \left( -\frac{1}{5} e^{-5x} \right) \Big|_0^1 = -e^{-5} - (-1) = 1 - e^{-5}$$

6. Evaluate  $\int_{-2}^4 f(x) dx$  if  $f(x) = \begin{cases} 0 & , x < -1 \\ 9x & , -1 \leq x < 3 \\ 3e^x & , 3 \leq x \leq 5 \\ 0 & , x > 5 \end{cases}$

$$\int_{-2}^{-1} 0 dx + \int_{-1}^3 9x dx + \int_3^4 3e^x dx$$

$$0 + \frac{9x^2}{2} \Big|_{-1}^3 + 3e^x \Big|_3^4 = 0 + \frac{9(9-1)}{2} + 3e^4 - 3e^3$$

$$1 - e^{-5} - (-1) = 1 - e^{-5} + 1 = 2 - e^{-5}$$

$$0 + 36 + 3e^4 - 3e^3$$

