For 3 - 8, find the complete factorization and all zeros of the following polynomials using the information given.

3. $P(x)=2x^5-5x^4+x^3+4x^2-4x$

PRZ: $\pm1, \pm2, \pm4, \pm\frac{1}{2}$

$\times(2x^4-5x^3+x^2+4x-4)$

2	2	-5	1	4	-4
		4	-2	-2	4
	2	-1	-1	2	0
-1		-2	3	-2	
	2	-3	2	0	← remainder

not factorable need to use Quadratic Formula or completing the square

$2x^2-3x-2$ we want $2x^2-3x+2$

CF: $x(2x^2-3x+2)(x-2)(x+1)$

Z: $\{0, -1, 2, \frac{3\pm i\sqrt{7}}{2}\}$

$x(2x+1)(x-2)(x-2)(x+1)$ $x=0$ $x=2$ $x=-1$ $x=-\frac{1}{2}$

$x(2x+1)(x-2)^2(x+1)$ $\{-1, -\frac{1}{2}, 0, 2\}$

4. $P(x)=x^4+6x^3+2x^2-18x-15$

PRZ: $\pm1, \pm3, \pm5, \pm15$ $\pm1, \pm3, \pm5, \pm15$

±1

-5 | 1 | 6 | 2 | -18 | -15

	-5	-5	15	15
1	1	-3	-3	0

x^3+x^2-3x-3 Z: $\{-1, -5, \pm\sqrt{3}\}$

$x^2(x+1)-3(x+1)$

CF: $(x^2-3)(x+1)(x+5)$

$x^2-3=0$

$\sqrt{x^2}=\sqrt{3}$

$\pm\sqrt{3}$

5. $P(x)=x^4-5x^3+3x^2+15x-18$

PRZ: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

±1

3 | 1 | -5 | 3 | 15 | -18

	3	-6	-9	18
1	-2	-3	6	0

CF: $(x-3)(x-2)(x^2-3)$

Z: $\{2, 3, \pm\sqrt{3}\}$

6. $P(x) = x^4 + 6x^3 + 7x^2 - 12x - 18$

PZR: $\frac{\pm1, \pm2, \pm3, \pm6, \pm9, \pm18}{\pm1} = \pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

2	1	6	7	-12	-18
		2	16	46	68
	1	8	23	34	50

-3	1	6	7	-12	-18
		-3	-9	6	18
	1	3	-2	-6	

CF: $(x+3)^2(x^2-2)$

$x^3 + 3x^2 - 2x - 6$

$x^2(x+3) - 2(x+3) = 0$

$(x^2-2)(x+3) = 0$

Z: $\{-3 \text{ (mult of 2)}, \pm\sqrt{2}\}$

7. $P(x) = x^4 + 3x^3 + 3x^2 + x$

$x(x^3 + 3x^2 + 3x + 1)$

PRZ: $\frac{\pm 1}{\pm 1} = \pm 1$

-1	1	3	3	1
		-1	-2	-1
	1	2	1	0
-1		-1	-1	
	1	1	0	
-1		-1		
	1	0		

CF: $x(x+1)^3$

Z: {0, -1 (multiplicity of 3)}

triple

8. $P(x) = 3x^4 - 11x^3 - 3x^2 - 6x + 8$

PRZ: $\frac{\pm 1, \pm 2, \pm 4, \pm 8}{\pm 1, \pm 3} = \pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{3}, \pm \frac{2}{3}, \pm \frac{4}{3}, \pm \frac{8}{3}$

$P(1) = 3 - 11 - 3 - 6 + 8 \neq 0$

$P(-1) = 3 + 11 - 3 + 6 + 8 \neq 0$

4	3	-11	-3	-6	8
		12	4	4	-8
$\frac{2}{3}$	3	1	1	-2	0
		2	2	2	
	3	3	3	0	

$(3x^2 + 3x + 3)(x - 4)(x - \frac{2}{3})$

$3(x^2 + x + 1)(x - 4)(x - \frac{2}{3})$

CF: $(x^2 + x + 1)(x - 4)(3x - 2)$

$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2(1)}$

$= \frac{-1 \pm \sqrt{1 - 4}}{2}$

$= \frac{-1 \pm \sqrt{-3}}{2}$

Z: $\{\frac{2}{3}, 4, \frac{-1 \pm i\sqrt{3}}{2}\}$