Homework 11-27

Evaluate each limit. Use L'Hôpital's Rule where appropriate.

1. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4} = \lim\limits_{x\to 2}\dfrac{3x^2}{2x} = \dfrac{12}{4} = 3$

2. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x} = \lim\limits_{x\to 0}\dfrac{5\cos 5x}{1} = 5$

3. $\lim\limits_{x\to 2}\dfrac{\sqrt{2+x}-2}{x-2} = \lim\limits_{x\to 2}\dfrac{\frac{1}{2}(2+x)^{-\frac{1}{2}}}{1} = \dfrac{1}{4}$

4. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{x-1} = \lim\limits_{x\to 1}\dfrac{\frac{1}{3}x^{-2/3}}{1}$ $\quad \dfrac{1}{3}$

5. $\frac{0}{0}$ $\lim\limits_{x\to 2}\dfrac{x^2-4x+4}{x^3-12x+16}$ $\quad \lim\limits_{x\to 2}\dfrac{2x-4}{3x^2-12} = \lim\limits_{x\to 2}\dfrac{2}{6x} = \dfrac{2}{12}$ $\quad \dfrac{1}{6}$

$4-8+4$ $\quad 8-24+16$

6. $\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1-\sin x}{1+\cos 2x}$ $\quad \lim\limits_{x\to \frac{\pi}{2}}\dfrac{+\cos x}{+2\sin 2x} = \lim\limits_{x\to \pi/2}\dfrac{\sin x}{-4\cos 2x}$ $\quad \dfrac{1}{4}$

7. $\lim\limits_{x\to 1}\dfrac{x^3-1}{4x^3-x-3}$ $\quad \lim\limits_{x\to 1}\dfrac{3x^2}{12x^2-1} = \dfrac{3}{11}$

$4-1-3$

Can't use Hopital's*

8. $\lim\limits_{x\to 3}\dfrac{x-4}{x-2} = \dfrac{-1}{1} = -1$

9. $\lim\limits_{x\to 0}\dfrac{x}{\tan x}$ $\quad \lim\limits_{x\to 0}\dfrac{1}{\sec^2 x} = \dfrac{1}{1} = 1$

10. $\frac{0}{0}$ $\lim\limits_{x\to 1}\dfrac{1-\frac{1}{x}}{1-\frac{1}{x^2}}$ $\quad \lim\limits_{x\to 1}\dfrac{+\frac{1}{x^2}}{\frac{2}{x^3}}$

x^{-1} $\quad -x^{-2}$ $\quad x^3$

$\lim\limits_{x\to 1}\dfrac{+x}{2} = \dfrac{1}{2}$

11. $\lim\limits_{x\to\infty}\dfrac{\log_2 x}{\log_3 x}$ $\quad \lim\limits_{x\to\infty}\dfrac{\frac{1}{x\ln 2}}{\frac{1}{x\ln 3}} = \lim\limits_{x\to\infty}\dfrac{\cancel{x}\ln 3}{\cancel{x}\ln 2}$

$\dfrac{\ln 3}{\ln 2}$

12. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1}{\tan x}$ $\quad \lim\limits_{x\to 0}\dfrac{2e^{2x}}{\sec^2 x} = \dfrac{2}{1} = 2$

13. $\lim\limits_{x\to 0}\dfrac{\arctan x}{2x}$ $\quad \lim\limits_{x\to 0}\dfrac{\frac{1}{1+x^2}}{2} = \dfrac{1}{2}$

14. $\lim\limits_{x\to \pi^+}\dfrac{2x-2\pi}{\sin(x-\pi)}$ $\quad \lim\limits_{x\to\pi^+}\dfrac{2}{\cos(x-\pi)\cdot 1} = \dfrac{2}{1} = 2$

15. $\lim\limits_{x\to 0}\pi^2\dfrac{\tan 2x}{x\cos 2x}$ $\quad \pi^2\lim\limits_{x\to 0}\dfrac{2\sec^2 2x}{2x\sin 2x+\cos 2x} = \dfrac{2}{1}\pi^2$

$= 2\pi^2$

16. $\lim\limits_{x\to 0}\dfrac{e^x-e^{-x}}{x}$ $\quad \lim\limits_{x\to 0}\dfrac{e^x+e^{-x}}{1} = 2$

17. $\lim\limits_{x\to 1}\dfrac{2\ln x}{x-1}$ $\quad \lim\limits_{x\to 1}\dfrac{\frac{2}{x}}{1} = \dfrac{2}{1} = 2$

18. $\lim\limits_{x\to 0}\dfrac{3(e^x-e^{-x})}{\sin x}$ $\quad \lim\limits_{x\to 0}\dfrac{3(e^x+e^{-x})}{\cos x} = \dfrac{3(2)}{1} = 6$

19. $\lim\limits_{x\to 0}\dfrac{2x^2}{e^x-1-x}$ $\quad \lim\limits_{x\to 0}\dfrac{4x}{e^x-1} = \lim\limits_{x\to 0}\dfrac{4}{e^x} = 4$

20. $\lim\limits_{x\to\infty}\dfrac{e^{2x}}{2x^2} = \lim\limits_{x\to\infty}\dfrac{2e^{2x}}{4x}$

$\lim\limits_{x\to\infty}\dfrac{4e^{2x}}{4}$ $\quad \infty$