A **hyperbola** is the locus of all points in the plane such that the absolute value of the differences of the distance from two given points in the plane, called foci, is constant. The center of the hyperbola is the midpoint of the segment whose endpoints are the foci. The asymptotes of a hyperbola are lines that the curve approaches as it recedes from the center. A hyperbola has two axes of symmetry. The **transverse** axis joins the vertices and has a length of $2a$. The segment perpendicular to the transverse axis through the center is called the **conjugate** axis and has a length of $2b$. The distance from the center to a vertex is a units, and the distance from the center to a foci is c units.

The standard form of the equation of a hyperbola with center (h, k) and transverse axis of length $2a$ units, where $c^2 = a^2 + b^2$ is as follows:

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1, \text{ when the transverse axis is parallel to the } x\text{-axis}$$

(opens left and right)

with asymptotes $y - k = \pm \frac{b}{a}(x - h)$

vertices $(h \pm a, k)$ and foci $(h \pm c, k)$

OR

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1, \text{ when the transverse axis is parallel to the } y\text{-axis}$$

(opens up and down)

with asymptotes $y - k = \pm \frac{a}{b}(x - h)$

vertices $(h, k \pm a)$ and foci $(h, k \pm c)$

1. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\dfrac{x^2}{25} - \dfrac{y^2}{16} = 1$. Then graph the hyperbola.

2. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\dfrac{y^2}{25} - \dfrac{x^2}{16} = 1$. Then graph the hyperbola.

3. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\dfrac{(y-3)^2}{25} - \dfrac{(x-2)^2}{16} = 1$. Then graph the hyperbola.

4. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $\dfrac{(x-5)^2}{25} - \dfrac{(y+1)^2}{9} = 1$. Then graph the hyperbola.

5. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $25y^2 - 9x^2 - 100y - 72x - 269 = 0$.. Then graph the hyperbola.

6. Find the coordinates of the center, foci, and vertices, and the equations of the asymptotes of the graph of $4x^2 - y^2 + 24x + 4y + 28 = 0$. Then graph the hyperbola.

Practice Exercises

Identify the vertices, foci, and direction of opening of each.

1) $\dfrac{x^2}{81} - \dfrac{y^2}{4} = 1$

2) $\dfrac{x^2}{121} - \dfrac{y^2}{81} = 1$

3) $\dfrac{y^2}{25} - \dfrac{x^2}{16} = 1$

4) $\dfrac{x^2}{121} - \dfrac{y^2}{36} = 1$

5) $\dfrac{(x+2)^2}{169} - \dfrac{(y+8)^2}{4} = 1$
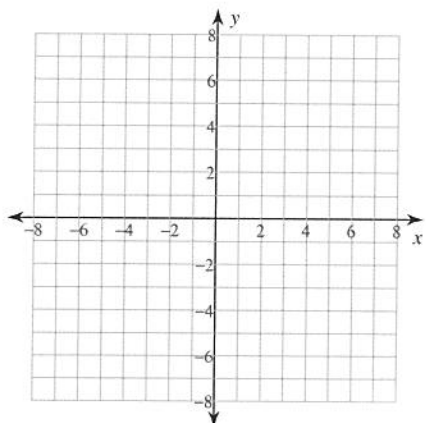
6) $\dfrac{(y+8)^2}{36} - \dfrac{(x+2)^2}{25} = 1$

Identify the vertices and foci of each. Then sketch the graph.

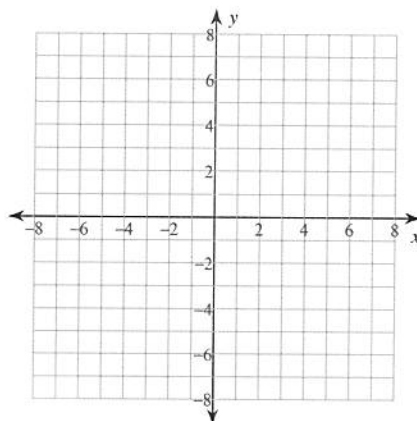7) $\dfrac{x^2}{20} - \dfrac{(y+1)^2}{10} = 1$

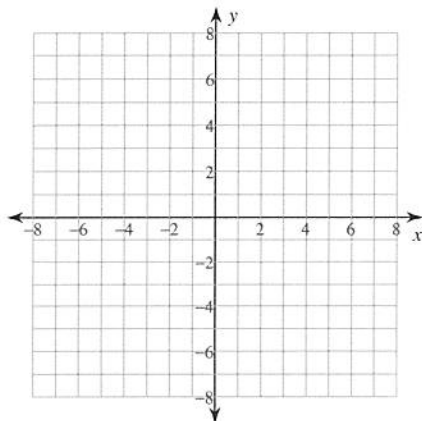8) $\dfrac{(x-3)^2}{4} - \dfrac{(y+1)^2}{9} = 1$
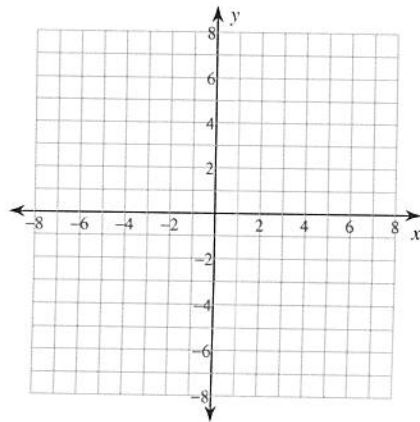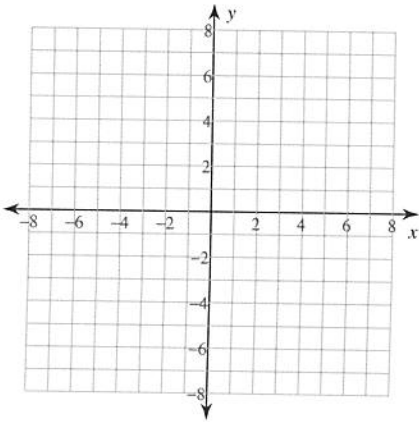
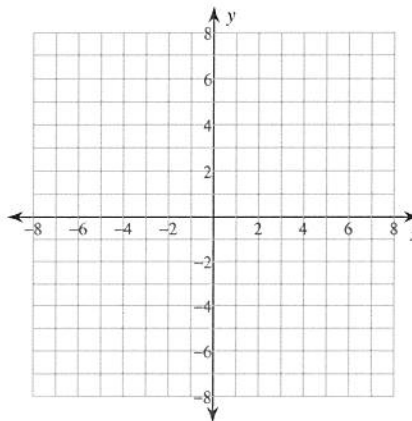9) $\dfrac{(y-1)^2}{9} - \dfrac{(x+1)^2}{16} = 1$

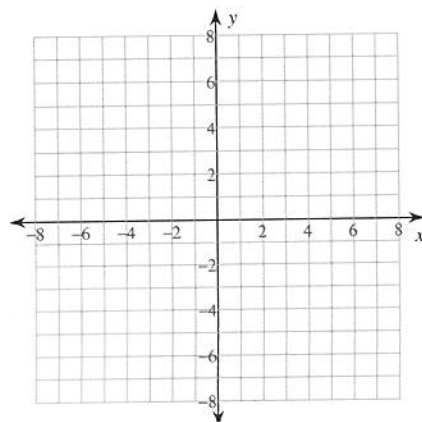10) $\dfrac{y^2}{9} - \dfrac{(x-2)^2}{9} = 1$

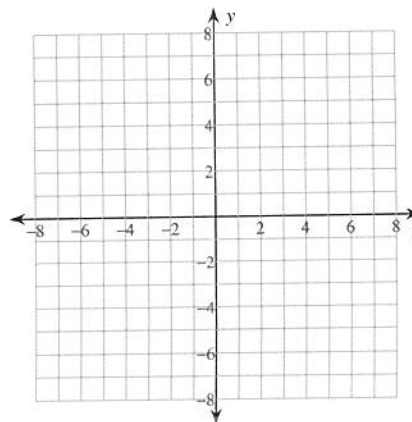11) $\dfrac{y^2}{25} - \dfrac{x^2}{25} = 1$

12) $\dfrac{x^2}{25} - \dfrac{(y-2)^2}{4} = 1$

13) $\dfrac{(x-1)^2}{4} - \dfrac{(y-3)^2}{4} = 1$

14) $\dfrac{y^2}{9} - \dfrac{x^2}{25} = 1$

Identify the center, vertices, foci, asymptotes and direction of opening of each.

15) $-10y - y^2 = -4x^2 - 72x - 199$

16) $-y^2 + 12y - 19 = 18x - x^2$