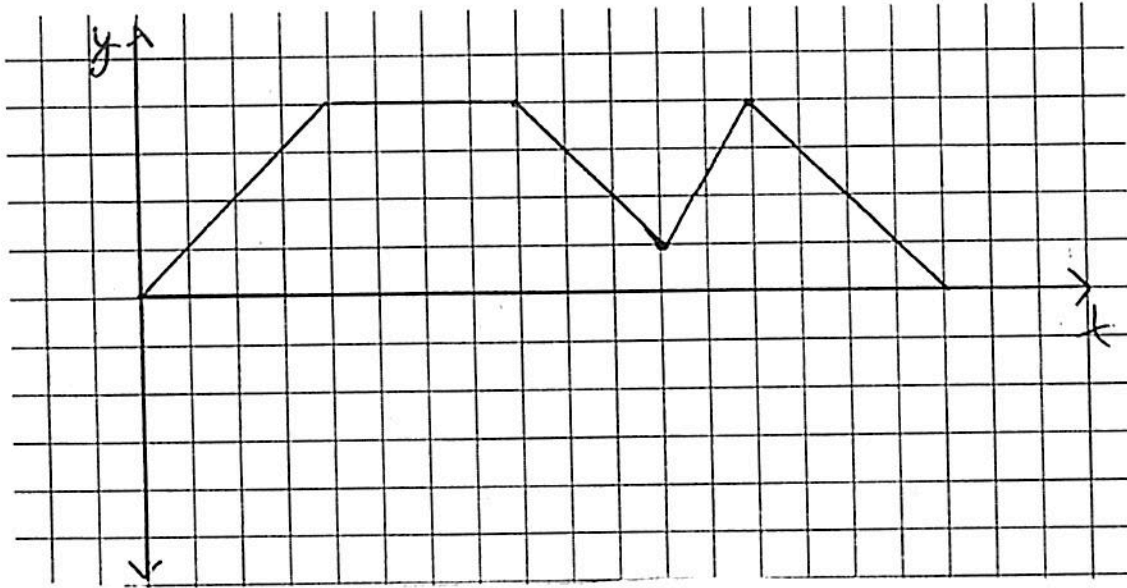


Name: _____
AP Calc AB: Introduction to Area Under a Curve

Date: _____

1. Find the area between the function and the x -axis (otherwise known as the area under the curve.)



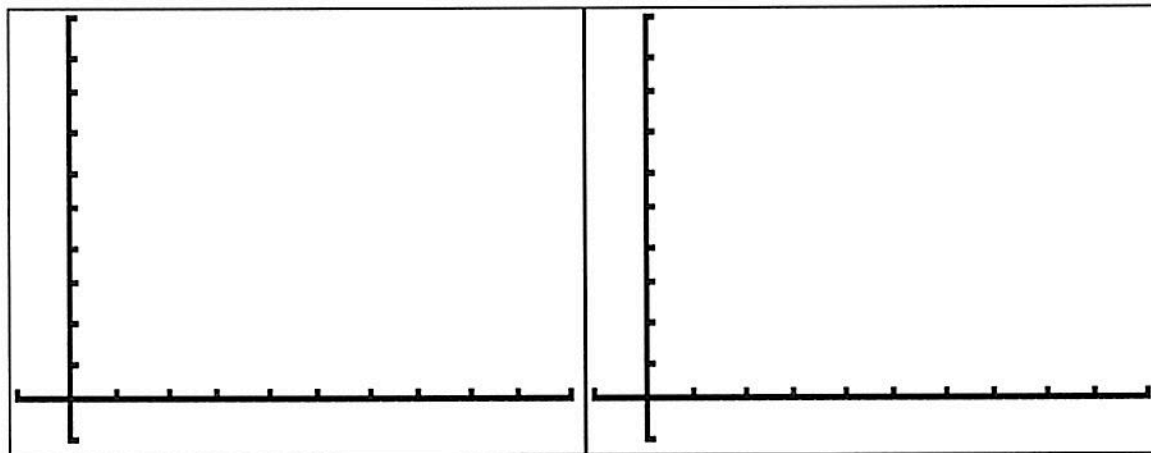
2. Sketch the graph of $x + y = 4$. Find the area under the curve.

3. Sketch the graph of $y = x^2 + 1$ in the interval $[0,3]$. Find the area under the curve.

We cannot find the area of this region using any formula we know since this is not a shape we have an area formula for. HOWEVER, we can make rectangles under this curve since we know the area of a rectangle.

How? By drawing in rectangles with equal widths.

Let's start by using 3 equal subintervals. There are two ways we can approach this.



RIGHT ENDPOINT

To draw these correctly, the right endpoint of each rectangle must be on the graph.

LEFT ENDPOINT

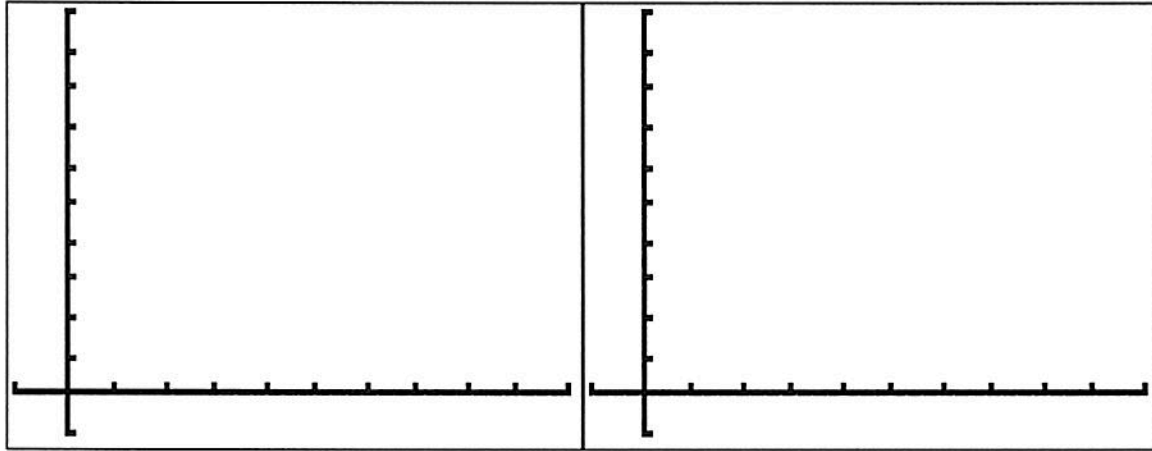
To draw these correctly, the left endpoint of each rectangle must be on the graph.

Let's let 2 boxes represent one unit on the x axis.

Using the same example, let's calculate the area if we made 6 rectangles.

RIGHT ENDPOINT

LEFT ENDPOINT



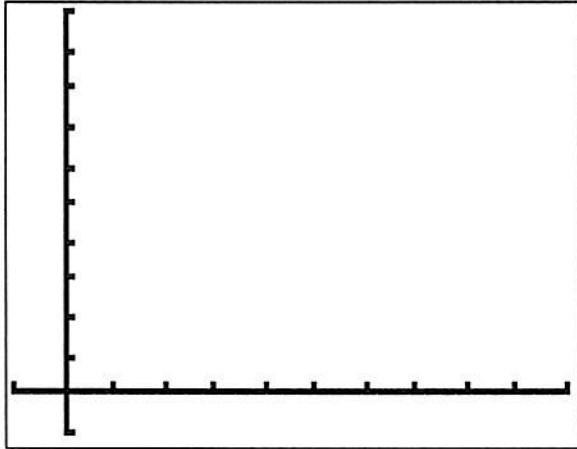
Calculations:

Right Endpoint

Left Endpoint

Midpoint Rule

Let's use the same example but instead of a right or left endpoint approximation, let's use 3 equal subintervals and do a midpoint approximation. The only difference is how we define the heights of the rectangles again.



The process of using rectangles to approximate area under a curve is called **Riemann sums**. A Riemann sum is an approximation of area calculated using rectangles

Trapezoid Rule

When you use the trapezoid rule, you get the average of the left and right hand approximation for the area under the curve. Let's prove it!

Example: Graph $f(x) = x^2 + 1$ $0 \leq x \leq 3$

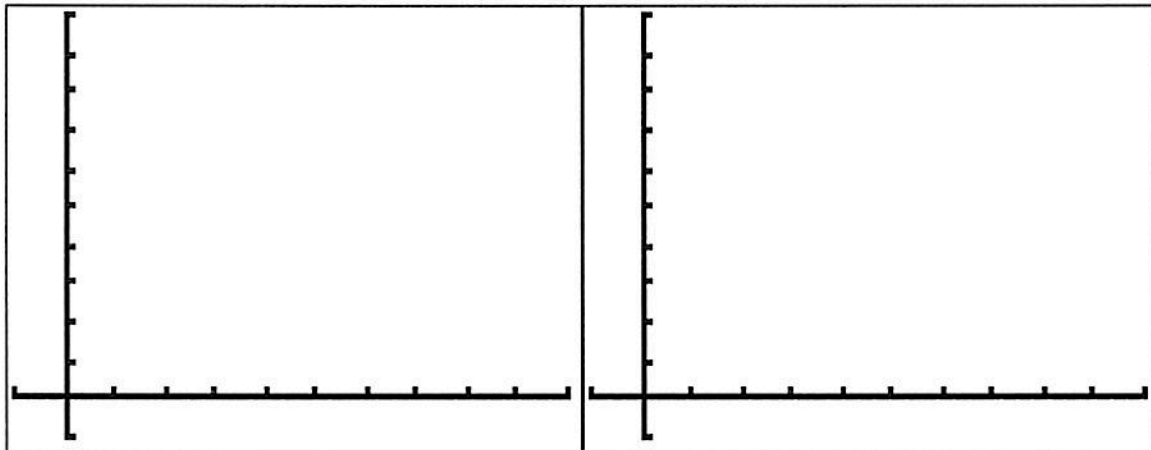
To make the trapezoids, first draw a line from each interval endpoint to the curve.

Our intervals are: (0, 1) (1, 2) and (2, 3), so the endpoints of these intervals are 1, 2, and 3.

Now the bottom of all of the trapezoids is the x axis. To get the top of each trapezoid connect the y values of each interval.

Graph with 3 trapezoids

Graph with 6 trapezoids



Area of a trapezoid is: $A = \frac{1}{2}(b_1 + b_2)h$

The b values in the formula are the parallel bases and h is the height (we can also say the height is the length between each trapezoid)

Calculations:

Area with 3 Trapezoids

Area with 6 Trapezoids