Name:____________________________________ Date:________

PCH: Inverses of Matrices Ms. Loughran

Do Now:

1. Find AB and BA, if possible.

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

The identity matrix of a square matrix has entries of 1 on its main diagonal and 0's as all other entries.

I_2 means the identity matrix of a 2×2 matrix , I_3 means the identity matrix of a 3×3 matrix and so on.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let A be an $n \times n$ matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, A^{-1} is called the **inverse** of A.

1. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

2. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

To find the inverse of a 2×2 matrix we are going to use the determinant.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A is $ad - bc$, and $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

3. Find A^{-1} and verify that $AA^{-1} = A^{-1}A = I_2$

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$$

4. Find the inverse of A.

$$A = \begin{bmatrix} 7 & -4 \\ 8 & 0 \end{bmatrix}$$

5. Find the inverse of B, if it exists.

$$B = \begin{bmatrix} 8 & 4 \\ -4 & -2 \end{bmatrix}$$

We can use inverses to solve systems of linear equations.

If A is an invertible matrix (if A has an inverse), the system of linear equations represented by $AX = B$ has a unique solution:

$$AX = B$$

$$X =$$

6. Solve the system using the inverse, if possible.

$$\begin{aligned} 2x - 5y &= 15 \\ 3x - 6y &= 36 \end{aligned}$$

7. Solve the system using the inverse, if possible.

$$\begin{aligned} 3x + 4y &= -2 \\ 5x + 3y &= 4 \end{aligned}$$

For 8 and 9, verify if *B* is the inverse of *A*.

8. $A = \begin{bmatrix} 3 & -2 \\ -4 & 6 \end{bmatrix}$ $\quad B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ \frac{2}{5} & \frac{3}{10} \end{bmatrix}$

9.
$$A = \begin{bmatrix} 1 & 1 & -2 \\ -3 & -2 & 5 \\ -6 & 4 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} -14 & -6 & \frac{1}{2} \\ -9 & 4 & \frac{1}{2} \\ -12 & -5 & \frac{1}{2} \end{bmatrix}$$

10.

For what value(s) of x does the matrix M have an inverse?

$$M = \begin{bmatrix} x & 1 \\ 2 & x+1 \end{bmatrix}$$