Name: Key

Pre-Calculus Review Q3

1) Using Matrices, find the area of the triangle with vertices (2/3, 4), (3, -6), and (1/2, -3)

$$A = \pm\frac{1}{2}\det(A)$$

$$A = -\frac{1}{2}(-18) = 9$$

$-3-2+12$

$$A = \begin{bmatrix} \frac{2}{3} & 4 & 1 \\ 3 & -6 & 1 \\ \frac{1}{2} & -3 & 1 \end{bmatrix} \begin{matrix} \frac{2}{3} & 4 \\ 3 & -6 \\ \frac{1}{2} & -3 \end{matrix}$$

$\det(A) = -11 - 7 \quad -4 + 2 - 9$

-18

-11

2) Which value of x for the equation below is true?

$$\begin{bmatrix} 3x \\ y \end{bmatrix} = \begin{bmatrix} 10+2y \\ 5-x \end{bmatrix}$$

$3x = 10 + 2y$

$y = 5 - x$

$3x = 10 + 2y$

$2(x = 5 - y)$

$3x = 10 + \cancel{2y}$

$2x = 10 - \cancel{2y}$

$5x = 20$

$x = 4$

$x = 4$

$y = 5 - x = 5 - 4 = 1$

3) Find the sum of the given matrices:

$$\begin{bmatrix} 3 & -2 \\ 5 & 6 \end{bmatrix} + \begin{bmatrix} 1 & -5 \\ 8 & 4 \end{bmatrix} = \begin{bmatrix} 4 & -7 \\ 13 & 10 \end{bmatrix}$$

4) Find the difference of the given matrices:

$$\begin{bmatrix} 5 & 0 \\ -2 & 1 \\ 4 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -6 \\ 2 & -2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ -2 & 1 \\ 4 & -3 \end{bmatrix} + \begin{bmatrix} -4 & 6 \\ -2 & 2 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ -4 & 3 \\ 5 & -6 \end{bmatrix}$$

5) Find the scalar product of the given matrix and coefficient:

$$-3\begin{bmatrix} 1 & -3 & 6 \\ 9 & -1 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 9 & -18 \\ -27 & 3 & -12 \end{bmatrix}$$

6) Find the product of the given matrices:

$2\times2 \quad 2\times3 \quad 2\times3$

$$\begin{bmatrix} 2 & 4 \\ 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 & -2 & 7 \\ 6 & 0 & -5 \end{bmatrix} = \begin{bmatrix} 30 & -4 & -6 \\ 3 & -6 & 26 \end{bmatrix}$$

7) Find the inverse of $\begin{bmatrix} 5 & -6 \\ -3 & 4 \end{bmatrix}$.

$$\det = 5(4) - (-3)(-6)$$
$$20 - 18 = 2$$

$$\frac{1}{2}\begin{bmatrix} 4 & 6 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ \frac{3}{2} & \frac{5}{2} \end{bmatrix}$$

8) Simplify: $\begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -3 & 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ 5 & -1 \end{bmatrix}$

$$\begin{bmatrix} 7 & -1 \\ -7 & 5 \end{bmatrix} + \begin{bmatrix} -11 & 9 \\ 13 & 11 \end{bmatrix} = \begin{bmatrix} -4 & 8 \\ 6 & 16 \end{bmatrix}$$

9) Evaluate: $\begin{vmatrix} 1 & 8 & -2 \\ 3 & -1 & 4 \\ 2 & -3 & -1 \end{vmatrix}$

Remember this notation means to find the determinant.

$$\begin{array}{ccccc} & & 4 & -12 & -24 \\ 1 & 8 & -2 & 1 & 8 \\ 3 & -1 & 4 & 3 & -1 \\ 2 & -3 & -1 & 2 & -3 \\ & & 1 & 64 & 18 \end{array}$$

$$83 - (-32)$$
$$= 115$$

10) For what value of w is the following statement true?

$$\begin{vmatrix} 5 & -2 \\ 3 & w \end{vmatrix} = w + 14$$

$$\det = 5w - (-6)$$
$$5w + 6$$

$$5w + 6 = w + 14$$
$$4w = 8$$
$$w = 2$$

Questions 11 through 13 refer to the following:

Use Cramer's Rule to solve the given linear system of equations:

11) $x - 4y = 22$
$2x - 7y = 39$

$\begin{bmatrix} 1 & -4 \\ 2 & -7 \end{bmatrix}$ $\det = 1(-7) - (-4)(2) = -7 + 8 = 1$

$\begin{bmatrix} 22 & -4 \\ 39 & -7 \end{bmatrix}$ $\det = 22(-7) - (-4)(39) = 2$ x value $= \frac{2}{1} = 2$

$\begin{bmatrix} 1 & 22 \\ 2 & 39 \end{bmatrix}$ $\det = 1(39) - 2(22) = -5$ y value $= \frac{-5}{1} = -5$

12) $2x - 4y = 7$
$-x + y = 1$

$\begin{bmatrix} 2 & -4 \\ -1 & 1 \end{bmatrix}$ $\det = 2(1) - (-4)(-1)$
$2 - 4 = -2$

$\begin{bmatrix} 7 & -4 \\ 1 & 1 \end{bmatrix} = 7 - (-4)(1) = 11$ x-value $= \frac{11}{-2}$
$= -5.5$

$\begin{bmatrix} 2 & 7 \\ -1 & 1 \end{bmatrix} = 2(1) - (7)(-1) = 9$ y-value $= \frac{9}{-2}$
$= -4.5$

13) $x + 3y - z = 8$
$2x - y + 2z = 0$
$-3x + y - 3z = -2$

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14) Write the system of equations represented by the matrix equation below:

$$\begin{bmatrix} 3 & -2 & 5 \\ 1 & 1 & -4 \\ -2 & 2 & 7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

$3x - 2y + 5z = 3$
$x + y - 4z = 2$
$-2x + 2y + 7z = -5$

15) Solve the system of linear equations by using an inverse matrix:

$6x - 5y = 3$
$3x - 2y = 3$

$$\overset{A}{\begin{bmatrix} 6 & -5 \\ 3 & -2 \end{bmatrix}} \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} \overset{B}{\begin{bmatrix} 3 \\ 3 \end{bmatrix}}$$

$X = A^{-1}B$

$\det(A) = 6(-2) - (-5)(3) = 3$

$A^{-1} = \frac{1}{3}\begin{bmatrix} -2 & 5 \\ -3 & 6 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} -2/3 & 5/3 \\ -1 & 2 \end{bmatrix}$ $B \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$X = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$ $x = 3$ $y = 3$

16) Are the points (3, 1/2), (-2, 2), and (5,-3) collinear?

$$\begin{bmatrix} 3 & \frac{1}{2} & 1 \\ -2 & 2 & 1 \\ 5 & -3 & 1 \end{bmatrix} \begin{matrix} 3 & \frac{1}{2} \\ -2 & 2 \\ 5 & -3 \end{matrix}$$

$\det = 14.5 - (0) = 14.5$

Since $d \neq 0$, the point are not collinear

17) Using Matrices, find an equation of the line that passes through (2, -2) and (1/2, 3)

18) What is the inverse of the following matrix?

$$\begin{bmatrix} i & m \\ g & o \end{bmatrix}$$

$\det = io - mg$

$= \frac{1}{io - mg}\begin{bmatrix} o & -m \\ -g & i \end{bmatrix}$

$= \begin{bmatrix} \frac{o}{io-mg} & \frac{-m}{io-mg} \\ \frac{-g}{io-mg} & \frac{i}{io-mg} \end{bmatrix}$

17

$$\begin{bmatrix} x & y & 1 \\ 2 & -2 & 1 \\ \frac{1}{2} & 3 & 1 \end{bmatrix} \begin{matrix} x & y \\ 2 & -2 \\ \frac{1}{2} & 3 \end{matrix}$$

$\det = -2x + \frac{1}{2}y + 6 - (2y + 3x - 1)$

$\det = -5x - \frac{3}{2}y + 7$

Eq. of line: $-5x - \frac{3}{2}y + 7 = 0$ or $-10x - 3y + 14 = 0$

19) What is the determinant of the following matrix?

$$\begin{array}{ccc} iat & brn & ecg \end{array}$$

$$\begin{bmatrix} b & a & t \\ c & e & r \\ i & n & g \end{bmatrix} \begin{matrix} b & a \\ c & e \\ i & n \end{matrix}$$

$$\begin{array}{ccc} bag & eri & tcn \end{array}$$

bag + eri + tcn − (iat + brn + ecg)

bag + eri + tcn − iat − brn − ecg

Questions 20 and 21 refer to the following:

Using the matrices below, find the matrix equal to the given expression:

$$S = \begin{bmatrix} 4 & 6 \\ 1 & 2 \end{bmatrix} \quad T = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix} \quad U = \overset{4 \times 2}{\begin{bmatrix} 1 & 4 \\ 6 & -3 \\ 2 & -1 \\ 5 & 0 \end{bmatrix}} \quad V = \overset{1 \times 4}{\begin{bmatrix} 9 & 6 & 4 & -1 \end{bmatrix}}$$

20) $S - T = \overset{S}{\begin{bmatrix} 4 & 6 \\ 1 & 2 \end{bmatrix}} + \overset{-T}{\begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}} = \begin{bmatrix} 4 & 7 \\ 0 & 0 \end{bmatrix}$

21) $T + U$ undefined

22) UV = undefined

13

$$\begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 2 \\ -3 & 1 & -3 \end{bmatrix} \begin{matrix} 1 & 3 \\ 2 & -1 \\ -3 & 1 \end{matrix}$$

det = −17 − (−19) = 17 + 19 = 2

$$\begin{bmatrix} 8 & 3 & -1 \\ 0 & -1 & 2 \\ -2 & 1 & -3 \end{bmatrix} \begin{matrix} 8 & 3 \\ 0 & -1 \\ -2 & 1 \end{matrix}$$

det = 12 − (14) = −2

x value = $\frac{-2}{2} = -1$

$$\begin{bmatrix} 1 & 8 & -1 \\ 2 & 0 & 2 \\ -3 & -2 & -3 \end{bmatrix} \begin{matrix} 1 & 8 \\ 2 & 0 \\ -3 & -2 \end{matrix}$$

det = −44 − (−52) = 8

y-value = $\frac{8}{2} = +4$

$$\begin{bmatrix} 1 & 3 & 8 \\ 2 & -1 & 0 \\ -3 & 1 & -2 \end{bmatrix} \begin{matrix} 1 & 3 \\ 2 & -1 \\ -3 & 1 \end{matrix}$$

det = 18 − (12) = 6

z-value = $\frac{6}{2} = 3$

Given:

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \text{ and } B = \begin{bmatrix} m & o & p \\ q & r & s \\ t & y & z \end{bmatrix}$$

Find:

23. $A + B$

$$\begin{bmatrix} a+m & b+o & c+p \\ d+q & e+r & f+s \\ g+t & h+y & k+z \end{bmatrix}$$

24. $A - B$

$$\begin{bmatrix} a-m & b-o & c-p \\ d-q & e-r & f-s \\ g-t & h-y & k-z \end{bmatrix}$$

25. AB

$$\begin{bmatrix} am+bq+ct & ao+br+cy & ap+bs+cz \\ dm+eq+ft & do+er+fy & dp+es+fz \\ gm+hq+kt & go+hr+ky & gp+hs+kz \end{bmatrix}$$

26. $2B$

$$\begin{bmatrix} 2m & 2o & 2p \\ 2q & 2r & 2s \\ 2t & 2y & 2z \end{bmatrix}$$

27. det (A)

28. $|B|$ ← find determinant

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \end{bmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$$B = \begin{bmatrix} m & o & p \\ q & r & s \\ t & y & z \end{bmatrix} \begin{matrix} m & o \\ q & r \\ t & y \end{matrix}$$

$$|B| = mrz + ost + pqy - (prt + msy + oqz)$$

$$|B| = mrz + ost + pqy - prt - msy - oqz$$

$$\det = aek + bfg + cdh - (ceg + afh + bdk)$$

$$\det = aek + bfg + cdh - ceg - afh - bdk$$

Using matrices

Area of Δ : $\pm \frac{1}{2}$ det

Collinearity

If det = 0, collinear.

det $\neq 0$, not collinear

To find eq. of the line passing through 2 points.

$$\begin{bmatrix} x & y & 1 \\ & \vdots & \\ & & \end{bmatrix}$$

find det.

set det = 0

Solve using an inverse matrix

$$X = A^{-1} \cdot B$$

To add matrices :

dimensions have to match

To multiply matrices :

$$\underbrace{a \times b \quad b \times c}_{\text{final dimensions}}$$

match

think row by column

To find an inverse matrix

① Find determinant

② $\frac{1}{\det}$ [switch] [negate]

③ mult. $\frac{1}{\det}$ by the new matrix