Name Key

Pre-Calculus

Quarter 2

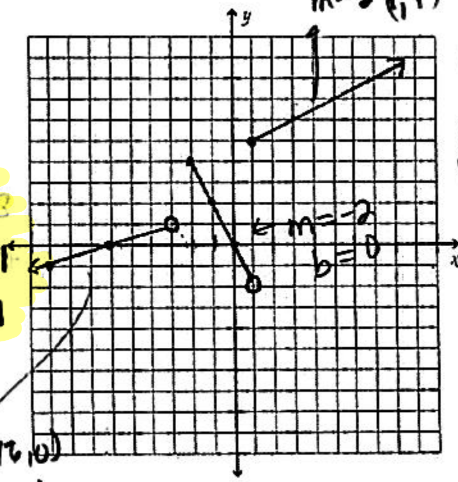
Test 1 - Review

$g(x)=\begin{cases} 2 & x\le -2 \\ (x+1)^2+1 & -2<x<1 \\ -\frac{1}{2}x+\frac{7}{2} & x\ge 1 \end{cases}$

1. Write a piecewise function for each of the graphs below:

a)

$m=\frac{1}{2}$ (1,5)



$y-5=\frac{1}{2}(x-1)$

$y-5=\frac{1}{2}x-\frac{1}{2}$

$y=\frac{1}{2}x+\frac{9}{2}$

$m=-2$

$b=0$

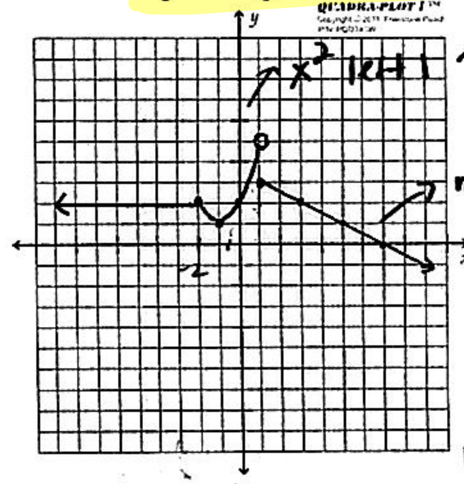
$f(x)=\begin{cases} \frac{1}{3}x+2 & x<-3 \\ -2x & -2\le x<1 \\ \frac{1}{2}x+\frac{9}{2} & x\ge 1 \end{cases}$

(-6,0)

$y-0=\frac{1}{3}(x+6)$

$y=\frac{1}{3}x+2$

b)



x^2 left 1 $\uparrow 1$

$m=-\frac{1}{2}$

(1,3)

$y-3=-\frac{1}{2}(x-1)$

$y-3=-\frac{1}{2}x+\frac{1}{2}$

$y=-\frac{1}{2}x+\frac{3}{2}$

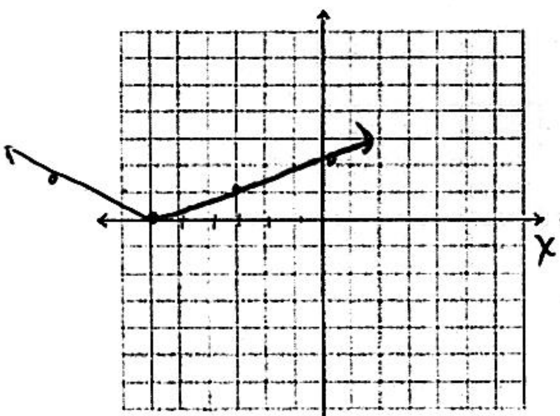
2. Use the algebraic definition of absolute value to rewrite each expression as a piecewise function and then sketch each graph.

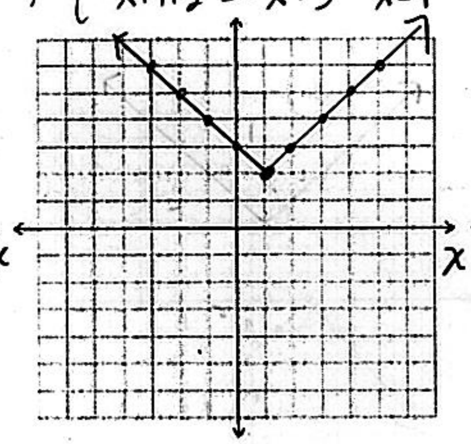
a) $f(x)=\left|\frac{1}{3}x+2\right|$

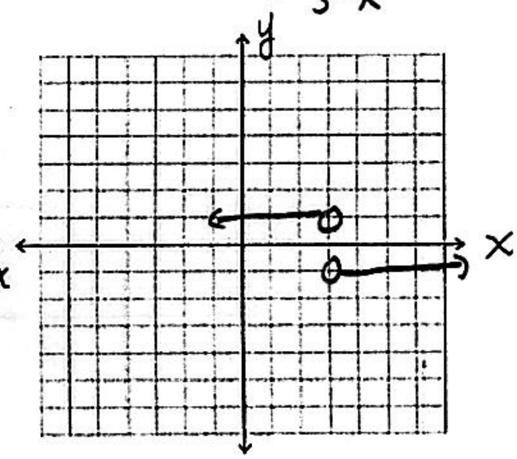
b) $f(x)=|x-1|+2$

$f(x)=\begin{cases} x-1+2=x+1 & x\ge 1 \\ -x+1+2=-x+3 & x<1 \end{cases}$

c) $f(x)=\frac{|x-3|}{3-x}=\begin{cases} \frac{x-3}{3-x}=-1 & x>3 \\ \frac{-(x-3)}{3-x}=1 & x<3 \end{cases}$







a) $f(x)=\begin{cases} \frac{1}{3}x+2 & \frac{1}{3}x+2\ge 0,\ x\ge -6 \\ -\frac{1}{3}x-2 & x<-6 \end{cases}$

a) (-1,1) (2,1) (2,-1)
(0,0) (3,0) (3,0)
(1,1) (4,1) (4,-1)

b) (-1,1) (-3,1) (-3,-1) (-3,2)
(0,0) (-2,0) (-2,0) (-2,3)
(1,1) (-1,1) (-1,-1) (-1,2)

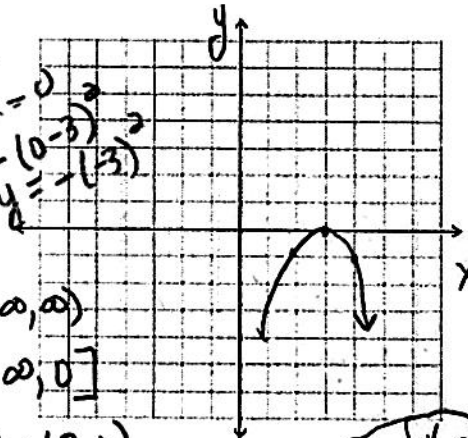
2

3) Describe each transformation in terms of the original function $f(x)=x^2$ then graph each function. State the domain, range, and any x- or y-intercepts

a) $f(x)=-(x-3)^2$

right 3
reflect over x-axis

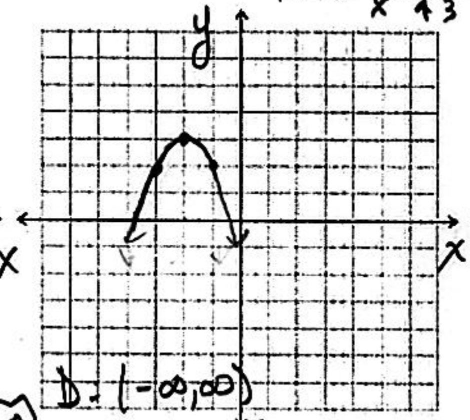
y-int
let x = 0
$y=-(0-3)^2$
$y=-(3)^2$



D: $(-\infty,\infty)$
R: $(-\infty,0]$
x-int : (3,0)
y-int : (0,-9)

b) $f(x)=3-(x+2)^2$

$f(x)=-(x+2)^2+3$
left 2 reflect over x ↑3



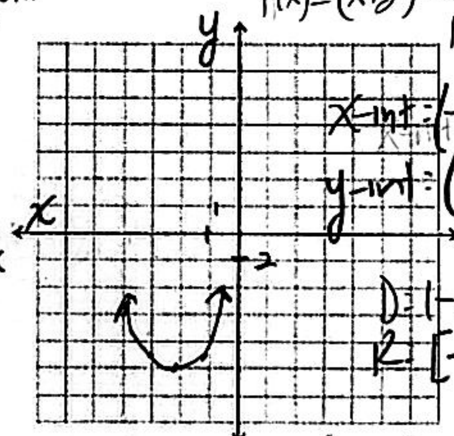
* work for intercepts on last page!

D: $(-\infty,\infty)$
R: $(-\infty,3]$
x-int : $(-2\pm\sqrt{3},0)$
y-int : (0,-1)

c) $f(x)=x^2+4x-6$

$f(x)=x^2+4x+4-4-6$
$f(x)=(x+2)^2-10$
left 2 ↓1

x-int: $(-2\pm\sqrt{10},0)$
y-int: (0,-6)



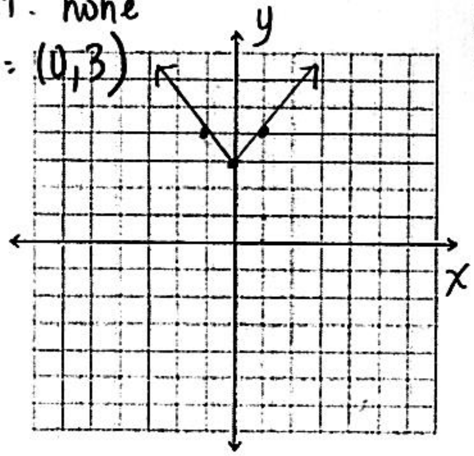
D: $(-\infty,\infty)$
R: $[-10,\infty)$

(-1,1) (-3,1) (-3,-9)
(0,0) (-2,0) (-2,-10)
(1,1) (-1,1) (-1,-9)

4) Describe each transformation in terms of the original function $f(x)=|x|$ then graph each function. State the domain, range, and any x- or y-intercepts.

a) $f(x)=|-x|+3$

D: $(-\infty,\infty)$
R: $[3,\infty)$
x-int: none
y-int: (0,3)

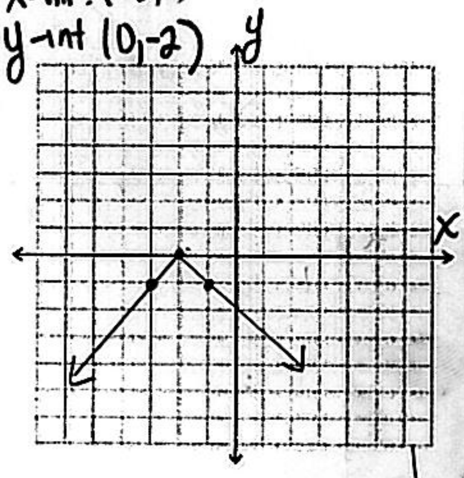


a) reflect over y-axis ↑3

(-1,1) (1,1) (1,4)
(0,0) (0,0) (0,3)
(1,1) (-1,1) (-1,4)

b) $f(x)=-|x+2|$

D: $(-\infty,\infty)$
R: $(-\infty,0]$
x-int: (-2,0)
y-int (0,-2)

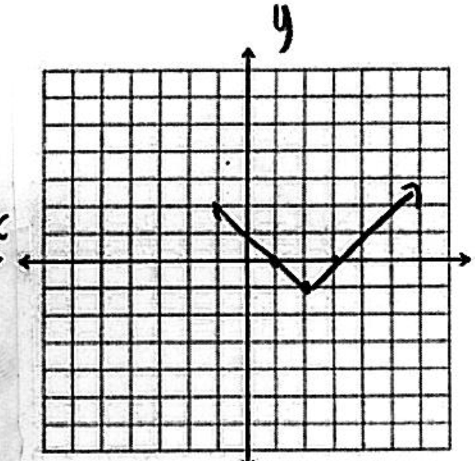


b) left 2 reflect over x

(-1,1) (-3,1) (-3,-1)
(0,0) (-2,0) (-2,0)
(1,1) (-1,1) (-1,-1)

c) $f(x)=|x-2|-1$

right 2 ↓1



|x| right 2 ↓1

(-1,1) (1,1) (1,0)
(0,0) (2,0) (2,-1)
(1,1) (3,1) (3,0)

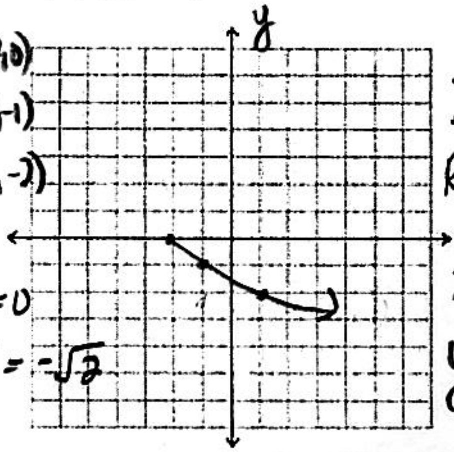
D: $(-\infty,\infty)$
R: $[-1,\infty)$
x int: (3,0) (1,0)
y-int (0,1)

5) Describe each function in terms of the basic function $f(x)=\sqrt{x}$ then graph each function. State the domain, range, x- and y-intercepts.

a) $f(x)=-\sqrt{x+2}$

left 2 reflect over x

(0,0) (-2,0) (-2,0)
(1,1) (-1,1) (-1,-1)
(4,2) (2,2) (2,-2)



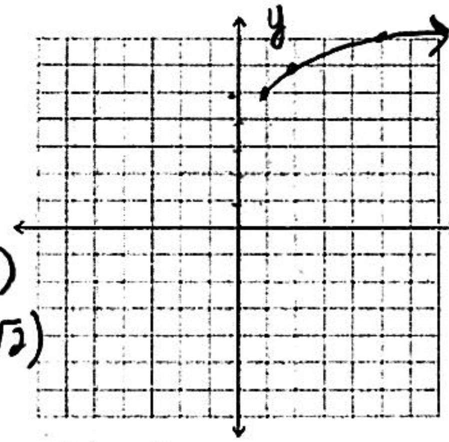
D: $[-2,\infty)$
R: $(-\infty,0]$
x-int: $(-2,0)$
y-int: $(0,-\sqrt{2})$

y-int let $x=0$
$y=-\sqrt{0+2}=-\sqrt{2}$

b) $f(x)=\sqrt{x-1}+5$

right one ↑5

(0,0) (1,0) (1,5)
(1,1) (2,1) (2,6)
(4,2) (5,2) (5,7)



D: $[1,\infty)$
R: $[5,\infty)$
x-int: none
y-int: none

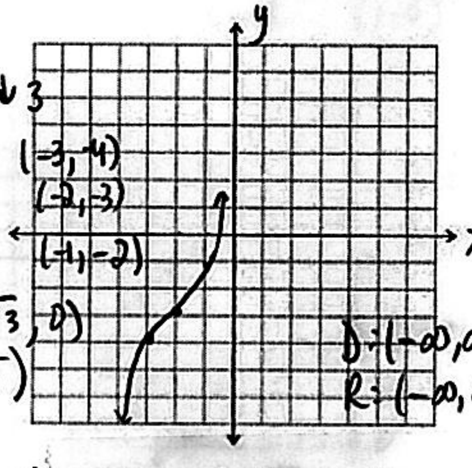
6) Graph each function as a transformation of the basic function $f(x)=x^3$. State the domain, range, x- and y-intercepts.

a) $f(x)=(x+2)^3-3$

left 2 ↓3

y^3 left 2 ↓3
(-1,-1) (-3,-1) (-3,-4)
(0,0) (-2,0) (-2,-3)
(1,1) (-1,1) (-1,-2)

x-int: $(-2+\sqrt[3]{3},0)$
y-int: $(0,5)$



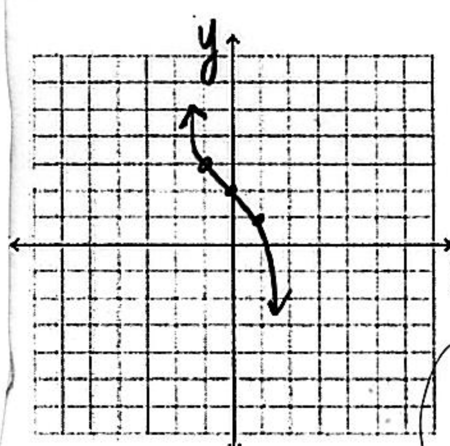
D: $(-\infty,\infty)$
R: $(-\infty,\infty)$

* work for intercept on last page *

b) $f(x)=(-x)^3+2$

reflect over y-axis ↑2

(-1,-1) (1,-1) (1,1)
(0,0) (0,0) (0,2)
(1,1) (-1,1) (-1,3)



D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
x-int $(\sqrt[3]{2},0)$
y-int: $(0,2)$

x-int let $y=0$
$0=(-x)^3+2$
$0=-x^3+2$
$-2=-x^3$
$2=x^3$
$x=\sqrt[3]{2}$

7) Write an equation for the indicated transformation that is applied to the given function.

a) $f(x)=x^4$; shift 3 units to the right, reflect over the x-axis, shifted up 5 units

$y=-(x-3)^4+5$

b) $f(x)=\sqrt{x}$; shift to the left 2, shift down 4 units

$y=\sqrt{x+2}-4$

c) $f(x)=x^2$; shift to the right 4, reflected over the x-axis, shift up 2 units

$y=-(x-4)^2+2$

d) $f(x)=x^3$; reflect over the y-axis, shifted up 3 unit

$y=(-x)^3+3$

$f(x) = -(x^2 - 4x + 4 - 4) - 3$
$f(x) = -(x-2)^2 + 4 - 3$
$f(x) = -(x-2)^2 + 1$

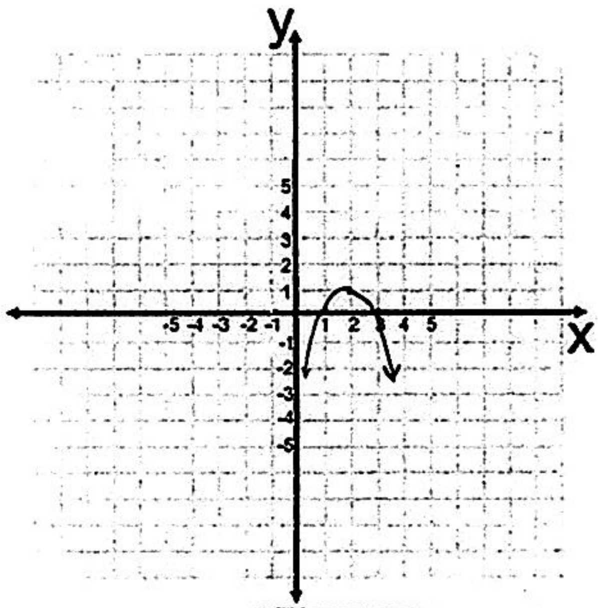
4

8) Write $f(x) = -x^2 + 4x - 3$ in vertex form.

9) For each, find the axis of symmetry, vertex, x-intercepts, y-intercepts and graph it

a. $f(x) = -x^2 + 4x - 3$ → $f(x) = -(x-2)^2 + 1$ see #8

x^2 shifted right 2, reflected over x axis, up 1



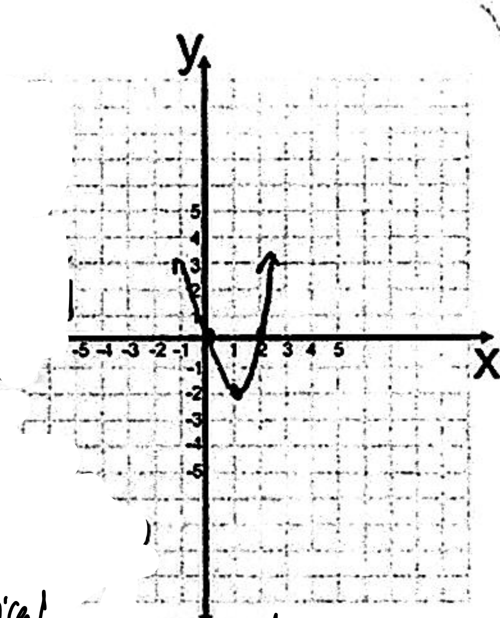
x^2	right 2	reflect over x-axis	up 1
(-1,1)	(1,1)	(1,-1)	(1,0)
(0,0)	(2,0)	(2,0)	(2,1)
(1,1)	(3,1)	(3,-1)	(3,0)

axis of sym: $x = 2$ vertex: (2,1)

y-int. $y = -(0-2)^2 + 1$
$y = -(4) + 1 = -3$
$y = -3$ (0,-3)

X-int: (1,0), (3,0)
D: $(-\infty, \infty)$
R: $(-\infty, 1]$

b. $f(x) = 2(x-1)^2 - 2$



D: $(-\infty, \infty)$
R: $[-2, \infty)$

x-int: (0,0), (2,0)
y-int: (0,0)
axis of sym: $x = 1$
vertex: (1,-2)

x^2	right 1	vertical stretch by a factor of 2	↓ 2
(-1,1)	(0,1)	(0,2)	(0,0)
(0,0)	(1,0)	(1,0)	(1,-2)
(1,1)	(2,1)	(2,2)	(2,0)

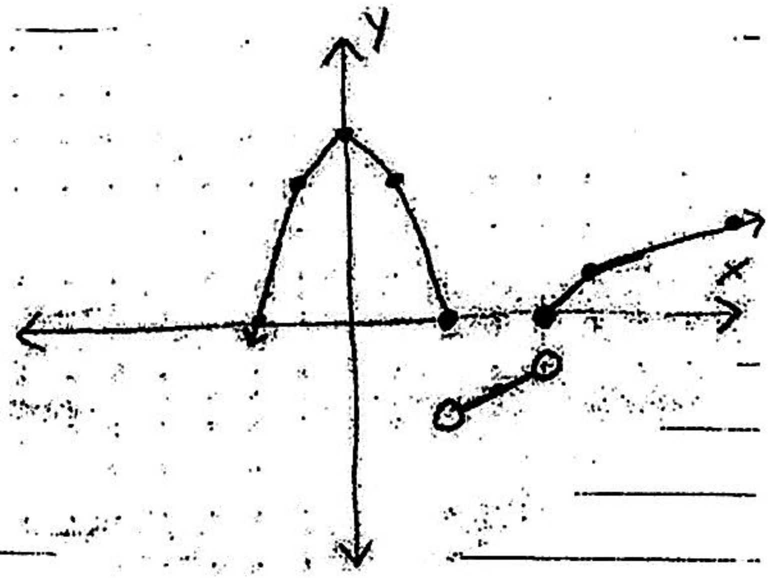
10)

$$f(x)=\begin{cases} -x^2+4 & x\leq 2 \\ \frac{1}{2}x-3 & 2<x<4 \\ \sqrt{x-4} & x\geq 4 \end{cases}$$

	$-x^2+4$	
closed	2	0
	1	3
	0	4
	-1	3
	-2	0

	$\frac{1}{2}x-3$	
open	2	-2
	3	$-\frac{3}{2}$
open	4	-1

	$\sqrt{x-4}$	
closed	4	0
	5	1
	8	2



D: $(-\infty,\infty)$

R: $(-\infty,\infty)$

11)

$f(-2)=0$

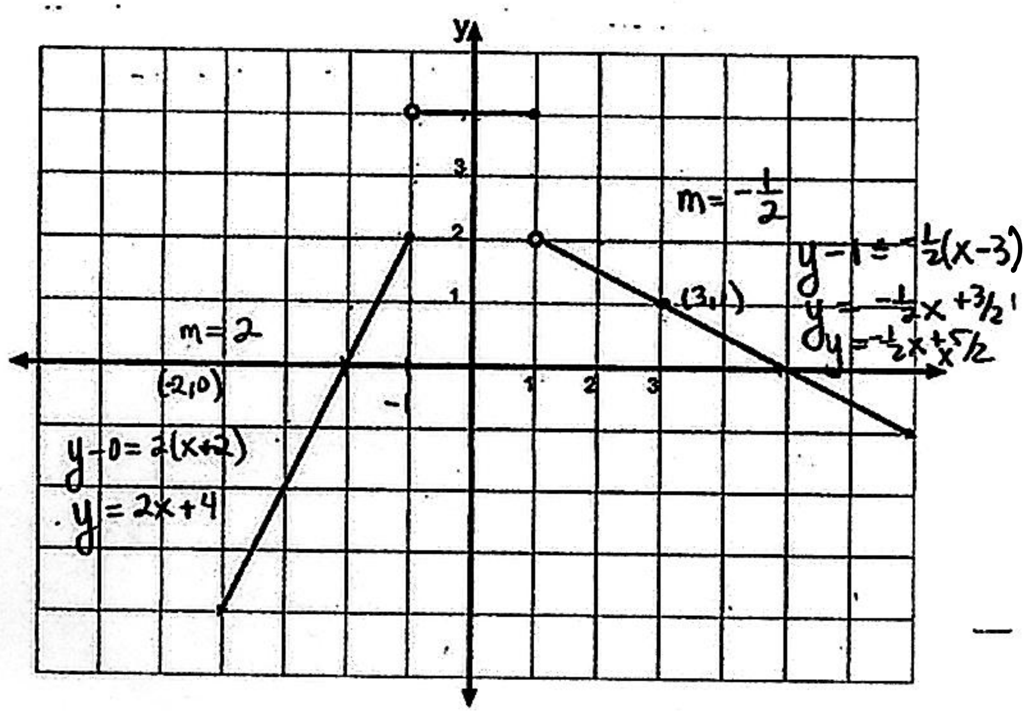
$f(-1)=2$

$f(0)=4$

$f(1)=4$

$f(3)=1$

$f(5)=0$



$$f(x)=\begin{cases} 2x+4 & x\leq -1 \\ 4 & -1<x\leq 1 \\ -\frac{1}{2}x+\frac{5}{2} & x>1 \end{cases}$$

Intercepts work for #3b and 6a

3b) $f(x) = 3-(x+2)^2$

x-int let $y=0$

$0 = 3-(x+2)^2$

$-3 = -(x+2)^2$

$3 = (x+2)^2$

$\pm\sqrt{3} = x+2$

$-2\pm\sqrt{3} = x$

$(-2\pm\sqrt{3}, 0)$

y-int let $x=0$

$y = 3-(0+2)^2$

$y = 3-4 = -1$

$(0,-1)$

3c) $f(x) = (x+2)^2-10$

let $y=0$ to find x-int

$0 = (x+2)^2-10$

$+10 = (x+2)^2$

$\pm\sqrt{10} = x+2$

$-2\pm\sqrt{10} = x$

$(-2\pm\sqrt{10}, 0)$

y-int let $x=0$

$y = (0+2)^2-10$

$y = 4-10 = -6$

$(0,-6)$

6 a) x-int:

let $y=0$

$(-2+\sqrt[3]{3}, 0)$

$0 = (x+2)^3-3$

$3 = (x+2)^3$

$\sqrt[3]{3} = x+2$

$x = -2+\sqrt[3]{3}$

y int:

let $x=0$

$y = (0+2)^3-3$

$y = 2^3-3 = 8-3 = 5$

y int: $(0,5)$

(12) (a) $f(-x) = -3(-x)^2 + 4 = -3x^2 + 4$ EVEN

(b) $f(-x) = 2(-x)^3 - 4(-x) = -2x^3 + 4x$ ODD

(c) $f(-x) = \dfrac{-x}{(-x)^2 - 1} = \dfrac{-x}{x^2 - 1} = -1 \cdot \dfrac{x}{x^2 - 1}$ ODD

(d) $f(x) = x^2 - 4x + 4 + 1 = x^2 - 4x + 5$

$f(-x) = (-x)^2 - 4(-x) + 5 = x^2 + 4x + 5$ NEITHER

(13) a) $g(x) = 3\sqrt{x+2} - 4$

b) $h(x) = \left(-\frac{1}{2}x\right)^3$

(14)

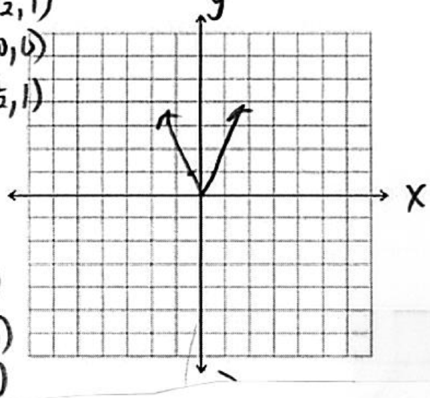
a. Original $f(x) = |x|$

horizontal shrink by a factor of $\frac{1}{2}$

mult. x's by $\frac{1}{2}$

$f(x) = |2x|$

$\lvert x \rvert$	
(-1, 1)	$(-\frac{1}{2}, 1)$
(0, 0)	(0, 0)
(1, 1)	$(\frac{1}{2}, 1)$



D: $(-\infty, \infty)$
R: $[0, \infty)$
x-int: (0, 0)
y-int: (0, 0)

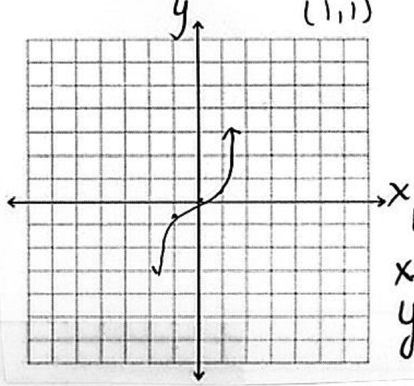
b. Original $f(x) = x^3$

vertical shrink by a factor of $\frac{1}{2}$

mult. y's by $\frac{1}{2}$

$f(x) = \frac{1}{2}x^3$

x^3	
(-1, -1)	(-1, -1/2)
(0, 0)	(0, 0)
(1, 1)	(1, 1/2)



D: $(-\infty, \infty)$
R: $(-\infty, \infty)$
x-int: (0, 0)
y-int: (0, 0)

15

a)
$$\begin{array}{r|rrr} \frac{5}{3} & 6 & -7 & -5 \\ & & 10 & 5 \\ \hline & 6 & 3 & 0 \end{array}$$

$\underbrace{6 \quad 3}_{\div 3}$

$2x + 1$

b)
$$\begin{array}{r|rrr} 2 & 2 & 13 & -8 \\ & & 4 & 34 \\ \hline & 2 & 17 & 26 \end{array}$$

$2x + 17 + \frac{26}{x-2}$

c)
$$\begin{array}{r|rrrrrr} -2 & 5 & 0 & 3 & 0 & 0 & 1 \\ & & -10 & 20 & -46 & 92 & -184 \\ \hline & 5 & -10 & 23 & -46 & 92 & -183 \end{array}$$

$5x^4 - 10x^3 + 23x^2 - 46x + 92 - \frac{183}{x+2}$

d)
$$\begin{array}{r|l} & 7x - 2 \\ \hline x-3 & 7x^2 - 23x + 6 \\ & \underline{7x^2 - 21x} \\ & \quad -2x + 6 \\ & \quad \underline{-2x + 6} \\ & \qquad 0 \end{array}$$

e)
$$\begin{array}{r|l} & 2x^2 - 2x + 5 \\ \hline x^2 + x - 3 & 2x^4 \qquad - 3x^2 + 7x - 8 \\ & \underline{2x^4 + 2x^3 - 6x^2} \\ & \quad -2x^3 + 3x^2 + 7x \\ & \quad \underline{2x^3 - 2x^2 + 6x} \\ & \qquad 5x^2 + x - 8 \\ & \qquad \underline{5x^2 + 5x - 15} \\ & \qquad\quad -4x + 7 \end{array}$$

$2x^2 - 2x + 5 + \frac{-4x+7}{x^2+x-3}$

or

$2x^2 - 2x + 5 - \frac{4x-7}{x^2+x-3}$

or

$$\begin{array}{r|rrrrr} & 2 & 0 & -3 & 7 & -8 \\ -1 & & -2 & 2 & -5 & \\ 3 & & & 6 & -6 & 15 \\ \hline & 2 & -2 & 5 & -4 & 7 \end{array}$$

$2x^2 - 2x + 5 - \frac{4x-7}{x^2+x-3}$