

Name: \_\_\_\_\_  
AP Calculus AB L'Hôpital's Rule

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Do Now:

Evaluate each of the following

1.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

2.  $\lim_{x \rightarrow 1} \frac{x - 1}{\sqrt{x} - 1}$

3.  $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x + 2}$

### L'Hôpital's Rule

Let  $f$  and  $g$  be differentiable functions on some interval containing  $a$  (except possibly at  $a$ ) such that  $g'(x) \neq 0$  when  $x \neq a$ . Then, if  $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$  (or  $\pm \infty$ ), then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}, \text{ provided that } \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \text{ exists or is } \pm \infty.$$

Note:

- L'Hôpital's Rule also works if  $x \rightarrow \pm \infty$
- Several consecutive applications of L'Hôpital's Rule, such as  $\lim_{x \rightarrow a} \frac{f''(x)}{g''(x)}$ , may be needed to evaluate a limit.
- You may NOT use L'Hôpital's Rule without  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ .
- There is no guarantee that L'Hôpital's Rule will help. L'Hôpital's Rule helps only if we eventually obtain a limit which exists or is infinite.

Evaluate each limit. Use L'Hôpital's Rule where appropriate.

$$1. \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^2 - 4}$$

$$11. \lim_{x \rightarrow \infty} \frac{\log_2 x}{\log_3 x}$$

$$2. \lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

$$12. \lim_{x \rightarrow 0} \frac{e^{2x} - 1}{\tan x}$$

$$3. \lim_{x \rightarrow 2} \frac{\sqrt{2+x} - 2}{x - 2}$$

$$13. \lim_{x \rightarrow 0} \frac{\arctan x}{2x}$$

$$4. \lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - 1}{x - 1}$$

$$14. \lim_{x \rightarrow \pi^+} \frac{2x - 2\pi}{\sin(x - \pi)}$$

$$5. \lim_{x \rightarrow 2} \frac{x^2 - 4x + 4}{x^3 - 12x + 16}$$

$$15. \lim_{x \rightarrow 0} \pi^2 \frac{\tan 2x}{x \cos 2x}$$

$$6. \lim_{x \rightarrow \frac{\pi}{2}} \frac{1 - \sin x}{1 + \cos 2x}$$

$$16. \lim_{x \rightarrow 0} \frac{e^x - e^{-x}}{x}$$

$$7. \lim_{x \rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3}$$

$$17. \lim_{x \rightarrow 1} \frac{2 \ln x}{x - 1}$$

$$8. \lim_{x \rightarrow 3} \frac{x - 4}{x - 2}$$

$$18. \lim_{x \rightarrow 0} \frac{3(e^x - e^{-x})}{\sin x}$$

$$9. \lim_{x \rightarrow 0} \frac{x}{\tan x}$$

$$19. \lim_{x \rightarrow 0} \frac{2x^2}{e^x - 1 - x}$$

$$10. \lim_{x \rightarrow 1} \frac{1 - \frac{1}{x}}{1 - \frac{1}{x^2}}$$

$$20. \lim_{x \rightarrow \infty} \frac{e^{2x}}{2x^2}$$