$f'(c) = \lim_{x \to 0} \frac{f(c+h) - f(c)}{h}$

Midterm Review Packet Key

1. (B) (for (D) pt of inflection would make $f''(x) = 0$.) 2. (A) 3. (B)

4. $f'(1)$ for $f(x) = x^6$, $f'(x) = 6x^5$, $f'(1) = 6$ (C)

5. $f'(8)$ for $f(x) = x^{1/3}$, $f'(x) = \frac{1}{3\sqrt[3]{x^2}}$, $f'(8) = \frac{1}{3\sqrt[3]{8^2}} = \frac{1}{12}$ (B)

6. $f'(e)$ for $f(x) = \ln x$, $f'(x) = \frac{1}{x}$, $f'(e) = \frac{1}{e}$ (B)

7. $\lim_{x \to 0} \frac{\cos(0+x) - \cos 0}{x} = f'(0)$ for $f(x) = \cos x$, $f'(x) = -\sin x$, $f'(0) = 0$ (B)

8.

(a)

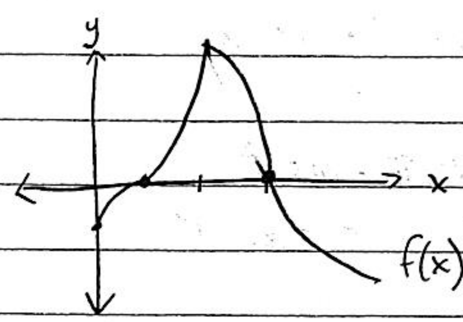
0		1		2		3		4	
	+		+		−		−		$f'(x)$
	↗		↗		↘		↘		$f(x)$

f has a relative maximum at $x = 2$ b/c f' changes from ⊕ to ⊖ at $x = 2$

(b)

0		1		2		3		4	
	−		+		−		+		f''
	∩		∪		∩		∪		$f(x)$

pt of infl. (at 1), pt of in (at 2), pt of inf. (at 3)



9. $y^2 = 2 + xy$

(a) $2y\, dy/dx = x\, dy/dx + y$

$dy/dx(2y - x) = y$

$dy/dx = \frac{y}{2y - x}$

(b) $\frac{y}{2y-x} = \frac{1}{2}$

$2y = 2y - x$

$0 = -x$

$x = 0$

If $x = 0$, $y^2 = 2$, $y = \pm\sqrt{2}$ so $(0, \pm\sqrt{2})$

(c) $\frac{y}{2y-x} \neq 0$

$y = 0$

$0^2 \neq 2 + 0$ so the curve does not have a horizontal tangent.

$y^2 = 2 + xy$

or $2y\frac{dy}{dt} = x\frac{dy}{dt} + y\frac{dx}{dt} \Rightarrow 2(3)(6) = \frac{7}{3}(6) + 3\frac{dx}{dt}$

$36 = 14 + 3\frac{dx}{dt}$

when $y = 3$, $3^2 = 2 + 3x$ $\qquad 22 = 3\frac{dx}{dt}$ $\qquad$ (2)

$7 = 3x$, $x = 7/3$ $\qquad \frac{22}{3} = \frac{dx}{dt}$

(d) $dy/dt = dy/dx \cdot dx/dt$

$6 = \frac{y}{2y-x} \cdot \frac{dx}{dt}$

At $t=5$

$6 = \frac{3}{6-7/3} \cdot dx/dt$

$6 = \frac{3}{11/3} \cdot dx/dt$

$6 = 9/11 \, dx/dt$

$22/3 = dx/dt$

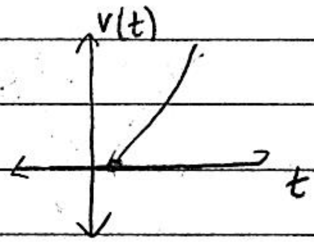
(10) (D)

(11) (A)

(12) (a) $\frac{x(3)-x(1)}{3-1} = \frac{e^3 - \sqrt{3} - (e-1)}{2} = \frac{e^3 - e - \sqrt{3} + 1}{2} = 8.318$ ft/sec

(b) $v(t) = e^t - \frac{1}{2\sqrt{t}}$, $v(1) = e - \frac{1}{2\sqrt{1}} = e - \frac{1}{2} = 2.218$ ft/sec to the right

(c)



particle moves to the right when $v(t) > 0$

$t > 0.176$

(d) velocity is zero when $t = 0.17586786\ldots$

$x(0.17586786\ldots) = 0.773$ ft

(13) $x = -.3906462 \approx -0.391$ (C)

(14) $f'(x) = 0$ or dne Three (B)

(15) $f'(x) = 4x^3 + 4x$ $\quad f'(x) = 1$

$x \approx .237$ $\qquad$ pt $(.237, \overset{f(.237)}{.115})$

$y - .115 = 1(x - .237)$ $\qquad$ (D)

$y = x - .237 + .115 \Rightarrow y = x - .122$

(16) $f(x) = \cos(2x) + \ln(3x)$ $\quad$ NDeriv twice

$f''(x) = 0$ $\qquad x \approx .932$ (B)

(17) $f(x) = \sqrt[5]{x^3 - 2x}$

$f'(\sqrt{3}) = .90215$ (B)

18 $f(x)=5x^3+x$ $\quad g(x)=f^{-1}(x)$, find $g'(6)$

$$5x^3+x=6$$
$$x=1$$

$$f'(x)=15x^2+1$$
$$f'(1)=15(1)^2+1=16$$

$$g'(6)=\frac{1}{f'(1)}=\frac{1}{16}$$

19) $f(x) = \begin{cases} 2x+3, & x \le 4 \\ 7 + \frac{16}{x}, & x > 4 \end{cases}$

$2(4)+3 = 11$ no points of

$7 + \frac{16}{4} = 11$ discontinuity

20) (a) $f(x) = \begin{cases} 7x-2, & x \le 1 \\ kx^2, & x > 1 \end{cases}$

$7(1)-2 = k(1)^2$

$5 = k$

(b) $f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x+k, & x > 2 \end{cases}$

$k(2)^2 = 2(2)+k$

$4k = 4+k$

$3k = 4$

$k = 4/3$

21) $\lim_{x \to +\infty} \sin\left(\frac{\pi x}{2-3x}\right) = \sin(-\pi/3) = -\sqrt{3}/2$

22) $\lim_{\theta \to 0} \frac{\sin 3\theta}{\theta} = 3$

23) $\lim_{x \to 0} \frac{\tan 7x}{\sin 3x} = 7/3$

24) $\lim_{\theta \to 0} \frac{\theta^2}{1-\cos\theta} \cdot \frac{1+\cos\theta}{1+\cos\theta} = \lim_{\theta \to 0} \frac{\theta^2(1+\cos\theta)}{\sin^2\theta} = 2$

$1-\cos^2\theta$

$\lim_{\theta \to 0} \left(\frac{\theta}{\sin\theta} \cdot \frac{\theta}{\sin\theta} \cdot 1+\cos\theta\right)$

$1 \cdot 1 \cdot 2$

25) $f(x) = \begin{cases} \frac{\tan kx}{x} & x < 0 \\ 3x+2k^2 & x \ge 0 \end{cases}$

Find a nonzero value.

$\lim_{x \to 0^-} \frac{\tan kx}{x} = k$

$\lim_{x \to 0^+} 3x+2k^2 = 2k^2$

$2k^2 = k$

$2k^2 - k = 0$

$k(2k-1) = 0$

$\cancel{k=0}$, $k = 1/2$

26) (a) $\frac{s(3)-s(1)}{3-1} = \frac{30-4}{2} = \frac{26}{2} = 13$ mph $= v_{avg}[1,3]$

(b) $s'(t) = 6t+1$, $s'(1) = 6(1)+1 = 7$ mph

(c) $6t+1 = 13$, $6t = 12$, $t = 2$ hrs.

27) (a) D (d) C

(b) F (e) A

(c) B (f) E

28) (a) $g'(2)$ $g(x) = [f(x)]^3$

$g'(x) = 3(f(x))^2 \cdot f'(x)$

$g'(2) = 3(f(2))^2 \cdot f'(2)$

$= 3(1)^2 \cdot 7$

$= 21$

(b) $h'(2)$, $h(x) = f(x^3)$

$h'(x) = f'(x^3) \cdot 3x^2$

$h'(2) = f'(8) \cdot 3(2)^2$

$-3 \cdot 12$

-36