29) $f(x) = \tan(4x^2)$

$f'(x) = \sec^2(4x^2) \cdot 8x$

$f'(x) = 8x\sec^2(4x^2)$

30) $f(x) = \sin\left(\frac{1}{x^2}\right)$

$f'(x) = \cos\left(\frac{1}{x^2}\right) \cdot \left(-\frac{2}{x^3}\right)$

31)

	dy/dx	d^2y/dx^2
A	−	+
B	+	−
C	−	−

32) $x = -1, 0, 1, 2$ b/c f'' changes sign

33) (a) $<$ (b) $>$ (c) $>$ $f'(1)$ is above x-axis

(d) $=$ (e) $<$ (f) $\geq$ slope of $f'(2) = 0$

34) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$f''(x) = 36x^2 - 24x$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$0 \mid x = 1$

$f'(x)$: − (0) − (1) +

$f(x)$: ↘ ↘ ↗

$36x^2 - 24x = 0$

$12x(3x-2) = 0$

$x = 0 \mid x = 2/3$

f'': + (0) − (2/3) +

f: ∪ ∩ ∪

(a) increasing on $(1, \infty)$

(b) decreasing $(-\infty, 1)$

(c) concave ↑: $(-\infty, 0), (2/3, \infty)$

(d) concave ↓: $(0, 2/3)$

(e) infl. pts: $x = 0, 2/3$

35) $f(x) = e^{-x^2/2}$

$f'(x) = e^{-x^2/2} \cdot \left(-\frac{1}{2}(2x)\right)$

$f'(x) = -xe^{-x^2/2}$

$-xe^{-x^2/2} = 0$

$x = 0 \mid e^{-x^2/2} \neq 0$

$f'(x)$: + (0) −

$f(x)$: ↗ ↘

$f''(x) = -e^{-x^2/2} + (-x)e^{-x^2/2} \cdot -x$

$f''(x) = -e^{-x^2/2} + x^2e^{-x^2/2}$

$= e^{-x^2/2}(-1 + x^2)$

$e^{-x^2/2} \neq 0 \mid x^2 - 1 = 0 \quad x = \pm 1$

f'': + (−1) − (1) +

f: ∪ ∩ ∪

(a) inc. $(-\infty, 0)$

(b) dec. $(0, \infty)$

(c) concave ↑ $(-\infty, -1), (1, \infty)$

(d) concave ↓ $(-1, 1)$

(e) infl. pts $x = -1, 1$

$$* \quad \frac{d}{dx}[\sin^{-1}u] = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \frac{d}{dx}[\cos^{-1}u] = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}[\tan^{-1}u] = \frac{1}{1+u^2}\frac{du}{dx}$$

6

(36) $y = \sin^{-1}(\frac{1}{3}x)$

$$y' = \frac{1}{\sqrt{1-\frac{1}{9}x^2}} \cdot \frac{1}{3} = \frac{1}{3\sqrt{\frac{9-x^2}{9}}} = \frac{1}{\sqrt{9-x^2}}$$

(37) $f(x) = \sqrt{x+1}$; $[0,3]$

$f'(x) = \frac{1}{2\sqrt{x+1}}$ cont. on $[0,3]$ diff on $(0,3)$

$$\frac{f(3)-f(0)}{3-0} = \frac{1}{2\sqrt{c+1}}$$

$$\frac{2-1}{3} = \frac{1}{2\sqrt{c+1}}$$

$2\sqrt{c+1} = 3$

$\sqrt{c+1} = 3/2$

$c+1 = 9/4$

$c = 5/4$

(38) $y = \sqrt{\ln x} = (\ln x)^{1/2}$

$y' = 1/2(\ln x)^{-1/2} \cdot \frac{1}{x}$

$y' = \frac{1}{2x\sqrt{\ln x}}$

$$y'(e^5) = \frac{1}{2e^5\sqrt{\ln e^5}} = \frac{1}{2e^5\sqrt{5}}$$

(39) $y = e^{7x}$

$y' = e^{7x} \cdot 7 = 7e^{7x}$

$y'(\ln 5) = 7e^{7\ln 5} = 7 \cdot 5^7$

(40) $f(x) = \pi^{\sin x}$

$f'(x) = \pi^{\sin x} \cdot \ln\pi \cdot \cos x$

$f'(\pi) = \pi^{\overset{0}{\sin\pi}} \cdot \ln\pi \cdot \cos\pi$

$= 1 \cdot \ln\pi \cdot -1 = -\ln\pi$

(41) $f(x) = \sin x - \cos x$ $[0,\pi]$

$f'(x) = \cos x + \sin x$

$\cos x + \sin x = 0$

$\cos x = -\sin x$

$x = 3\pi/4$

x	f(x)
0	-1
$3\pi/4$	$\sqrt{2}$
π	1

abs. max is $\sqrt{2}$ at $x = 3\pi/4$

abs min is -1 at $x = 0$

(42) $A = \pi r^2$

$dA/dt = \pi(2r\, dr/dt)$

$6 = \pi \cdot 2(3/\sqrt{\pi})\, dr/dt$

$6 = 6\sqrt{\pi}\, dr/dt$

$dr/dt = \frac{1}{\sqrt{\pi}}$ mph

$9 = \pi r^2$

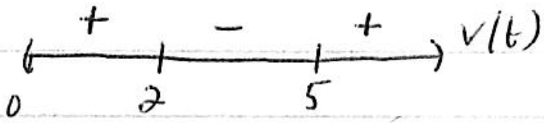
$9/\pi = r^2$

$3/\sqrt{\pi} = r$

(43) $x(t) = 2t^3 - 21t^2 + 60t - 50$

a) $v(t) = 0$

$v(t) = 6t^2 - 42t + 60$

b) 

$6t^2 - 42t + 60 = 0$

$6(t^2 - 7t + 10) = 0$

$6(t-5)(t-2) = 0$

$t = 5, 2$ s

b) Particle is moving right: $[0,2), (5,\infty)$
b/c $v(t) > 0$ in those intervals

c) Particle is moving left: $(2,5)$
b/c $v(t) < 0$ in that interval

d) Need to find max/min velocity first

$v'(t) = a(t) = 12t - 42$

$12t - 42 = 0$

$12t = 42$

$t = \frac{42}{12} = \frac{21}{6}$ or $\frac{7}{2}$

$v(1) = 6(1-5)(1-2) = 6(-4)(-1) = 24$

$v(\frac{7}{2}) = 6(\frac{7}{2}-5)(\frac{7}{2}-2) = \overset{3}{\cancel{6}}(\frac{-3}{\cancel{2}})(\frac{3}{2}) = \frac{-27}{2} = -13.5$

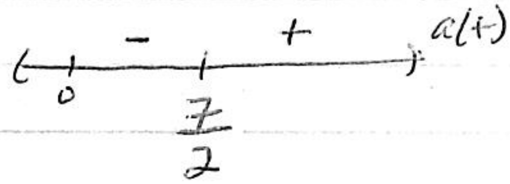
$v(6) = 6(6-5)(6-2) = 6(1)(4) = 24$

Speed is $|v(t)|$. Max speed is 24 ft/s

e) Velocity is increasing where acceleration is +

$a(t) = 12t - 42$

$a(t) = 0$ when $t = \frac{21}{6} = \frac{7}{2}$



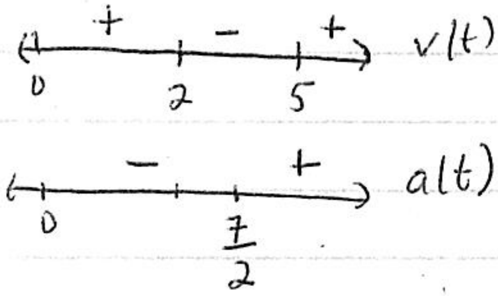
So velocity is increasing $(\frac{7}{2}, \infty)$

f) $x(1) = -9$
$x(2) = 2$
$x(5) = -25$
$x(6) = -14$

11
27
11

TD: $11 + 27 + 11 = 49$ ft

g)



speeding up: $(2,\frac{7}{2}), (5,\infty)$ b/c $a(t)$ and $v(t)$ have same sign

h) slowing down: $[0,2), (7/2,5)$ b/c $a(t)$ and $v(t)$ have opposite signs