

Name: _____
PCH: More General Solutions to Trig Equations

Date: _____
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Do Now:

Find all solutions of the equation.

1. $2 \cos 2x + 1 = 0$

2. $\sec 4x - 2 = 0$

Classwork:

Find all solutions of each equation.

1. $\csc 3x = \sin 3x$

2. $2 \sin \frac{x}{3} + \sqrt{3} = 0$

For this section, find:

(a) all solutions of the equation.

(b) all solutions of the equation in the interval $(0, 2\pi]$.

3. $3\sec^2 x + 4\cos^2 x = 7$

4. $\tan x - 3\cot x = 0$

5. $\sec x \tan x - \cos x \cot x = \sin x$

From your textbook:

7.5 Exercises

1–40 ■ Find all solutions of the equation.

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| 1. $\cos x + 1 = 0$ | 2. $\sin x + 1 = 0$ |
| 3. $2 \sin x - 1 = 0$ | 4. $\sqrt{2} \cos x - 1 = 0$ |
| 5. $\sqrt{3} \tan x + 1 = 0$ | 6. $\cot x + 1 = 0$ |
| 7. $4 \cos^2 x - 1 = 0$ | 8. $2 \cos^2 x - 1 = 0$ |
| 9. $\sec^2 x - 2 = 0$ | 10. $\csc^2 x - 4 = 0$ |

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| 21. $\sin^2 x = 2 \sin x + 3$ | 22. $3 \tan^3 x = \tan x$ |
| 23. $\sin^2 x = 4 - 2 \cos^2 x$ | 24. $2 \cos^2 x + \sin x = 1$ |
| 25. $2 \sin 3x + 1 = 0$ | 26. $2 \cos 2x + 1 = 0$ |
| 27. $\sec 4x - 2 = 0$ | 28. $\sqrt{3} \tan 3x + 1 = 0$ |
| 29. $\sqrt{3} \sin 2x = \cos 2x$ | 30. $\cos 3x = \sin 3x$ |
| 31. $\cos \frac{x}{2} - 1 = 0$ | 32. $2 \sin \frac{x}{3} + \sqrt{3} = 0$ |
| 33. $\tan \frac{x}{4} + \sqrt{3} = 0$ | 34. $\sec \frac{x}{2} = \cos \frac{x}{2}$ |
| 35. $\tan^5 x - 9 \tan x = 0$ | |
| 36. $3 \tan^3 x - 3 \tan^2 x - \tan x + 1 = 0$ | |
| 37. $4 \sin x \cos x + 2 \sin x - 2 \cos x - 1 = 0$ | |
| 38. $\sin 2x = 2 \tan 2x$ | 39. $\cos^2 2x - \sin^2 2x = 0$ |
| 40. $\sec x - \tan x = \cos x$ | |

41–48 ■ Find all solutions of the equation in the interval $[0, 2\pi)$.

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| 41. $2 \cos 3x = 1$ | 42. $3 \csc^2 x = 4$ |
| 43. $2 \sin x \tan x - \tan x = 1 - 2 \sin x$ | |
| 44. $\sec x \tan x - \cos x \cot x = \sin x$ | |
| 45. $\tan x - 3 \cot x = 0$ | 46. $2 \sin^2 x - \cos x = 1$ |
| 47. $\tan 3x + 1 = \sec 3x$ | 48. $3 \sec^2 x + 4 \cos^2 x = 7$ |

49–56 ■ (a) Find all solutions of the equation. (b) Use a calculator to solve the equation in the interval $[0, 2\pi)$, correct to five decimal places.

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| 49. $\cos x = 0.4$ | 50. $2 \tan x = 13$ |
| 51. $\sec x - 5 = 0$ | 52. $3 \sin x = 7 \cos x$ |
| 53. $5 \sin^2 x - 1 = 0$ | 54. $2 \sin 2x - \cos x = 0$ |
| 55. $3 \sin^2 x - 7 \sin x + 2 = 0$ | |
| 56. $\tan^4 x - 13 \tan^2 x + 36 = 0$ | |

57–60 ■ Graph f and g on the same axes, and find their points of intersection.

57. $f(x) = 3 \cos x + 1$, $g(x) = \cos x - 1$

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| 11. $3 \csc^2 x - 4 = 0$ | 12. $1 - \tan^2 x = 0$ |
| 13. $\cos x (2 \sin x + 1) = 0$ | 14. $\sec x (2 \cos x - \sqrt{2}) = 0$ |
| 15. $(\tan x + \sqrt{3})(\cos x + 2) = 0$ | |
| 16. $(2 \cos x + \sqrt{3})(2 \sin x - 1) = 0$ | |
| 17. $\cos x \sin x - 2 \cos x = 0$ | 18. $\tan x \sin x + \sin x = 0$ |
| 19. $4 \cos^2 x - 4 \cos x + 1 = 0$ | 20. $2 \sin^2 x - \sin x - 1 = 0$ |

63. $\sin 2x \cos x + \cos 2x \sin x = \sqrt{3}/2$

64. $\sin 3x \cos x - \cos 3x \sin x = 0$

65–68 ■ Use a double- or half-angle formula to solve the equation in the interval $[0, 2\pi)$.

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| 65. $\sin 2x + \cos x = 0$ | 66. $\tan \frac{x}{2} - \sin x = 0$ |
| 67. $\cos 2x + \cos x = 2$ | 68. $\tan x + \cot x = 4 \sin 2x$ |

69–72 ■ Solve the equation by first using a sum-to-product formula.

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| 69. $\sin x + \sin 3x = 0$ | 70. $\cos 5x - \cos 7x = 0$ |
| 71. $\cos 4x + \cos 2x = \cos x$ | 72. $\sin 5x - \sin 3x = \cos 4x$ |

73–78 ■ Use a graphing device to find the solutions of the equation, correct to two decimal places.

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| 73. $\sin 2x = x$ | 74. $\cos x = \frac{x}{3}$ |
| 75. $2^{\sin x} = x$ | 76. $\sin x = x^3$ |

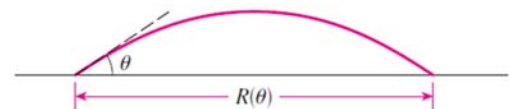
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| 77. $\frac{\cos x}{1 + x^2} = x^2$ | 78. $\cos x = \frac{1}{2}(e^x + e^{-x})$ |
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Applications

79. **Range of a Projectile** If a projectile is fired with velocity v_0 at an angle θ , then its *range*, the horizontal distance it travels (in feet), is modeled by the function

$$R(\theta) = \frac{v_0^2 \sin 2\theta}{32}$$

(See page 818.) If $v_0 = 2200$ ft/s, what angle (in degrees) should be chosen for the projectile to hit a target on the ground 5000 ft away?



58. $f(x) = \sin 2x$, $g(x) = 2 \sin 2x + 1$

59. $f(x) = \tan x$, $g(x) = \sqrt{3}$

60. $f(x) = \sin x - 1$, $g(x) = \cos x$

61–64 ■ Use an addition or subtraction formula to simplify the equation. Then find all solutions in the interval $[0, 2\pi)$.

61. $\cos x \cos 3x - \sin x \sin 3x = 0$

62. $\cos x \cos 2x + \sin x \sin 2x = \frac{1}{2}$

80. **Damped Vibrations** The displacement of a spring vibrating in damped harmonic motion is given by

$$y = 4e^{-3t} \sin 2\pi t$$

Find the times when the spring is at its equilibrium position ($y = 0$).

81. **Refraction of Light** It has been observed since ancient times that light refracts or “bends” as it travels from one medium to another (from air to water, for example). If v_1 is