

What You Need to Know About Motion Along the x -axis (Part 1)

In discussing motion, there are three closely related concepts that you need to keep straight. These are:

If $x(t)$ represents the position of a particle along the x -axis at any time t , then the following statements are true.

1. "Initially" means when _____ = 0.
2. "At the origin" means _____ = 0.
3. "At rest" means _____ = 0.
4. If the velocity of the particle is positive, then the particle is moving to the _____.
5. If the velocity of the particle is _____, then the particle is moving to the left.
6. To find average velocity over a time interval, divide the change in _____ by the change in time.
7. Instantaneous velocity is the velocity at a single moment (instant!) in time.
8. If the acceleration of the particle is positive, then the _____ is increasing.
9. If the acceleration of the particle is _____, then the velocity is decreasing.
10. In order for a particle to change direction, the _____ must change signs.
11. One way to determine total distance traveled over a time interval is to find the sum of the absolute values of the differences in position between all resting points. Here's an example: If the position of a particle is given by:

$$x(t) = \frac{1}{3}t^3 - t^2 - 3t + 4,$$

find the total distance traveled on the interval $0 \leq t \leq 6$.

Curriculum Module: Calculus: Motion

Example 1 (analytic).

A particle moves along the x -axis so that at time t its position is given by:

$$x(t) = t^3 - 6t^2 + 9t + 11$$

1. At $t = 0$, is the particle moving to the right or to the left? Explain your answer.
2. At $t = 1$, is the velocity of the particle increasing or decreasing? Explain your answer.
3. Find all values of t for which the particle is moving to the left.
4. Find the total distance traveled by the particle over the time interval $0 \leq t \leq 5$.

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Example 2 (analytic)

A particle is moving along a horizontal line according to the function:

$$S(t) = \frac{t^3}{3} - 3t^2 + 8t + 4 \quad \text{for } t \geq 0$$

where t is time in seconds and s is measured in feet.

- a) At $t = 1$, is the particle moving to the right or left? Explain.

- b) During what time interval(s) is the particle moving to the left? Explain.

- c) What is the average velocity from $t=0$ to $t=3$ sec?

- d) Is the particle speeding up or slowing down at $t=1$? Explain.

- e) What is the total distance traveled from $t=0$ to $t=4$ sec?