Midterm Review Packet Key

1. $f(x-3) = (x-3)^3 + 3(x-3) - 2$

$= x^3 - 9x^2 + 27x - 27 + 3x - 9 - 2$

$= x^3 - 9x^2 + 30x - 38$

$(x-3)(x-3)(x-3)$

$(x^2 - 6x + 9)(x-3)$

$x^3 - 6x^2 + 9x - 3x^2 + 18x - 27$

$x^3 - 9x^2 + 27x - 27$

2. $(f \circ h \circ g)(x)$

$g(x) = 2x^2$

$h(2x^2) = \sqrt{2x^2 - 9}$

$f(\sqrt{2x^2-9}) = \sqrt{2x^2-9} + 3$

3. $\dfrac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$ $\quad h \neq 0$

$\dfrac{\cancel{x^2} + 2xh + h^2 \cancel{- 2x} - 2h \cancel{+ 5} \cancel{- x^2} \cancel{+ 2x} \cancel{- 5}}{h} = \dfrac{2xh + h^2 - 2h}{h} = 2x + h - 2, \; h \neq 0$

4. $f(x) = \sqrt{2x+3}$

$y = \sqrt{2x+3}$

$(x)^2 = (\sqrt{2y+3})^2$

$x^2 = 2y + 3$

$\dfrac{x^2 - 3}{2} = \dfrac{2y}{2}$

$\dfrac{x^2-3}{2} = y = f^{-1}(x)$

5. $m = \dfrac{4-2}{4-(-7)} = \dfrac{2}{11}$

point slope $\quad y - 4 = \frac{2}{11}(x-4)$ or $y - 2 = \frac{2}{11}(x+7)$

$y - 4 = \frac{2}{11}x - \frac{8}{11}$

$y = \frac{2}{11}x - \frac{8}{11} + 4$

slope intercept $\quad y = \frac{2}{11}x + \frac{36}{11}$

$11\left(\frac{2}{11}x - y = -\frac{36}{11}\right)$

standard form $\quad 2x - 11y = -36$

6. $f(x) = x + 12$

$g(x) = \sqrt{x}$

$h(x) = \dfrac{12}{x}$ as $h(g(f(x)))$

answers can vary

7. $f(g(x)) = g(f(x)) = x$

$f(x^2-3)$ $\qquad$ $g(\sqrt{x+3})$

$\sqrt{x^2 \cancel{-3} \cancel{+3}}$ $\qquad$ $(\sqrt{x+3})^2 - 3$

$\sqrt{x^2}$ $\qquad$ $x \cancel{+3} \cancel{-3}$

$x = x$ $\qquad$ $\therefore$ f and g are inverses

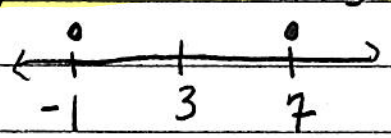
8. $\dfrac{2x^4}{x^3-x^2} = \dfrac{2x^{\cancel{4}\,2}}{\cancel{x^2}(x-1)} = \dfrac{2x^2}{x-1} \quad x \neq 0, 1$

9. $|3-x| = 4$

Remember $|3-x| = |x-3|$

(a) $|x-3| = 4$

x's distance from 3 is 4



$-4 \qquad +4$

$\{-1, 7\}$

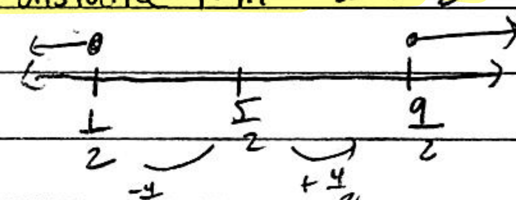
$|5-2x| \geq 4$ $\quad$ * $|5-2x| = |2x-5|$

(b) $|2x-5| \geq 4$

$2\left|x - \frac{5}{2}\right| \geq 4$

$\left|x - \frac{5}{2}\right| \geq \frac{4}{2}$

x's distance from $5/2 \geq \frac{4}{2}$



$\{x \mid x \leq 1/2 \text{ or } x \geq 9/2\}$ set builder

$(-\infty, \frac{1}{2}] \cup [\frac{9}{2}, \infty)$ interval

10. (a) $\dfrac{4-x^{-2}}{2x^{-1}-x^{-2}} = \dfrac{4 - \frac{1}{x^2}}{\frac{2}{x} - \frac{1}{x^2}} \cdot \dfrac{x^2}{x^2}$

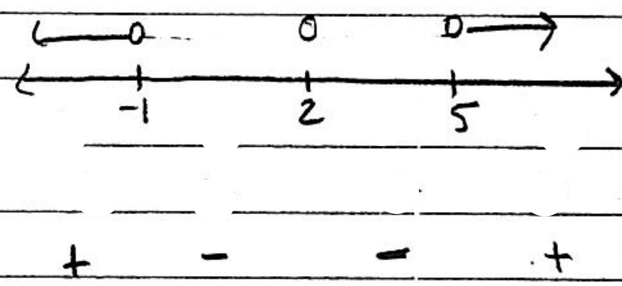
$\dfrac{4x^2-1}{2x-1} = \dfrac{(2x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}} = 2x+1 \quad x \neq 0, \frac{1}{2}$

10 (b) $\dfrac{x^2-xy}{xy+2y^3} \div \dfrac{x^2+xy}{xy+y^2}$

$\dfrac{\cancel{x}(x-y)}{\cancel{y}(x+2y^2)} \cdot \dfrac{\cancel{y}\cancel{(x+y)}}{\cancel{x}\cancel{(x+y)}} = \dfrac{x-y}{x+2y^2}$

$y \neq 0,\ x \neq 0$
$x \neq -2y^2, -y$

11 $\dfrac{\overset{(x-5)(x+1)}{x^2-4x-5}}{(x-2)^2} > 0$

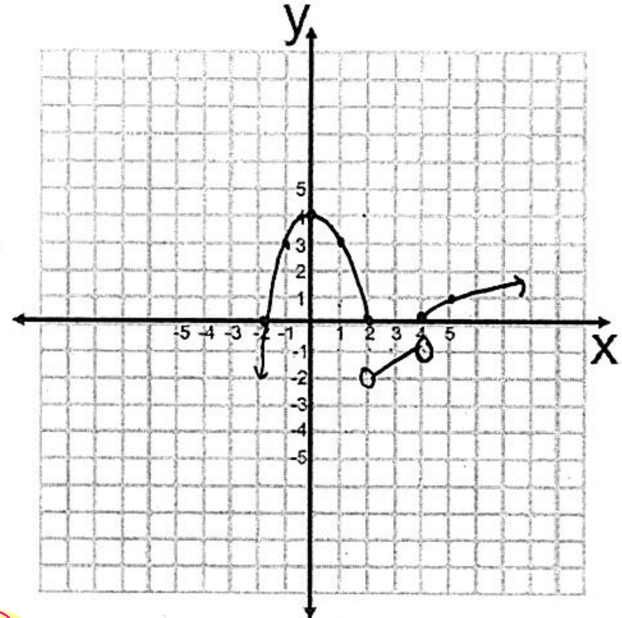


SB (a) $\{x \mid x < -1 \ \vee\ x > 5\}$
IN (b) $(-\infty, -1) \cup (5, \infty)$

12 $f(x) = -x^2 + 4x + 6$
$f(x) = -(x^2 - 4x + 4 - 4 - 6)$
$f(x) = -(x-2)^2 - (-10)$
$f(x) = -(x-2)^2 + 10$

13. Sketch the function **without** using a graphing calculator. Find the domain and range of each function.

a. $f(x) = \begin{cases} -x^2 + 4, & x \le 2 \\ \frac{1}{2}x - 3, & 2 < x < 4 \\ \sqrt{x - 4}, & x \ge 4 \end{cases}$



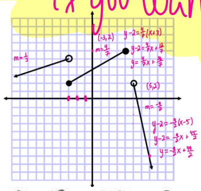
D: $(-\infty, \infty)$

R: $(-\infty, \infty)$

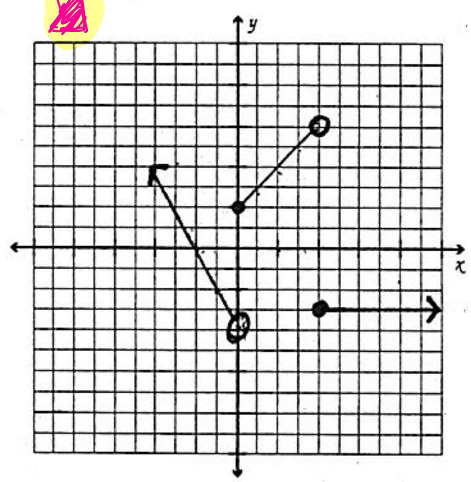
Original #14 but some pts were hard to see, so if you want try this new one

14. Write a piecewise function for the graph

$f(x) = \begin{cases} \frac{1}{3}x + 6 & x < -3 \\ \frac{4}{7}x + \frac{26}{7} & -3 \le x \le 4 \\ \frac{-9}{2}x + \frac{49}{2} & x > 5 \end{cases}$



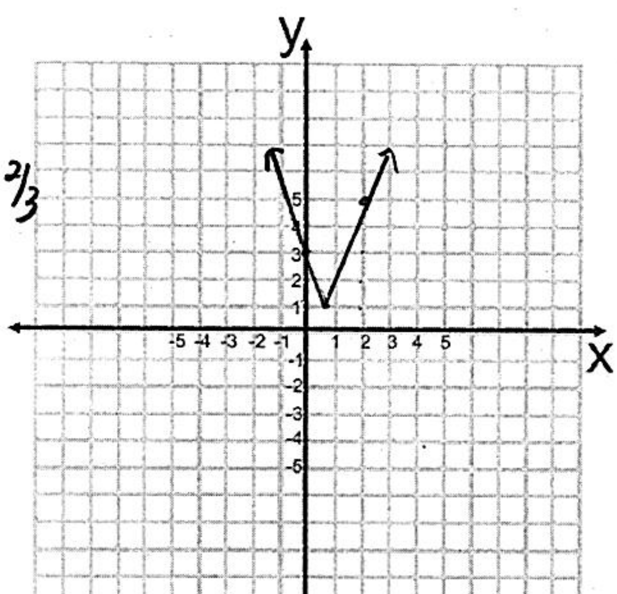
14. Write a piecewise function for the graph



$f(x) = \begin{cases} -2x - 4 & x < 0 \\ x + 2 & 0 \le x < 4 \\ -3 & x \ge 4 \end{cases}$

15. Use the algebraic definition of absolute value to rewrite $f(x) = |3x - 2| + 1$ as a piecewise function and then sketch each graph.

$f(x) = \begin{cases} 3x - 2 + 1 = 3x - 1 & \text{if } 3x - 2 \ge 0,\ x \ge 2/3 \\ -3x + 2 + 1 = -3x + 3 & \text{if } x < 2/3 \end{cases}$



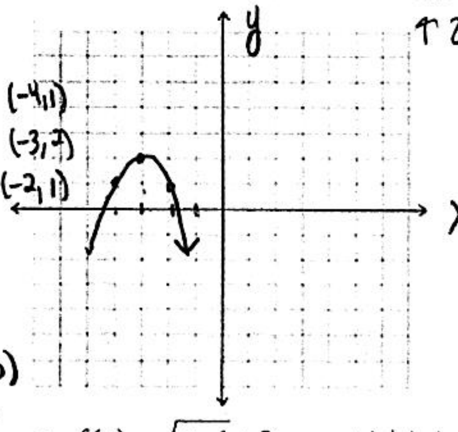
(a) X-int
$0 = 2-(x+3)^2$
$-2 = -(x+3)^2$
$2 = (x+3)^2$
$\pm\sqrt{2} = x+3$
$-3 \pm \sqrt{2} = x$

16. Describe each transformation in terms of the parent function and then graph the function. State the domain, range, and any x- or y- intercepts.

a. $f(x) = 2-(x+3)^2$

left 3
reflect over x
↑2

(-1,1) (-4,1) (-4,-1) (-4,1)
(0,0) (-3,0) (-3,0) (-3,2)
(1,1) (-2,1) (-2,-1) (-2,1)

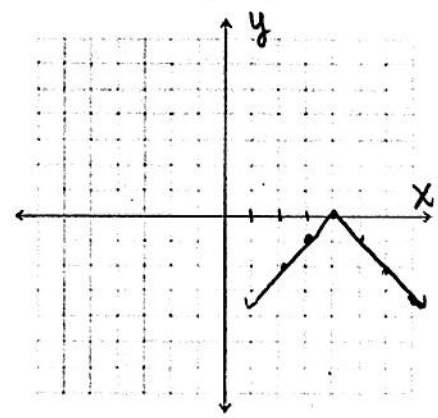


D: $(-\infty, \infty)$
R: $(-\infty, 2]$
x-int $(-3\pm\sqrt{2}, 0)$
y-int $(0,-7)$

b. $f(x) = -|x-4|$

right 4
reflect over x

(-1,1) → (3,1) → (3,-1)
(0,0) (4,0) (4,0)
(1,1) (5,1) (5,-1)

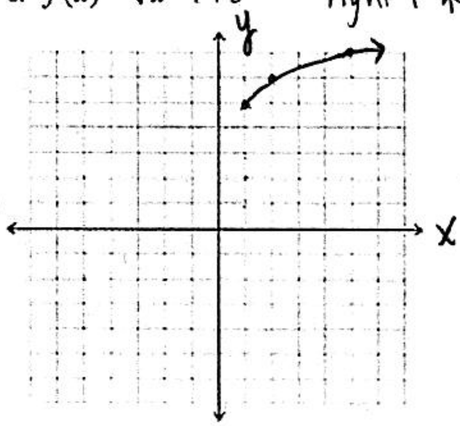


D: $(-\infty, \infty)$
R: $(-\infty, 0]$
x-int (4,0)
y-int (0, -4)

c. $f(x) = \sqrt{x-1}+5$

right 1 ↑5

(0,0) → (1,0) → (1,5)
(1,1) (2,1) (2,6)
(4,2) (5,2) (5,7)

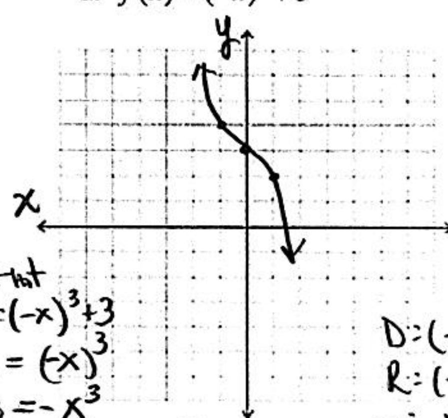


D: $[1, \infty)$
R: $[5, \infty)$
x-int: none
y-int: none

d. $f(x) = (-x)^3+3$

reflect over y
↑3

(-1,-1) → (1,-1)
(0,0) (0,0)
(1,1) (-1,1)



x-int
$0 = (-x)^3+3$
$-3 = (-x)^3$
$-3 = -x^3$
$3 = x^3$ $x = \sqrt[3]{3}$

D: $(-\infty, \infty)$
R: $(-\infty, \infty)$
x-int $(\sqrt[3]{3}, 0)$
y-int (0,3)

(1,2)
(0,3)
(-1,4)

17. Determine algebraically if the following functions are even, odd, or neither

a. $f(x) = -x^4 + 4x^2$

$f(-x) = -(-x)^4 + 4(-x)^2$

$f(-x) = -x^4 + 4x^2$

even

b. $f(x) = \dfrac{x^3}{x^2 - 4}$

$f(-x) = \dfrac{(-x)^3}{(-x)^2 - 4}$

$f(-x) = \dfrac{-x^3}{x^2 - 4}$ odd

b/c its $-1 \cdot f(x)$

(18)

$$\begin{array}{r} x^2 - 7 \\ x^2+2 \enclose{longdiv}{x^4 + 0x^3 - 5x^2 + 6x - 7} \\ -x^4 \mp 2x^2 \\ \hline -7x^2 + 6x - 7 \\ \pm 7x^2 \pm 14 \\ \hline 6x + 7 \end{array}$$

$6x + 7$ remainder

(19)

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & -5 & 10 \\ & & 3 & 9 & 27 & 66 \\ \hline & 1 & 3 & 9 & 22 & 76 \end{array}$$

$x^3 + 3x^2 + 9x + 22 + \dfrac{76}{x-3}$

(20)

$$\begin{array}{r|rrrr} 3 & 1 & 0 & -7 & -6 \\ & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

$\therefore$ $(x-3)$ is a factor

$(x-3)(x^2 + 3x + 2)$

$(x-3)\boxed{(x+2)(x+1)}$ other two factors

(21) $f(2) = 2^3 - 13(2)^2 + 23(2) - 11 = 8 - 52 + 46 - 11 = -9$

Since $f(2) \neq 0$

$(x-2)$ is not a factor of $f(x)$

(22) $f(x) = -3x^2 + 5x + 4x^3 - 6$

$f(x) = 4x^3 - 3x^2 + 5x - 6$

possible rational zeros: $\dfrac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1, \pm 2, \pm 4}$

$= \pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{4}, \pm \frac{3}{4}$

23

$f(x) = x^3 - 13x - 12$ poss rational zeros: $\frac{\pm1, \pm2, \pm3, \pm4, \pm12}{\pm1} = \pm1, \pm2, \pm3, \pm4, \pm12$

$f(-1) = (-1)^3 - 13(-1) - 12 = -1 + 13 - 12 = 0$

$$\begin{array}{r|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$$

$(x^2 - x - 12)(x+1)$

$(x-4)(x+3)(x+1)$ complete factorization

24 $f(x) = x^3 - 13x - 12$

We found the complete factorization in 23, we set that $= 0$ and solve

$(x-4)(x+3)(x+1) = 0$

$x = 4$ | $x = -3$ | $x = -1$ roots are $\{-3, -1, 4\}$

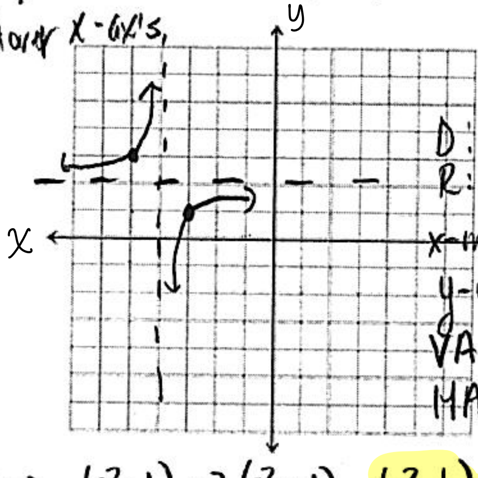
25 16

26 $x - 5$

27. Graph the following using a minimum of 2 points. For each graph, state the domain, range, intercepts, and the equations of any asymptotes.

a. $y=-\frac{1}{(x+4)}+2$

left 4
reflect over x-axis,
↑ 2



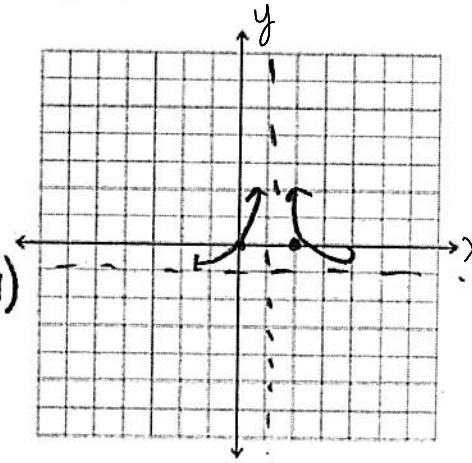
D: $x \neq -4$
R: $y \neq 2$
x-int $(-7/2, 0)$
y-int $(0, 1\frac{3}{4})$
VA: $x=-4$
HA $y=2$

$(1,1) \to (-3,1) \to (-3,-1)$, $(-3,1)$
$(-1,-1) \to (-5,-1) \to (-5,1)$, $(-5,3)$

b. $y=\frac{1}{(x-1)^2}-1$

right 1 ↓1

$(1,1) \to (2,1) \to (2,0)$
$(-1,1) \to (0,1) \to (0,0)$



D: $x \neq 1$
R: $y > -1$
x-int $(0,0), (2,0)$
y-int $(0,0)$
VA $x=1$
HA $y=-1$

28. Fill in the chart:

Reduced	Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	Oblique Asymptote	x-intercept(s)	y-intercept
$y=\frac{-1}{x+5}$	$y=\frac{\cancel{5-x}}{x^2-25}$ $(\cancel{x-5})(x+5)$	$(5, -\frac{1}{10})$	$x=-5$	$y=0$	none	none	$(0, -\frac{1}{5})$
$y=\frac{2x^3}{x^2+1}$	$y=\frac{2x^{3}}{x^3+x}$ $\cancel{x}(x^2+1)$	$(0,0)$	none	none	$y=2x$	none	none

x and y intercept would be a (0,0) but there is a hole there

* x-int for 27a

$0=-\frac{1}{(x+4)}+2$

$-2=\frac{-1}{(x+4)}$

$2=\frac{1}{x+4}$

$2x+8=1$

$2x=-7$

$x=-7/2$ or -3.5

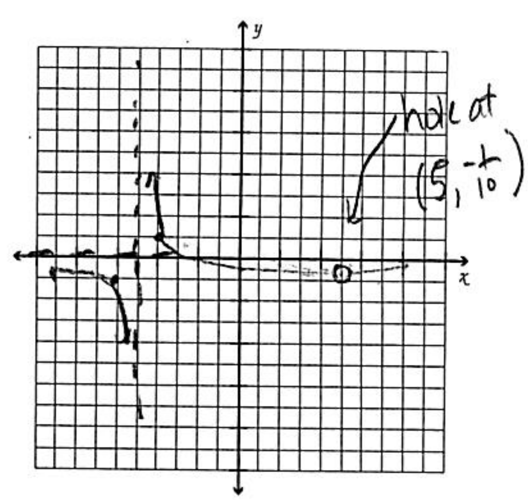
28

$y=\frac{-1}{x+5}$

left 5
reflect over x

y-intercept

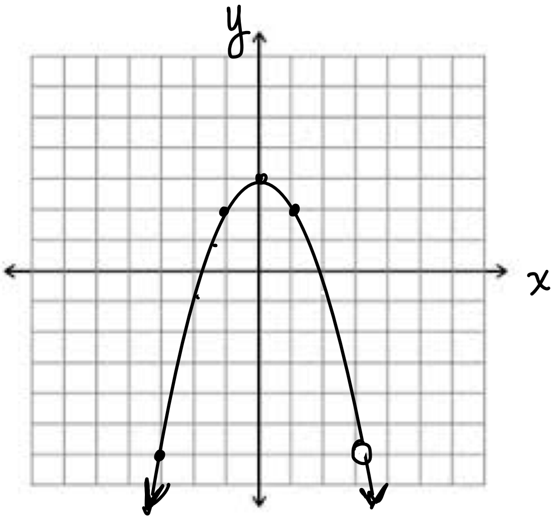
$y=\frac{-1}{0+5}=\frac{-1}{5}$

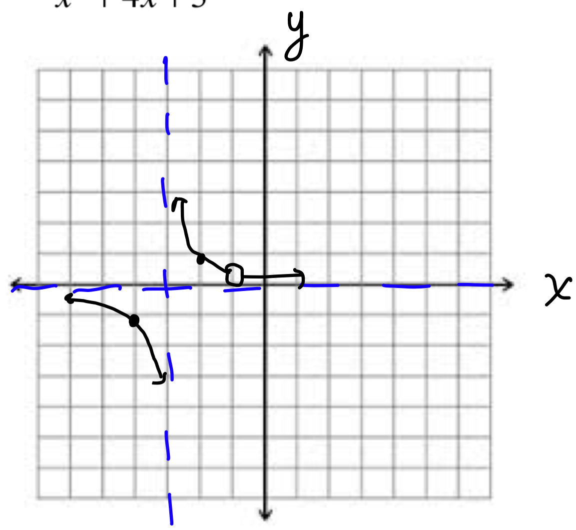


29. Graph the following using a minimum of 2 points. For each graph, state the domain, range, coordinates of any holes or intercepts, and the equations of any asymptotes.

a. $y=\dfrac{x^3-3x^2-3x+9}{3-x}$

b. $y=\dfrac{x+1}{x^2+4x+3}$





$y=\dfrac{x^2(x-3)-3(x-3)}{3-x}$

$y=\dfrac{(x^2-3)\overset{-1}{\cancel{(x-3)}}}{\cancel{3-x}}$

$y=-(x^2-3)$

$y=-x^2+3$

x^2 reflected over x-axis ↑3

(-1,1)	(-1,-1)	(-1,2)
(0,0)	(0,0)	(0,3)
(1,1)	(1,-1)	(1,2)

hole @ (3,-6)

VA: none
HA: none
OA: none

D: $\{x \mid x \neq 3\}$
R: $\{y \mid y \leq 3\}$

$0=-x^2+3$
$x^2=3$
$x=\pm\sqrt{3}$

x-int: $(\pm\sqrt{3}, 0)$
y-int: $(0, 3)$

$y=\dfrac{\cancel{x+1}}{\cancel{(x+1)}(x+3)}$

RF: $y=\dfrac{1}{x+3}$

hyperbola left 3

(-1,-1)	(-4,-1)
(1,1)	(-2,1)

hole: $(-1, \frac{1}{2})$

VA: $x=-3$
HA: $y=0$
OA: none

D: $\{x \mid x \neq -3, -1\}$
R: $\{y \mid y \neq 0, \frac{1}{2}\}$

x-int: none
y-int: $(0, \frac{1}{3})$

In 30 - 40, factor each completely if possible.

30. $x^3 - 3x^2 - 4x + 12$

$x^2(x-3) - 4(x-3)$

$(x^2-4)(x-3)$

$(x+2)(x-2)(x-3)$

31. $3x^2 - 75$

$3(x^2-25)$

$3(x+5)(x-5)$

32. $ax^2 + 15 - 5ax - 3x$ rearrange

$ax^2 - 5ax - 3x + 15$

$ax(x-5) - 3(x-5)$

$(ax-3)(x-5)$

33. $6x^2 - 11x - 10$

$6x^2 - 15x + 4x - 10$

$3x(2x-5) + 2(2x-5)$

$(3x+2)(2x-5)$

A m

34. $x^4 - x^2 - 12$

$(x^2-4)(x^2+3)$

$(x+2)(x-2)(x^2+3)$

35. $16x^2y^2 - 25$

$(4xy-5)(4xy+5)$

36. $8x^3 - 125y^3$

$(2x-5y)(4x^2+10xy+25y^2)$

37. $(x^2-3x)^2 - 38(x^2-3x) - 80$ substitution

let $y = x^2 - 3x$

$y^2 - 38y - 80$

$(y-40)(y+2)$

$(x^2-3x-40)(x^2-3x+2)$

$(x-8)(x+5)(x-2)(x-1)$

38. $x^2(x^2-1) - 9(x^2-1)$ substitution

let $y = x^2 - 1$

$x^2y - 9y$

$y(x^2-9)$

$(x^2-1)(x^2-9)$

$(x-1)(x+1)(x+3)(x-3)$

39. $4(x^2-1)^2 - 13(x^2-1) - 12$

let $y = x^2 - 1$

$4y^2 - 13y - 12$

$4y^2 - 16y + 3y - 12$

$4y(y-4) + 3(y-4)$

$(4y+3)(y-4)$

$(4(x^2-1)+3)(x^2-1-4)$

$(4x^2-4+3)(x^2-5)$

$(4x^2-1)(x^2-5)$

$(2x+1)(2x-1)(x^2-5)$

40. $7x^2 + 10xy + 3y^2$

multiply to $21x^2y^2$
add to $10xy$

$7x^2 + 7xy + 3xy + 3y^2$

$7x(x+y) + 3y(x+y)$

$(7x+3y)(x+y)$