Homework 12-14

Name: Key Date: ____________

AP Calculus AB: Extreme Value Theorem HW Ms. Loughran

$h''(x) = \frac{x(2x) - (x^2-2)}{x^2}$

$\frac{2x^2 - x^2 + 2}{x^2} = \frac{x^2+2}{x^2}$ + +

2001: AB-4; BC-4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$. $\frac{16-2}{4}$ $\frac{14}{4}$

$x^2 - 2 = 0$ $x^2 = 2$ $x = \pm\sqrt{2}$

h' − + − + $-\sqrt{2}$ 0 $\sqrt{2}$

(a) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

$-\sqrt{2}$ rel min
$\sqrt{2}$ rel min

(b) On what intervals, if any, is the graph of h concave up? Justify your answer.

(c) Write an equation for the line tangent to the graph of h at $x = 4$.

(d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

(b) $h''(x) > 0$ so $h(x)$ is concave up always

(c) $y + 3 = \frac{7}{2}(x-4)$

below, b/c this graph is always concave up

For 1 and 2, find the absolute maximum and minimum values of f on the given closed interval, and state where those values occur.

$1 - 2 + 1$

1. $f(x) = 4x^2 - 4x + 1$; $[0,1]$

$f'(x) = 8x - 4$

$8x - 4 = 0$

$x = \frac{1}{2}$

$f(0) = 1$

$f(\frac{1}{2}) = 0$

$f(1) = 1$

Abs max of 1 at $x = 0, 1$

Abs min of 0 at $x = 1/2$

2. $f(x) = \frac{3x}{\sqrt{4x^2+1}}$; $[-1,1]$

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$\frac{9}{4} - \frac{9}{2} - 1$

$\frac{9 - 18 - 4}{4}$

3. Find the absolute maximum and minimum values of f, if any, on the given interval and state where those values occur.

$f(x) = x^2 - 3x - 1$; $(-\infty, \infty)$

$f''(x) = 2$

$f'(x) = 2x - 3$

$2x - 3 = 0$

$x = 3/2$

$f(3/2) = (3/2)^2 - 3(\frac{3}{2}) - 1$

$f''(\frac{3}{2}) > 0$ so there is a rel min at $3/2$

Since there is only one rel min in the interval it must be the absolute minimum of $-13/4$ there is no absolute maximum

② $f(x) = \frac{3x}{\sqrt{4x^2+1}}$

$$f'(x) = \frac{3\sqrt{4x^2+1} - 3x \cdot \frac{1}{\cancel{2}}(4x^2+1)^{-\frac{1}{2}} \cdot \overset{4}{\cancel{8}}x}{4x^2+1}$$

$$f'(x) = \frac{3\sqrt{4x^2+1} - \frac{12x^2}{\sqrt{4x^2+1}}}{4x^2+1}$$

$12x^2+1 \quad 4x^2+1$

$$f'(x) = \frac{3(4x^2+1) - 12x^2}{(4x^2+1)(\sqrt{4x^2+1})} = \frac{3}{(4x^2+1)^{3/2}}$$

there are no critical pts

$f(1) = \frac{3}{\sqrt{5}}$ abs max

$f(-1) = \frac{-\sqrt{3}}{5}$ abs min

1980 AB5/BC2
Solution

(a) $f(x) = \cos x \cdot (1 - \cos x)$ $\cos x = 1$

Either $\cos x = 0$ or $1 - \cos x = 0$, so the x-intercepts are $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$, and $x = 0$.

(b) $f'(x) = -\sin x + 2\sin x \cos x$

$0 = \sin x \,(-1 + 2\cos x)$

Either $\sin x = 0$ or $\cos x = \frac{1}{2}$, so the candidates are $x = \pm\pi$, $x = 0$, and $x = \pm\frac{\pi}{3}$.

The relative maximum points are at $\left(\pm\frac{\pi}{3}, \frac{1}{4}\right)$.

Justification:

Second Derivative Test

(i) $f''(x) = -\cos x + 2\cos 2x$

$f''(\pm\pi) = 3 \Rightarrow$ relative minimum

$f''(0) = 1 \Rightarrow$ relative minimum

$f''\left(\pm\frac{\pi}{3}\right) = -\frac{3}{2} \Rightarrow$ relative maximum

or

(ii) Selecting critical values:

x	$-\pi$	$-\frac{\pi}{3}$	0	$\frac{\pi}{3}$	π
$f(x)$	-2	$1/4$	0	$1/4$	-2

or

(iii) Sign chart:

First Derivative Test

$f'(x)$: $+$ on $(-\pi, -\pi/3)$, $-$ on $(-\pi/3, 0)$, $+$ on $(0, \pi/3)$, $-$ on $(\pi/3, \pi)$

$-\pi \quad -\pi/3 \quad 0 \quad \pi/3 \quad \pi$

$\frac{\pi}{6}$ $\frac{\pi}{2}$

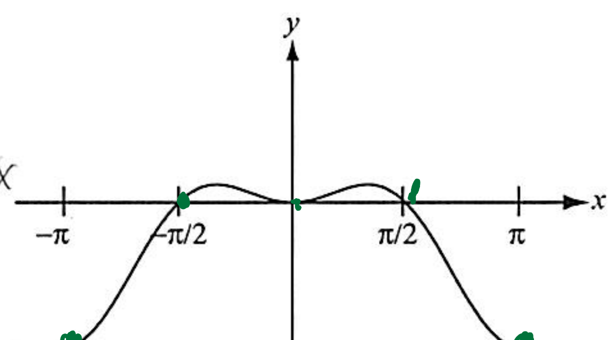
$-\frac{3\pi}{2}$ $\frac{\pi}{2}$ π -2π π 0 2π $\frac{3\pi}{2}$ $-\frac{\pi}{2}$

f: $\nearrow \searrow \nearrow \searrow$

X	f(x)	f''	
$-\pi$	-2	$+$	CU
$-\frac{\pi}{3}$	$\frac{1}{4}$	$-$	CD
0	0	$+$	CU
$\frac{\pi}{3}$	$\frac{1}{4}$	$-$	CD
π	-2	$+$	CU

(c) Graph of f increases on the intervals $-\pi < x < -\frac{\pi}{3}$ and $0 < x < \frac{\pi}{3}$.

(d)



$f'(x) = -\sin x + 2\sin x\cos x$

$f'(x) = -\sin x + \sin 2x$

$f''(x) = -\cos x + 2\cos 2x$

$f(\pi) = -2$

$f(-\pi) = -2$

$f\left(\frac{\pi}{3}\right) = \frac{1}{4}$ $f\left(-\frac{\pi}{3}\right) = \frac{1}{4}$