

Name: \_\_\_\_\_

PC: More Polynomial Practice

Date: \_\_\_\_\_

Ms. Loughran

1. If  $f(-6) = 0$ , then  $x+6$  is a factor of  $f(x)$ .

2. If  $3x-2$  is a factor of  $f(x)$ , then  $\frac{2}{3}$  is a zero of  $f(x)$ .

3. If  $f(x) = (5x-2)(2x+1)(3x-1)$ , then the zeros of  $f(x)$  are:  $\frac{2}{5}, -\frac{1}{2}, \frac{1}{3}$

4. If  $f(7) = 0$ , then a factor of  $f(x)$  is:  $x-7$

5. If  $3x-4$  is a factor of  $f(x)$ , then  $f(\frac{4}{3}) = 0$

6. Show in 2 ways that  $x+1$  is a factor of  $x^3 - x^2 - 5x - 3$ .

S.D.

$$\begin{array}{r|rrrr} -1 & 1 & -1 & -5 & -3 \\ & & -1 & -2 & 3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

OR  
LD

Plug in  $(-1)^3 - (-1)^2 - 5(-1) - 3 = 0$

7. Show 2 ways that  $x-3$  is a factor of  $x^5 - 243$ .

S.D.

$$\begin{array}{r|rrrrrr} 3 & 1 & 0 & 0 & 0 & 0 & -243 \\ & & 3 & 9 & 27 & 81 & 243 \\ \hline & 1 & 3 & 9 & 27 & 81 & 0 \end{array}$$

OR  
Long  
Div.

Plug in  $(3)^5 - 243 = 0$

8. Factors of  $x^3 + 2x^2 - 5x - 6$  are  $(x-2)$ ,  $(x+3)$  and  $(x+1)$ . What are the zeros of the polynomial?

$$\{2, -3, -1\}$$

9. Given the zeros of  $x^3 - 3x^2 - 18x + 40$  are 2, -4, and 5. What are the factors of the polynomial? Check by multiplication.

$$\begin{aligned}
 & (x-2)(x+4)(x-5) \\
 & (x^2 + 2x - 8)(x-5) \\
 & x^3 + 2x^2 - 8x - 5x^2 - 10x + 40 \\
 & x^3 - 3x^2 - 18x + 40
 \end{aligned}$$

10. Show that -4 is a zero of  $f(x) = x^3 + 6x^2 + 11x + 12$ .

$$\begin{aligned}
 f(-4) &= (-4)^3 + 6(-4)^2 + 11(-4) + 12 \\
 &= -64 + 96 - 44 + 12 = 0
 \end{aligned}$$

OR  
Syn. Div. or Long Division  
to show remainder = 0

11. Given that  $(x+1)$  is a factor of  $f(x) = 3x^3 - 7x^2 - 18x - 8$  find all zeros of  $f(x)$ .

$$\begin{array}{r}
 -1 \overline{) 3 \quad -7 \quad -18 \quad -8} \\
 \underline{\phantom{-1} 3 \quad -3 \quad 10 \quad 8} \\
 3 \quad -10 \quad -8 \quad 0
 \end{array}$$

$0 = (x+1)(x-4)(3x+2)$

$$\begin{aligned}
 (x+1)(3x^2 - 10x - 8) &= 0 \\
 (x+1)(3x^2 - 12x + 2x - 8) &= 0 \\
 (x+1)[3x(x-4) + 2(x-4)] &= 0
 \end{aligned}$$

$x = 4, -1, -\frac{2}{3}$

12. One root of  $2x^3 + 7x^2 - 33x - 18 = 0$  is  $-6$ . Find the complete solution set of this equation.

$$\begin{array}{r}
 -6 \overline{) 2 \quad 7 \quad -33 \quad -18} \\
 \underline{-12 \quad 30 \quad 18} \\
 2 \quad -5 \quad -3 \quad 0
 \end{array}$$

$(2x+1)(x-3)(x+6) = 0$   
 $x = -\frac{1}{2}, 3, -6$

$$(x+6)(2x^2-5x-3) = 0$$

$$(x+6)[2x^2-6x+x-3] = 0$$

$$(x+6)[2x(x-3)+1(x-3)] = 0$$

13. Show that  $(x+2)$  is a factor of  $x^3 + 3x^2 + 4x + 4 = 0$ . Use this information to find the solution set of this equation.

$$\begin{array}{r}
 -2 \overline{) 1 \quad 3 \quad 4 \quad 4} \\
 \underline{-2 \quad -2 \quad -4} \\
 1 \quad 1 \quad 2 \quad 0
 \end{array}$$

$$(x+2)(x^2+x+2) = 0$$

$x = -2$

$$x = \frac{-1 \pm \sqrt{1-4(1)(2)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{-7}}{2} = \frac{-1 \pm i\sqrt{7}}{2}$$

$\left\{ -2, \frac{-1 \pm i\sqrt{7}}{2} \right\}$

14. One zero of  $2x^3 - 3x^2 - 23x + 12$  is  $\frac{1}{2}$ . Find the complete **factorization** of this polynomial and find the remaining zeros. (THE COMPLETE FACTORIZATION OF A POLYNOMIAL WILL INCLUDE FACTORS WITH ONLY INTEGRAL COEFFICIENTS.)

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & -3 & -23 & 12 \\ & & 1 & -1 & -12 \\ \hline & 2 & -2 & -24 & 0 \\ & \underbrace{\hspace{2cm}} & & & \\ & & \div 2 & & \end{array}$$

$$(2x-1)(x^2-x-12)$$

$$(2x-1)(x-4)(x+3) \leftarrow \text{complete factorization}$$

$$0 = (2x-1)(x-4)(x+3)$$

$$x = \frac{1}{2}, 4, -3$$

$$\text{remaining zeros: } \{4, -3\}$$

15. Find the remainder when  $x^{124} - 5x^{76} + 2x^{45} - 3x + 5$  is divided by  $x+1$ .

$$(-1)^{124} - 5(-1)^{76} + 2(-1)^{45} - 3(-1) + 5 = 1 - 5 - 2 + 3 + 5 = 2$$

16. If  $x+3$  is a factor of  $f(x) = x^3 + 4x^2 + x - 6$ , find the complete factorization of  $f(x)$ .

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 1 & -6 \\ & & -3 & -3 & 6 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

$$(x+3)(x^2+x-2)$$

$$(x+3)(x+2)(x-1)$$

17. One root of  $x^3 + 4x^2 - 4x - 1 = 0$  is 1. Find the other roots.

$$\begin{array}{r|rrrr} 1 & 1 & 4 & -4 & -1 \\ & & 1 & 5 & 1 \\ \hline & 1 & 5 & 1 & 0 \end{array}$$

$$(x-1)(x^2+5x+1) = 0$$

$x-1=0$ $x=1$	$x^2+5x+1=0$ $x = \frac{-5 \pm \sqrt{25-4(1)(1)}}{2(1)}$ $x = \frac{-5 \pm \sqrt{21}}{2}$
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other roots:  $\left\{ \frac{-5 \pm \sqrt{21}}{2} \right\}$