For 1-3, find $\frac{dy}{dx}$.

$$\left[\tan^{-1}u\right]' = \frac{1}{u^2+1}\cdot u'$$

$$\left[\arcsin u\right]' = \frac{1}{\sqrt{1-u^2}}\cdot u'$$

① $y = e^{\arctan(x)}$

$$\frac{dy}{dx} = e^{\arctan x}\cdot\frac{1}{1+x^2} = \frac{e^{\arctan x}}{1+x^2}$$

② $y = \arcsin\sqrt{1-x^2}$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-(\sqrt{1-x^2})^2}}\cdot\frac{1}{\cancel{2}}(1-x^2)^{-\frac{1}{2}}\cdot -\cancel{2}x$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{1-(1-x^2)}}\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\frac{-x}{\sqrt{1-1+x^2}}\cdot\frac{1}{\sqrt{1-x^2}}$$

$$\frac{-x}{\sqrt{x^2}}\cdot\frac{1}{\sqrt{1-x^2}} = \frac{-x}{|x|\sqrt{1-x^2}}$$

$$\left[\arccos u\right]' = -\frac{1}{\sqrt{1-u^2}}\cdot u'$$

③ $y = \arctan(\underset{u}{\cos(x)})$

$$\frac{dy}{dx} = \frac{1}{\cos^2 x+1}\cdot -\sin x = \frac{-\sin x}{\cos^2 x+1}$$

$$g'(y) = \frac{1}{f'(x)}$$ if f and g are inverses and (x,y) is a pt on f

④ If $f(x) = \frac{x+6}{x-2}$, $x > 2$ and $(f\circ g)(x) = x$, find $g'(3)$. $= \frac{1}{f'(6)}$ ↳ $\frac{1}{-\frac{1}{2}} = -2$

inverses

$$\frac{x+6}{x-2} = 3$$

$$x+6 = 3x-6$$

$$12 = 2x \qquad x = 6$$

$$f'(x) = \frac{(x-2)(1)-(x+6)(1)}{(x-2)^2}$$

$$f'(6) = \frac{4-12}{16} = \frac{-8}{16} = \frac{-1}{2}$$

(5) If $f(x)=\sqrt{x-4}$, and $f(g(x))=x$, find $g'(2)$. $= \dfrac{1}{f'(8)} = \dfrac{1}{\frac{1}{4}} = 4$

inverse

y on f

$$\sqrt{x-4}=2$$
$$x-4=4$$
$$x=8$$

$$f'(x)=\frac{1}{2}(x-4)^{-\frac{1}{2}}$$
$$f'(8)=\frac{1}{2}(8-4)^{-\frac{1}{2}}=\frac{1}{2}\left(\frac{1}{2}\right)=\frac{1}{4}$$

(6) Evaluate $\lim\limits_{x\to 0}\dfrac{3^x-2^x}{x^2-x} = \lim\limits_{x\to 0}\dfrac{3^x\ln 3-2^x\ln 2}{2x-1} = \dfrac{\ln 3-\ln 2}{-1}$

$$= -\ln 3+\ln 2$$

formal def of derivative

$y=\arcsin x$

$y'(x)=\dfrac{1}{\sqrt{1-x^2}}$

(7) Given the $\lim\limits_{h\to 0}\dfrac{\arcsin(a+h)-\arcsin(a)}{h}=2$, find $a$. $y'(a)=\dfrac{1}{\sqrt{1-a^2}}$

$$\frac{1}{\sqrt{1-a^2}}=2$$
$$\left(2\sqrt{1-a^2}\right)^2=(1)^2$$
$$4(1-a^2)=1$$
$$1-a^2=\frac{1}{4}$$
$$\frac{3}{4}=a^2$$
$$a=\pm\frac{\sqrt{3}}{2}$$

⑧ Find $y'$ if $y = \sin(\arccos x)$

$$y' = \cos(\arccos x) \cdot -\frac{1}{\sqrt{1-x^2}}$$

$$y' = x \cdot \frac{-1}{\sqrt{1-x^2}} = \frac{-x}{\sqrt{1-x^2}}$$

⑨ Find the eq. of the tangent line to the graph of

$$x^2 + \underbrace{x \arctan y}_{\text{product rule}} = y - 1 \quad \text{at } \left(-\frac{\pi}{4}, 1\right).$$

$$2x + \arctan y + x \cdot \frac{1}{y^2+1}\frac{dy}{dx} = \frac{dy}{dx}$$

$$2x + \arctan y = \frac{dy}{dx} - \frac{x}{y^2+1}\frac{dy}{dx}$$

$$2x + \arctan y = \frac{dy}{dx}\left(1 - \frac{x}{y^2+1}\right)$$

$$\frac{2x + \arctan y}{1 - \frac{x}{y^2+1}} = \frac{dy}{dx}$$

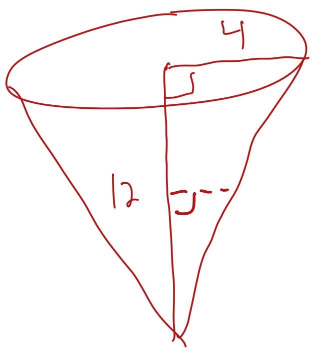
$$y - 1 = \frac{-2\pi}{8+\pi}\left(x + \frac{\pi}{4}\right)$$

$$\left.\frac{dy}{dx}\right|_{\left(-\frac{\pi}{4},1\right)} = \frac{2\left(-\frac{\pi}{4}\right) + \arctan(1)}{1 - \frac{\left(-\frac{\pi}{4}\right)}{1^2+1}} = \frac{-\frac{\pi}{2} + \frac{\pi}{4}}{1 + \frac{\pi}{8}} = \frac{-4\pi + 2\pi}{8 + \pi} = \frac{-2\pi}{8+\pi}$$

$1$

$\frac{\frac{\pi}{4}}{2}$

James is filling an ice cream cone. The cone is 12 cm tall and has a radius of 4 cm. If the ice cream fills the cone evenly at a rate of 1.5 $cm^3$ / s, what is the rate of change of the height of the ice cream when the cone is filled up to 5 cm?




$\frac{dV}{dt} = 1.5 cm^3/s$

$\frac{dh}{dt} = ?$ $\qquad h = 5$ cm

$\frac{r}{h} = \frac{4}{12}$

$\frac{r}{h} = \frac{1}{3}$

$3r = h$

$r = \frac{h}{3}$

Want the formula in terms of h

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi \left(\frac{h}{3}\right)^2 h$

$V = \frac{1}{3}\pi \left(\frac{h^2}{9}\right) h$

$V = \frac{\pi}{27} h^3$

$\frac{dV}{dt} = \cancel{3} \cdot \frac{\pi}{\cancel{27}_{9}} h^2 \frac{dh}{dt}$

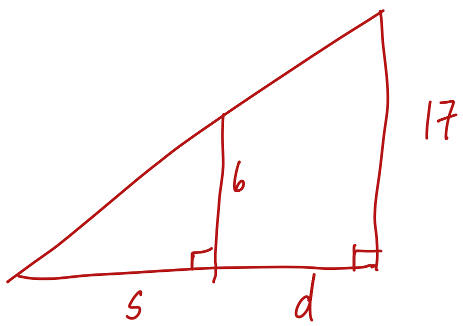
$$\frac{dV}{dt} = \frac{\pi}{9} h^2 \frac{dh}{dt}$$

or 1.5

$$\frac{3}{2} = \frac{\pi}{9}(5)^2 \frac{dh}{dt}$$

$$\frac{9}{25\pi} \cdot \frac{3}{2} = \frac{25\pi}{9} \frac{dh}{dt} \cdot \frac{9}{25\pi}$$

$$\frac{27}{50\pi} \text{ cm/s} = \frac{dh}{dt}$$




$$\frac{s}{6} = \frac{s+d}{17}$$

$$17s = 6s + 6d$$

$$11s = 6d$$

$$s = \frac{6}{17} d$$

$$\frac{ds}{dt} = \frac{6}{17} \frac{dd}{dt}$$