Name:____ Date:_____________

PCH: Operations on Matrices Ms. Loughran

Two matrices are equal if they have the same order $m \times n$ and their corresponding entries are equal.

1. Solve for $a_{11}, a_{12}, a_{21},$ and a_{22} in the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

Matrix Addition

You can add two matrices (of the same order) by adding their corresponding entries. The sum of two matrices of different orders is undefined.

2. $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} =$

3. $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$

4. $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} =$

5. The sum of $A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix}$ and $B \begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}$ is

Scalar Multiplication

In work with matrices, numbers are usually referred to as scalars. For our purposes, scalars will always be real numbers. You can multiply a matrix *A* by a scalar *c* by multiplying each entry in *A* by *c*.

The symbol $-A$ represents the scalar product $(-1)A$. Moreover, if *A* and *B* are of the same order, $A-B$ represents the sum of *A* and $(-1)B$. That is,

$$A-B = A+(-1)B \qquad \text{(Subtraction of matrices)}$$

6. For the following matrices, find (a) $3A$
(b) $-B$
(c) $3A-B$

$$A=\begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } B=\begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ and let c and d be scalars.

1. $A + B = B + A$ ( )

2. $A + (B + C) = (A + B) + C$ ( )

3. $(cd)A = c(dA)$ ( )

4. $IA = A$ ( )

5. $c(A + B) = cA + cB$ ( )

6. $(c + d)A = cA + dA$ ( )

7. $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} =$

8. Solve for X in the equation $3X + A = B$, where

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}.$$

Matrix Multiplication

To find the entries of the product, multiply each row of *A* by each column of *B*. Note that the number of columns of *A* must be equal to the of rows of *B*.

9. Find the product *AB* where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}.$$

10. $\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} =$

11. $\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$

12. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$

13. $\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} =$

14. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & -3 \end{bmatrix} =$

15. Find the product of *AB*. If $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$.

Practice

Perform the indicated operation when possible.

1) $-4\begin{bmatrix} 5 & 1 \\ 6 & 0 \end{bmatrix}$

2) $\begin{bmatrix} 6 & -5 & 3 & -5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$

3) $-4w\begin{bmatrix} -w & -4+u & 0 \\ v & 5v & 3wv \end{bmatrix}$

4) $\begin{bmatrix} -4 & -5 & 5 \\ 1 & 6 & 3 \\ -2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -6 & -1 & -6 \\ 6 & -3 & -2 \\ 4 & -1 & -3 \end{bmatrix}$

5) $\begin{bmatrix} -5wu \\ 6 \\ v-1 \end{bmatrix} - \left(\begin{bmatrix} -5v \\ 6v \\ 5u+6 \end{bmatrix} - \begin{bmatrix} -3v \\ -5 \\ 3vu \end{bmatrix} \right)$

6) $\begin{bmatrix} -3y & 3x \\ -2 & -4x+2 \\ y^2 & 2x \end{bmatrix} - \begin{bmatrix} x & x-2 \\ 4 & y \\ x-1 & xy \end{bmatrix}$

7) $\begin{bmatrix} -4b \\ 2b \\ 6b \end{bmatrix} + 2\begin{bmatrix} 3a \\ ab \\ a+4 \end{bmatrix}$

8) $-5\left(\begin{bmatrix} 1 & 0 \\ -2 & -3 \\ 6 & -6 \end{bmatrix} + \begin{bmatrix} -5 & 4 \\ -6 & 0 \\ 4 & 4 \end{bmatrix} \right)$

9) $\begin{bmatrix} 3 & -3 \\ 6 & 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 6 & 1 \\ 6 & -5 & 4 \end{bmatrix}$

10) $\begin{bmatrix} 3 & 1 \\ -3 & -4 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 \\ -4 & -1 \end{bmatrix}$

11) $\begin{bmatrix} 3 & 2 \\ 2 & 1 \\ 3 & 4 \\ -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 2 & -6 \end{bmatrix} \cdot \begin{bmatrix} -2 & 6 \\ 0 & 0 \end{bmatrix}$

12) $\begin{bmatrix} 4 & -2 \\ -3 & 6 \end{bmatrix} \cdot \left(\begin{bmatrix} 4 & -5 & 4 & 0 \\ 6 & 3 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 \\ -6 & -1 \end{bmatrix} \right)$