integrals can be easily evaluated by technology has made the world less reliant on antidifferentiation, and hence less reliant on u-substitution. Consequently, you have seen in this book only a sampling of the substitution tricks calculus students would have routinely studied in the past. You may see more of them in a differential equations course.

Quick Review 7.2 (For help, go to Sections 4.1 and 4.4.)

Exercise numbers with a gray background indicate problems that the authors have designed to be solved *without a calculator*.

In Exercises 1 and 2, evaluate the definite integral.

1. $\int_0^2 x^4\,dx$ 2. $\int_1^5 \sqrt{x-1}\,dx$

In Exercises 3–10, find dy/dx.

3. $y = \int_2^x 3^t\,dt$ 4. $y = \int_0^x 3^t\,dt$

5. $y = (x^3 - 2x^2 + 3)^4$

6. $y = \sin^2(4x - 5)$

7. $y = \ln \cos x$

8. $y = \ln \sin x$

9. $y = \ln(\sec x + \tan x)$

10. $y = \ln(\csc x + \cot x)$

Section 7.2 Exercises

In Exercises 1–6, find the indefinite integral.

1. $\int (\cos x - 3x^2)\,dx$ 2. $\int x^{-2}\,dx$

3. $\int \left(t^2 - \frac{1}{t^2}\right) dt$ 4. $\int \frac{dt}{t^2+1}$

5. $\int (3x^4 - 2x^{-3} + \sec^2 x)\,dx$

6. $\int (2e^x + \sec x \tan x - \sqrt{x})\,dx$

In Exercises 7–12, use differentiation to verify the antiderivative formula.

7. $\int \csc^2 u\,du = -\cot u + C$ 8. $\int \csc u \cot u = -\csc u + C$

9. $\int e^{2x}\,dx = \frac{1}{2}e^{2x} + C$ 10. $\int 5^x\,dx = \frac{1}{\ln 5}5^x + C$

11. $\int \frac{1}{1+u^2}\,du = \tan^{-1} u + C$ 12. $\int \frac{1}{\sqrt{1-u^2}}\,du = \sin^{-1} u + C$

In Exercises 13–16, verify that $\int f(u)\,du \neq \int f(u)\,dx$

13. $f(u) = \sqrt{u}$ and $u = x^2$ $(x > 0)$

14. $f(u) = u^2$ and $u = x^5$

15. $f(u) = e^u$ and $u = 7x$ 16. $f(u) = \sin u$ and $u = 4x$

In Exercises 17–24, use the indicated substitution to evaluate the integral. Confirm your answer by differentiation.

17. $\int \sin 3x\,dx,\quad u = 3x$

18. $\int x\cos(2x^2)\,dx,\quad u = 2x^2$

19. $\int \sec 2x \tan 2x\,dx,\quad u = 2x$

20. $\int 28(7x-2)^3\,dx,\quad u = 7x - 2$

21. $\int \frac{dx}{x^2+9},\quad u = \frac{x}{3}$ 22. $\int \frac{9r^2\,dr}{\sqrt{1-r^3}},\quad u = 1 - r^3$

23. $\int \left(1 - \cos\frac{t}{2}\right)^2 \sin\frac{t}{2}\,dt,\quad u = 1 - \cos\frac{t}{2}$

24. $\int 8(y^4 + 4y^2 + 1)^2(y^3 + 2y)\,dy,\quad u = y^4 + 4y^2 + 1$

In Exercises 25–46, use substitution to evaluate the integral.

25. $\int \frac{dx}{(1-x)^2}$ 26. $\int \sec^2(x+2)\,dx$

27. $\int \sqrt{\tan x}\,\sec^2 x\,dx$

28. $\int \sec\left(\theta + \frac{\pi}{2}\right)\tan\left(\theta + \frac{\pi}{2}\right)d\theta$

29. $\int \tan(4x+2)\,dx$ 30. $\int 3(\sin x)^{-2}\,dx$

31. $\int \cos(3z+4)\,dz$ 32. $\int \sqrt{\cot x}\,\csc^2 x\,dx$

33. $\int \frac{\ln^6 x}{x}\,dx$ 34. $\int \tan^7\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)dx$

35. $\int s^{1/3}\cos(s^{4/3} - 8)\,ds$ 36. $\int \frac{dx}{\sin^2 3x}$

37. $\int \frac{\sin(2t+1)}{\cos^2(2t+1)}\,dt$ 38. $\int \frac{6\cos t}{(2+\sin t)^2}\,dt$

39. $\int \frac{dx}{x\ln x}$ 40. $\int \tan^2 x \sec^2 x\,dx$