43. Which of the following could be true if $f''(x) = x^{-1/3}$?

(a) $f(x) = \frac{3}{2}x^{2/3} - 3$ (b) $f(x) = \frac{9}{10}x^{5/3} - 7$

(c) $f'''(x) = -\frac{1}{3}x^{-4/3}$ (d) $f'(x) = \frac{3}{2}x^{2/3} + 6$

44. Which of the following could be true if $g''(t) = 1/t^{3/4}$?

(a) $g'(t) = 4\sqrt[4]{t} - 4$ (b) $g'''(t) = -4/\sqrt[4]{t}$

(c) $g(t) = t - 7 + (16/5)t^{5/4}$ (d) $g'(t) = (1/4)t^{1/4}$

45. *The Eight Curve* (a) Find the slopes of the figure-eight-shaped curve

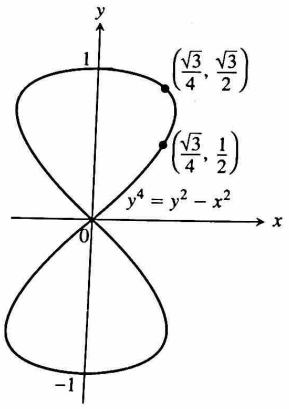
$$y^4 = y^2 - x^2$$

at the two points shown on the graph that follows.

(b) Use parametric mode and the two pairs of parametric equations

$$x_1(t) = \sqrt{t^2 - t^4}, \quad y_1(t) = t,$$
$$x_2(t) = -\sqrt{t^2 - t^4}, \quad y_2(t) = t,$$

to graph the curve. Specify a window and a parameter interval.



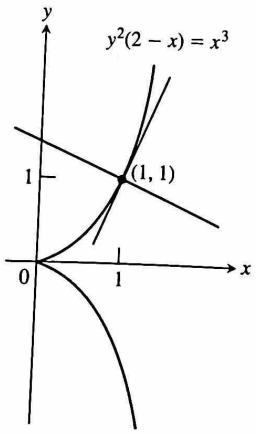
46. *The Cissoid of Diocles (dates from about 200 B.C.E.)*

(a) Find equations for the tangent and normal to the cissoid of Diocles,

$$y^2(2 - x) = x^3,$$

at the point $(1, 1)$ as pictured below.

(b) Explain how to reproduce the graph on a grapher.



47. (a) Confirm that $(-1, 1)$ is on the curve defined by $x^3y^2 = \cos(\pi y)$.

(b) Use part (a) to find the slope of the line tangent to the curve at $(-1, 1)$.

48. Group Activity

(a) Show that the relation

$$y^3 - xy = -1$$

cannot be a function of x by showing that there is more than one possible y-value when $x = 2$.

(b) On a small enough square with center $(2, 1)$, the part of the graph of the relation within the square will define a function $y = f(x)$. For this function, find $f'(2)$ and $f''(2)$.

49. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

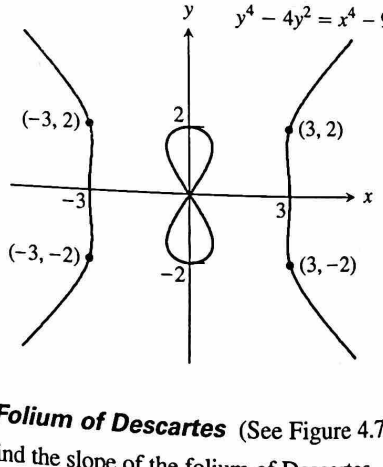
50. Find points on the curve $x^2 + xy + y^2 = 7$ (a) where the tangent is parallel to the x-axis and (b) where the tangent is parallel to the y-axis. (In the latter case, dy/dx is not defined, but dx/dy is. What value does dx/dy have at these points?)

51. *Orthogonal Curves* Two curves are *orthogonal* at a point of intersection if their tangents at that point cross at right angles. Show that the curves $2x^2 + 3y^2 = 5$ and $y^2 = x^3$ are orthogonal at $(1, 1)$ and $(1, -1)$. Use parametric mode to draw the curves and to show the tangent lines.

52. The position of a body moving along a coordinate line at time t is $s = (4 + 6t)^{3/2}$, with s in meters and t in seconds. Find the body's velocity and acceleration when $t = 2$ sec.

53. The velocity of a falling body is $v = 8\sqrt{s - t} + 1$ feet per second at the instant t (sec) the body has fallen s feet from its starting point. Show that the body's acceleration is 32 ft/sec^2.

54. *The Devil's Curve (Gabriel Cramer [the Cramer of Cramer's Rule], 1750)* Find the slopes of the devil's curve $y^4 - 4y^2 = x^4 - 9x^2$ at the four indicated points.



55. *The Folium of Descartes* (See Figure 4.7 on page 162)

(a) Find the slope of the folium of Descartes, $x^3 + y^3 - 9xy = 0$, at the points $(4, 2)$ and $(2, 4)$.

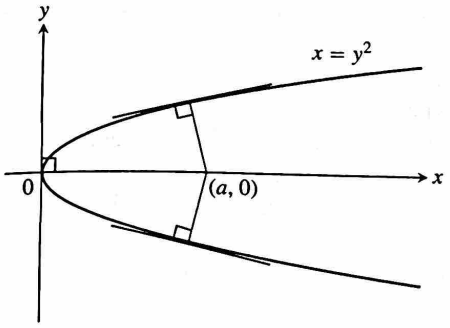
(b) At what point other than the origin does the folium have a horizontal tangent?

(c) Find the coordinates of point A in Figure 4.7, where the folium has a vertical tangent.

56. The line that is normal to the curve $x^2 + 2xy - 3y^2 = 0$ at $(1, 1)$ intersects the curve at what other point?

57. Find the normals to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$.

58. Show that if it is possible to draw these three normals from the point $(a, 0)$ to the parabola $x = y^2$ shown here, then a must be greater than $1/2$. One of the normals is the x-axis. For what value of a are the other two normals perpendicular?



Standardized Test Questions

59. **True or False** The slope of $xy^2 + x = 1$ at $(1/2, 1)$ is 2. Justify your answer.

60. **True or False** The derivative of $y = \sqrt[3]{x}$ is $\dfrac{1}{3x^{2/3}}$. Justify your answer.

In Exercises 61 and 62, use the curve $x^2 - xy + y^2 = 1$.

61. **Multiple Choice** Which of the following is equal to dy/dx?

(A) $\dfrac{y - 2x}{2y - x}$ (B) $\dfrac{y + 2x}{2y - x}$ (C) $\dfrac{2x}{x - 2y}$

(D) $\dfrac{2x + y}{x - 2y}$ (E) $\dfrac{y + 2x}{x}$

62. **Multiple Choice** Which of the following is equal to $\dfrac{d^2y}{dx^2}$?

(A) $-\dfrac{6}{(2y - x)^3}$ (B) $\dfrac{10y^2 - 10x^2 - 10xy}{(2y - x)^3}$

(C) $\dfrac{8x^2 - 4xy + 8y^2}{(x - 2y)^3}$ (D) $\dfrac{10x^2 + 10y^2}{(x - 2y)^3}$ (E) $\dfrac{2}{x}$

63. **Multiple Choice** Which of the following is equal to dy/dx if $y = x^{3/4}$?

(A) $\dfrac{3x^{1/3}}{4}$ (B) $\dfrac{4x^{1/4}}{3}$ (C) $\dfrac{3x^{1/4}}{4}$ (D) $\dfrac{4}{3x^{1/4}}$ (E) $\dfrac{3}{4x^{1/4}}$

64. **Multiple Choice** Which of the following is equal to the slope of the tangent to $y^2 - x^2 = 1$ at $(1, \sqrt{2})$?

(A) $-\dfrac{1}{\sqrt{2}}$ (B) $-\sqrt{2}$ (C) $\dfrac{1}{\sqrt{2}}$ (D) $\sqrt{2}$ (E) 0

Extending the Ideas

65. ***Finding Tangents***

(a) Show that the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_1, y_1) has equation

$$\frac{x_1x}{a^2} + \frac{y_1y}{b^2} = 1.$$

(b) Find an equation for the tangent to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at the point (x_1, y_1).

66. ***End Behavior Model*** Consider the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Show that

(a) $y = \pm\dfrac{b}{a}\sqrt{x^2 - a^2}$.

(b) $g(x) = (b/a)|x|$ is an end behavior model for

$$f(x) = (b/a)\sqrt{x^2 - a^2}.$$

(c) $g(x) = -(b/a)|x|$ is an end behavior model for

$$f(x) = -(b/a)\sqrt{x^2 - a^2}.$$

Quick Quiz for AP® Preparation: Sections 4.1–4.2

1. **Multiple Choice** Which of the following gives $\dfrac{dy}{dx}$ for $y = \sin^4(3x)$?

(A) $4\sin^3(3x)\cos(3x)$ (B) $12\sin^3(3x)\cos(3x)$

(C) $12\sin(3x)\cos(3x)$ (D) $12\sin^3(3x)$

(E) $-12\sin^3(3x)\cos(3x)$

2. **Multiple Choice** What is the slope of the line tangent to the curve $2x^2 - 3y^2 = 2xy - 6$ at the point $(3, 2)$?

(A) 0 (B) $\dfrac{4}{9}$ (C) $\dfrac{7}{9}$ (D) $\dfrac{6}{7}$ (E) $\dfrac{5}{3}$

3. **Multiple Choice** Which of the following gives $\dfrac{dy}{dx}$ for the parametric curve $x = 3\sin t$, $y = 2\cos t$?

(A) $-\dfrac{3}{2}\cos t$ (B) $\dfrac{3}{2}\cos t$ (C) $-\dfrac{2}{3}\tan t$

(D) $\dfrac{2}{3}\tan t$ (E) $\tan t$

4. **Free Response** A curve in the xy-plane is defined by $xy^2 - x^3y = 6$.

(a) Find $\dfrac{dy}{dx}$.

(b) Find an equation for the tangent line at each point on the curve with x-coordinate 1.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.