

PC Review Sheet for Exam 1 Q3

* Answers to Monday's
hw are attached at
the end *

①

① rows by columns in 2×4 (B)

② $2x+y=3$ (A)
 $3x+2y=1$

③ $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 4 \end{bmatrix} \xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \end{bmatrix} \quad \begin{array}{l} y=2 \\ x=3 \end{array}$ (3,2) (C)

④ 3×2

⑤ $\left| \begin{array}{ccc} 2 & 3 & 4 \\ -3 & -4 & 5 \end{array} \right|$

⑥ $\left| \begin{array}{ccc|c} 2 & -1 & 1 & 6 \\ -1 & 2 & 1 & 0 \\ 1 & 1 & -1 & -3 \end{array} \right| \xrightarrow{R_1 \leftrightarrow R_3} \left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ -1 & 2 & 1 & 0 \\ 2 & -1 & 1 & 6 \end{array} \right| \xrightarrow{R_1 + R_2} \left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 3 & 0 & -3 \\ 2 & -1 & 1 & 6 \end{array} \right|$

$\frac{1}{3}R_2$ $\left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 1 & 0 & -1 \\ 2 & -1 & 1 & 6 \end{array} \right| \xrightarrow{-2R_1 + R_3} \left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & -3 & 3 & 12 \end{array} \right| \xrightarrow{\frac{1}{3}R_3} \left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & -1 & 1 & 4 \end{array} \right|$

$R_2 + R_3$ $\left| \begin{array}{ccc|c} 1 & 1 & -1 & -3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right| \quad \begin{cases} 2 = 3 \\ y = -1 \end{cases} \quad \begin{array}{l} x + y - 2 = -3 \\ x - 1 - 3 = -3 \\ x - 4 = -3 \\ \hline x = 1 \end{array} \quad (1, -1, 3)$

(2)

$$\textcircled{7} \quad \begin{aligned} \textcircled{1} \quad & x + 3y + z = 3 \\ \textcircled{2} \quad & x + 5y + 5z = 1 \\ \textcircled{3} \quad & 2x + 6y + 3z = 8 \end{aligned}$$

$$\begin{aligned} x + 3y + z &= 3 \\ x + 3(-5) + z &= 3 \\ x - 15 + z &= 3 \end{aligned}$$

$$\begin{aligned} x - 15 &= 3 \\ x &= 16 \end{aligned}$$

1+2 eliminate x

$$\begin{aligned} -1(x + 3y + z = 3) \quad & -x - 3y - z = -3 \\ x + 5y + 5z = 1 \quad & \underline{x + 5y + 5z = 1} \\ & 2y + 4z = -2 \quad \textcircled{4} \end{aligned}$$

$$\begin{aligned} 2y + 4z &= -2 \\ 2y + 4(2) &= -2 \\ 2y + 8 &= -2 \end{aligned}$$

1+3 eliminate x

$$\begin{aligned} -2(x + 3y + z = 3) \quad & -2x - 6y - 2z = -6 \\ 2x + 6y + 3z = 8 \quad & \underline{2x + 6y + 3z = 8} \\ & z = 2 \end{aligned}$$

$$\begin{aligned} 2y &= -10 \\ y &= -5 \end{aligned}$$

$$(16, -5, 2)$$

(3)

$$③ \quad x - z - 3z = -2$$

$$3x + y - 2z = 5$$

$$2x + 2y + z = 4$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{array} \right] \xrightarrow{-3R_1+R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 2 & 2 & 1 & 4 \end{array} \right] \xrightarrow{-3R_1 - 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 1 & 4 \end{array} \right]$$

$$\xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{array} \right] \xrightarrow{-2R_2+R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{array} \right] \xrightarrow{-\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{-2R_3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 0 & -4 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{2=2} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{y+7z=11} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{y+7(2)=11} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{y+14=11} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{y=-3} \left[\begin{array}{ccc|c} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{array} \right] \xrightarrow{(4, -3, 2)}$$

$$x - 3z = -2$$

$$x - 3(2) = -2$$

$$x - 6 = -2$$

$$x = 4$$

(4)

$$⑨ \quad x - 2y + 3z = 9$$

$$-x + 3y = -4$$

$$2x - 5y + 5z = 17$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array} \right]$$

$$-2R_1 + R_3$$

$$2R_1 \left[\begin{array}{ccc|c} -2 & 4 & -6 & -18 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array} \right]$$

$$R_2 + R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array} \right]$$

$$\frac{1}{2}R_3$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\boxed{z=2}$$

$$y + 3z = 5$$

$$y + 3(2) = 5$$

$$y + 6 = 5$$

$$y = -1$$

$$x - 2y + 3z = 9$$

$$x - 2(-1) + 3(2) = 9$$

$$x + 2 + 6 = 9$$

$$x + 8 = 9$$

$$\boxed{x=1}$$

$$\boxed{(1, -1, 2)}$$

(5)

10

$$(a) \begin{array}{l} 7x - 2y = 14 \\ -3y + 7x = 21 \end{array} \rightarrow \begin{array}{l} 7x - 2y = 14 \\ -(7x - 3y = 21) \end{array} \quad \begin{array}{l} 7x - 2y = 14 \\ -7x + 3y = -21 \end{array}$$

$$y = -7$$

$$\begin{array}{l} 7x - 2y = 14 \\ 7x - 2(-7) = 14 \quad | (0, -7) \\ 7x + 14 = 14 \\ 7x = 0 \\ x = 0 \end{array}$$

$$(b) \begin{array}{l} -4x + 5y = 14 \\ 4x - 3y = -8 \end{array}$$

$$\begin{array}{l} 2y = 6 \\ y = 3 \end{array}$$

$$\begin{array}{l} -4x + 5y = 14 \\ -4x + 5(3) = 14 \quad | (\frac{1}{4}, 3) \\ -4x + 15 = 14 \\ -4x = -1 \\ x = \frac{1}{4} \end{array}$$

$$(c) \begin{array}{l} -a - 4b = -93 \\ a + 4b = 43 \quad \text{no solution} \\ 0 = -50 \end{array}$$

$$(d) \begin{array}{l} 3(8x + 12y = 48) \\ -4(6x + 9y = 36) \end{array} \quad \begin{array}{l} 24x + 36y = 144 \\ -24x - 36y = -144 \\ 0 = 0 \end{array} \quad \text{infinite solutions}$$

in terms of x $(x, -\frac{2}{3}x + 4)$ need y in terms of x

$$8x + 12y = 48$$

$$\begin{array}{l} 12y = -8x + 48 \\ y = \frac{-8}{12}x + \frac{48}{12} \end{array}$$

$$\text{in terms of } y \quad (-\frac{3}{2}y + 6, y)$$

need x in terms of y

$$y = -\frac{2}{3}x + 4$$

general solution:

$$8x + 12y = 98$$

$$8x = -12y + 48$$

$$x = -\frac{12}{8}y + \frac{48}{8} = -\frac{3}{2}y + 6 \quad (6)$$

$$(e) 4x - 5y = -6$$

$$-4(x - 3y = 2)$$

$$4x - 5y = -6$$

$$-4x + 12y = -8$$

$$7y = -14$$

$$y = -2$$

$$4x - 5y = -6$$

$$4x - 5(-2) = -6$$

$$4x + 10 = -6 \quad | \quad (-4, -2)$$

$$4x = -16$$

$$x = -4$$

D)

$$(a) G = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{array} \right]$$

$$-3R_1 + R_2$$

$$-3R_1 [-3 \ 6 \ -3 \ | \ -21]$$

$$(b) H = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 2 & 3 & 2 & 7 \end{array} \right]$$

$$-2R_1 + R_3$$

$$2R_1$$

$$[-2 \ 4 \ -2 \ | \ -14]$$

$$(c) J = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 7 & 0 & -7 \end{array} \right]$$

$$(d)$$

$$-R_2 + R_3$$

$$-R_2$$

$$[0 \ -7 \ 4 \ | \ 19]$$

$$(d) K = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 0 & 4 & 12 \end{array} \right]$$

$$(e)$$

$$3R_2$$

$$L = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & 0 & -7 \\ 0 & 0 & 4 & 12 \end{array} \right]$$

$$(f)$$

$$\frac{1}{7}R_2$$

$$P_1 =$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 4 & 12 \end{array} \right]$$

$$(g)$$

$$\frac{1}{4}R_3$$

$$Q = \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$(h) z = 3$$

$$y = -1$$

$$(2, -1, 3)$$

$$x - 2y + z = 7$$

$$x - 2(1) + 3 = 7$$

$$x + 2 + 3 = 7$$

$$x + 5 = 7$$

$$x = 2$$

(7)

$$\textcircled{2} \quad (a) \quad \frac{x}{x-2} - \frac{8}{x+3} = \frac{10}{x^2+x-6}$$

$$x^2 + 3x - 8x + 16 = 10$$

$$x^2 - 5x + 6 = 0$$

$$(x-3)(x-2) = 0$$

$$\boxed{x=3} \quad \boxed{x=2}$$

reject

$$(b) \quad \frac{9}{x} + \frac{9}{x-2} = \frac{12}{x(x-2)}$$

$$9x - 18 + 9x = 12x^2 - 24x$$

$$18x - 18 = 12x^2 - 24x$$

$$0 = 12x^2 - 42x + 18$$

$$0 = 6(2x^2 - 7x + 3)$$

$$0 = 6(2x^2 - 6x - x + 3)$$

$$0 = 6(2x(x-3) - 1(x-3))$$

$$0 = 6(2x-1)(x-3)$$

$$\boxed{x=\frac{1}{2}} \quad \boxed{x=3}$$

$$(c) \quad \frac{4}{x+1} = \frac{4}{5} \frac{(x+1)}{2x-2} + \frac{3x}{4}$$

$$2(x+1)$$

$$0 = 3(x-2)(x+1)$$

$$16 = 10 + 3x^2 - 3x$$

$$\boxed{x=2} \quad \boxed{x=-1}$$

$$0 = 3x^2 - 3x - 6$$

$$0 = 3(x^2 - x - 2)$$

$$0 = 3(x-2)(x+1)$$

(8)

$$(d) \frac{1}{2a} - \frac{9}{a^2 + 6a} = \frac{2-a}{2(a+b)}$$

$$a+b - 18 = 2a - a^2$$

$$a^2 - a - 12 = 0$$

$$(a-4)(a+3) = 0$$

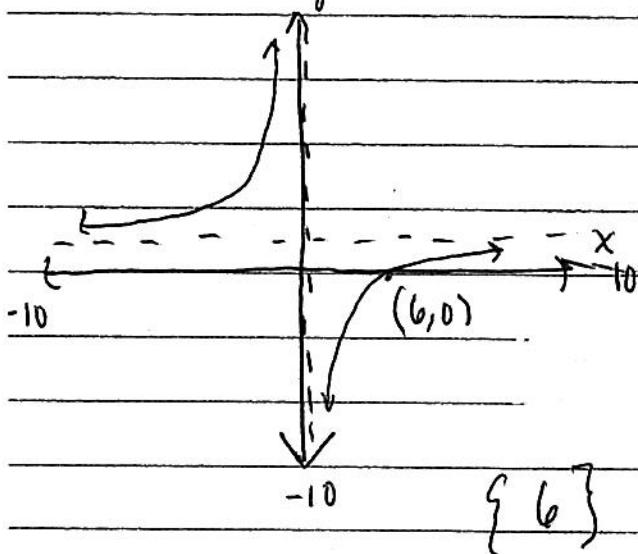
$$\boxed{a=4 \quad a=-3}$$

$$(13) (a) \frac{x-2}{x} - \frac{x+2}{2x}$$

$$\frac{x-2}{x} - \frac{x+2}{2x} = 0$$

possible vertical asymptotes: $x=0$

end behavior: $y = 1 - \frac{1}{2} = +\frac{1}{2}$



(9)

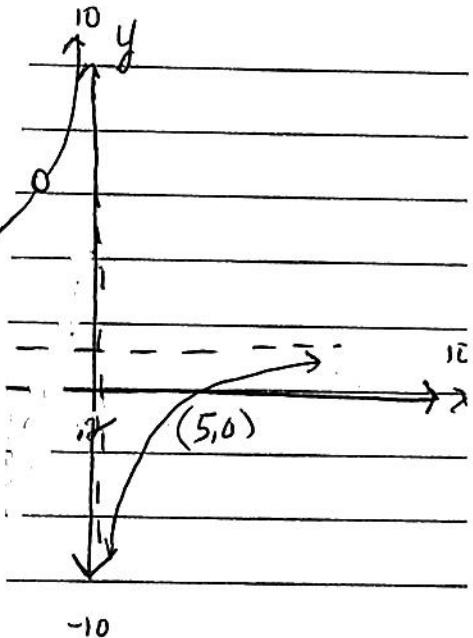
$$(b) \frac{x}{x+2} = \frac{3}{x} + \frac{4}{x^2+2x}$$

$$\frac{x}{x+2} - \frac{3}{x} - \frac{4}{x(x+2)} = 0$$

pVA $x=0, -2$ hole here
not an asymptote

$$EB: y = 1 - 0 - 0 = 1$$

{5}



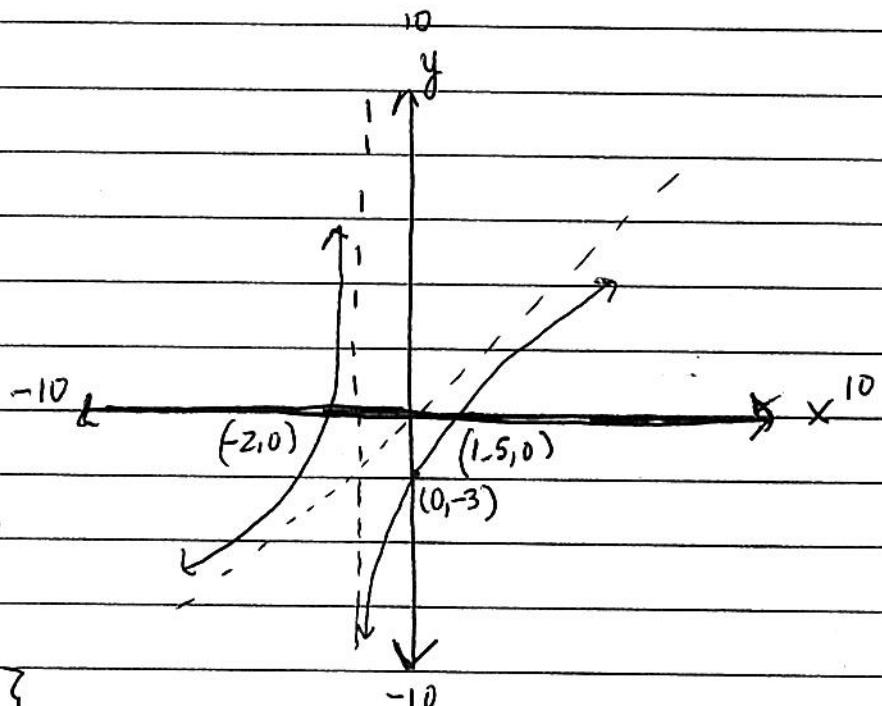
$$(c) \frac{x}{2} = \frac{3}{2x+1}$$

$$\frac{x}{2} - \frac{3}{2x+1} = 0$$

$$pVA \quad x = -\frac{1}{2}$$

$$EB \quad y = \frac{1}{2}x - 0 = \frac{1}{2}x$$

{-2, 1.5}

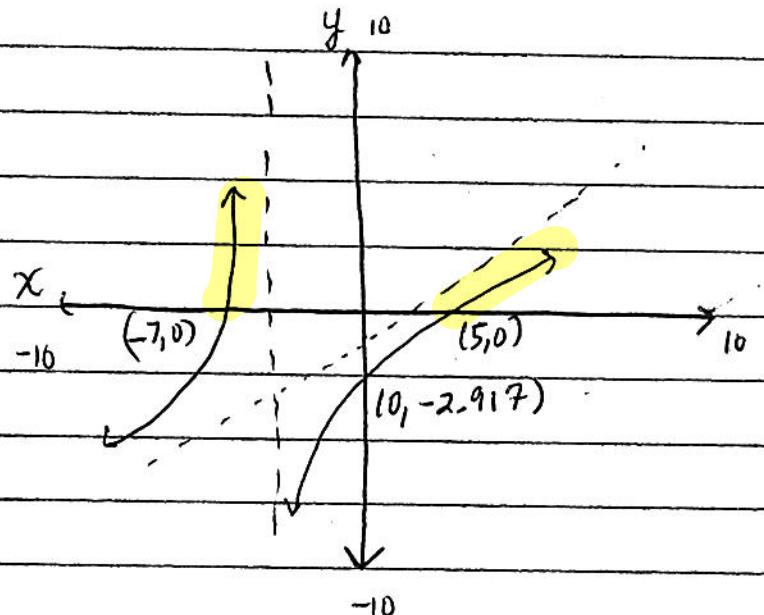


(10)

(14) $\frac{x-1}{4} \geq \frac{8}{x+3}$

(a) $4 \leq x+3$

$$\frac{x-1}{4} - \frac{8}{x+3} \geq 0$$



PVA $x = -3$

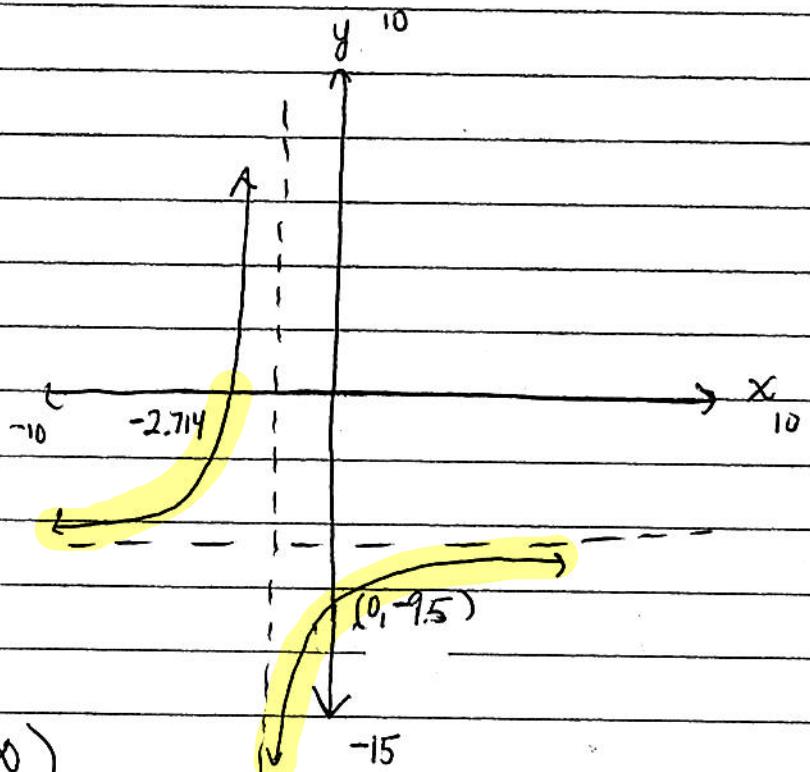
EB $y = \frac{1}{4}x - \frac{1}{4} - 0 = \frac{1}{4}x - \frac{1}{4}$

$$W [-7, -3) \cup [5, \infty)$$

$$SB \{x \mid -7 \leq x < -3 \vee x \geq 5\}$$

(b) $\frac{x-3}{x+2} < 8$

$$\frac{x-3}{x+2} - 8 < 0$$



PVA: $x = -2$

EB: $y = 1 - 8 = -7$

$$W (-\infty, -2.714) \cup (-2, \infty)$$

$$SB \{x \mid x < -2.714 \vee x > -2\}$$

$$(c) \frac{3}{x+1} + \frac{2}{x} < 6$$

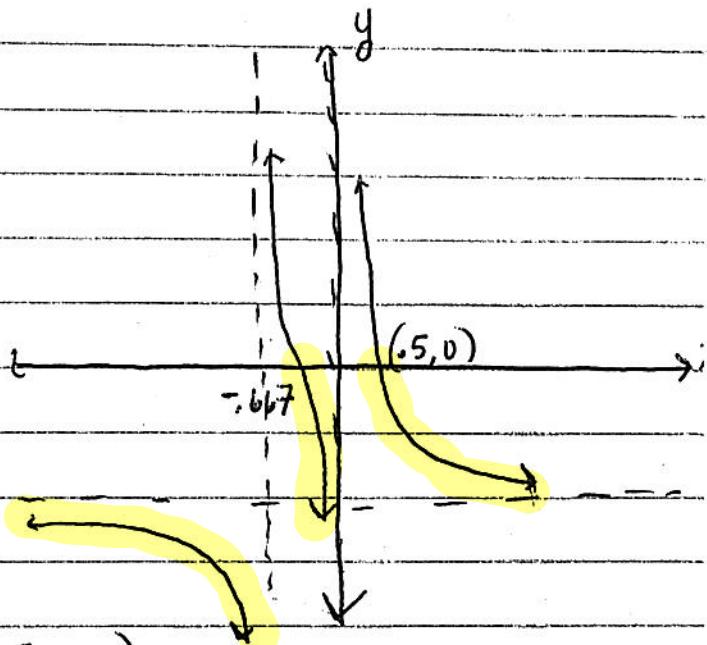
$$\frac{3}{x+1} + \frac{2}{x} - 6 < 0$$

PVA: $x=0, -1$

$$EB: y = 0 + 0 - 6 = -6$$

$$IV (-\infty, -1) \cup (-667, 0) \cup (.5, \infty)$$

$$SB \left\{ x \mid x < -1 \vee -667 < x < 0 \vee x > .5 \right\}$$



Homework 02-12

$$15. \begin{cases} x - 2y + z = 1 \\ y + 2z = 5 \\ x + y + 3z = 8 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 1 & 1 & 3 & 8 \end{array} \right] \xrightarrow{-R_1+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 1 & 3 & 8 \end{array} \right] \xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 3 \end{array} \right] \xrightarrow{-3R_2+R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 2 & 3 \end{array} \right] \xrightarrow{-3R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & -15 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & -15 \end{array} \right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & \frac{15}{4} \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 2 & 5 \\ 0 & 0 & 1 & \frac{15}{4} \end{array} \right] \quad \begin{aligned} z &= 2 \\ y + 2z &= 5 \\ y + 2(2) &= 5 \\ y &= 1 \end{aligned}$$

$$(1, 1, 2) \quad \begin{aligned} x - 2y + z &= 1 \\ x - 2(1) + 2 &= 1 \\ x &= 1 \end{aligned}$$

16.
$$\begin{cases} x + y + 6z = 3 \\ x + y + 3z = 3 \\ x + 2y + 4z = 7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 1 & 1 & 3 & 3 \\ 1 & 2 & 4 & 7 \end{array} \right] \xrightarrow[-R_1+R_2]{-R_3[-1-1-6-3]} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & -3 & 0 \\ 1 & 2 & 4 & 7 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & -3 & 0 \\ 1 & 2 & 4 & 7 \end{array} \right] \xrightarrow{-\frac{1}{3}R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 4 & 7 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & 1 & 0 \\ 1 & 2 & 4 & 7 \end{array} \right] \xrightarrow[-R_1+R_3]{-R_1[-1-1-6-3]} \left[\begin{array}{ccc|c} 1 & 1 & 6 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 4 \end{array} \right] \quad \begin{aligned} z &= 0 \\ y - 2z &= 4 \\ y - 2(0) &= 4 \\ y &= 4 \end{aligned}$$

$$(-1, 4, 0) \quad \begin{aligned} x + y + 6z &= 3 \\ x + 4 + 6(0) &= 3 \\ x + 4 &= 3 \\ x &= -1 \end{aligned}$$

$$17. \begin{cases} x + y + z = 2 \\ 2x - 3y + 2z = 4 \\ 4x + y - 3z = 1 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 2 & -3 & 2 & 4 \\ 4 & 1 & -3 & 1 \end{array} \right] \xrightarrow{-2R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -5 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{array} \right] \xrightarrow{-2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 4 & 1 & -3 & 1 \end{array} \right] \xrightarrow{-4R_1+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -3 & -7 \end{array} \right] \xrightarrow{-4R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -7 & -7 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & -3 & -7 & -7 \end{array} \right] \xrightarrow{+3R_2+R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -7 & -7 \end{array} \right] \xrightarrow{\frac{1}{7}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right] \quad \begin{aligned} z &= 1 \\ y &= 0 \\ x + y + z &= 2 \\ x + 0 + 1 &= 2 \\ x + 1 &= 2 \\ x &= 1 \end{aligned}$$

$$(1, 0, 1)$$

$$18. \begin{cases} x + y + z = 4 \\ -x + 2y + 3z = 17 \\ 2x - y = -7 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ -1 & 2 & 3 & 17 \\ 2 & -1 & 0 & -7 \end{array} \right] \xrightarrow{R_1+R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 2 & -1 & 0 & -7 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_3 \\ -2R_1 [-2 \ -2 \ -2 \ -8] \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & -3 & -2 & -15 \end{array} \right]$$

(-2, 3, 3)

$$R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 2 & 6 \end{array} \right] \xrightarrow{\frac{1}{2}R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 3 & 4 & 21 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\begin{aligned} z &= 3 \\ 3y + 4z &= 21 \\ 3y + 4(3) &= 21 \\ 3y + 12 &= 21 \\ 3y &= 9 \\ y &= 3 \end{aligned}$$

$$\begin{aligned} x + y + z &= 4 \\ x + 3 + 3 &= 4 \\ x + 6 &= 4 \\ x &= -2 \end{aligned}$$

row echelon
OR

$$\frac{1}{3}R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 0 & 1 & \frac{4}{3} & 7 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$\begin{aligned} y + \frac{4}{3}z &= 7 \\ y + \frac{4}{3}(3) &= 7 \\ y &= 3 \end{aligned}$$

$$19. \begin{cases} x + 2y - z = -2 \\ x + z = 0 \\ 2x - y - z = -3 \end{cases}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 1 & 0 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right] \xrightarrow{-R_1+R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 2 & 2 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right] \quad \xrightarrow{-R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 2 & 2 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 2 & 2 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right] \xrightarrow{-\frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 2 & -1 & -1 & -3 \end{array} \right] \xrightarrow{-2R_1+R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & -3 & -3 & -3 \end{array} \right] \quad \xrightarrow{-2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & -3 & -3 & -3 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & -3 & -3 & -3 \end{array} \right] \xrightarrow{5R_2+R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & -2 & -3 \end{array} \right] \quad \xrightarrow{5R_2} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & -5 & -5 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 5 & -5 & -5 \end{array} \right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & 2 & -1 & -2 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$z = 1$$

$$y - z = -1$$

$$y - 1 = -1$$

$$y = 0$$

$$x + 2y - z = -2$$

$$x + 2(0) - 1 = -2$$

$$x - 1 = -2$$

$$x = -1$$

$$(-1, 0, 1)$$