Show all work on a separate sheet of paper. Make sure to study your notes and homework as well.

1. Sketch each function using a minimum of 2 points. For each graph, state the domain, range, intercepts, and the equations of any asymptotes.

    a. $y = \dfrac{1}{(x+4)^2} - 2$ volcano · left 4 · ↓ 2

    b. $y = -\dfrac{1}{x+2}$ hyperbola · left 2 · reflect over x

    c. $y = \dfrac{1}{-(x+3)} + 1$ hyperbola · left 3 · reflect over x · ↑ 1

    d. $y = -\dfrac{3}{x^2}$ volcano · mult. y values by -3

2. Sketch each function. For each graph, state the domain, range, intercepts, coordinates of any holes, and the equations of any asymptotes.   hole $(3, -5)$

    a. $y = \dfrac{x^3-1}{x-1} = \dfrac{(x-1)(x^2+x+1)}{(x-1)}$ hole $(1,3)$ $x^2 + x + \frac{1}{4} - \frac{1}{4} + 1$ $y = (x+\frac{1}{2})^2 + \frac{3}{4}$

    b. $y = \dfrac{x-4}{x^2-16} = \dfrac{1}{(x-4)(x+4)} = \dfrac{1}{x+4}$ hole $(4, \frac{1}{8})$

    c. $y = \dfrac{x^3-3x^2-4x+12}{-(3x^2+5x-2)} = \dfrac{x^2(x-3)-4(x-3)}{3-x}$ $= \dfrac{(x^2-4)(x-3)}{(3-x)} = -x^2+4$

    d. $y = \dfrac{2-5x-3x^2}{x+2} = \dfrac{-(3x-1)(x+2)}{x+2} = -3x+1$ hole $(-2, 7)$
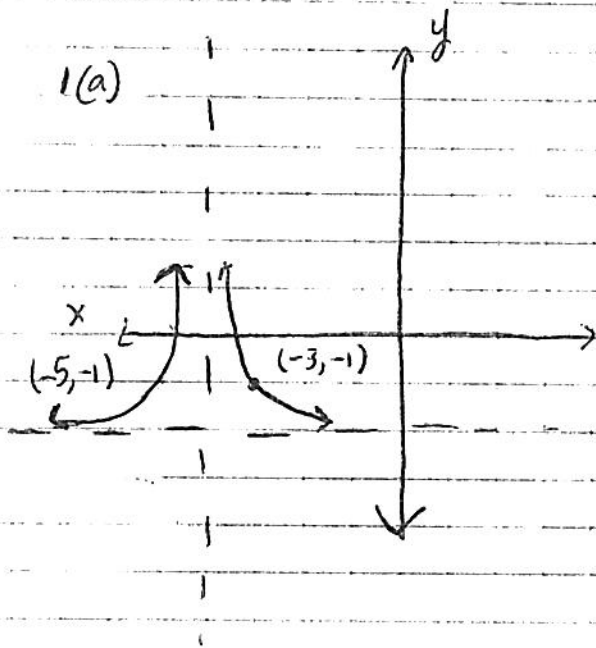
3. Complete the chart for each of the following:

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	Oblique Asymptote	x-intercept(s)	y-intercept
$y = \dfrac{3-x}{x^2-9} = \dfrac{-1}{(x-3)(x+3)} = \dfrac{-1}{x+3}$	$(3, -\frac{1}{6})$	$x = -3$	$y = 0$	none	none	$(0, -\frac{1}{3})$
$y = \dfrac{3x^3 \cdot 3}{x^4+x} = \dfrac{3x^3}{x(x^3+1)}$	$(0,0)$	$x^3+1=0$ $x^3=-1$ $x=-1$	$y=3$	none	none	none
$y = \dfrac{x^2+2x-24}{x+6} = \dfrac{(x+6)(x-4)}{x+6} = x-4$	$(-6, -10)$	none	none	none	$(4,0)$	$(0,-4)$
$y = \dfrac{x^2+3x+2}{x-2} = \dfrac{(x+1)(x+2)}{x-2}$	none	$x=2$	none	$y=x+5$	$(-1,0)$ $(-2,0)$	$(0,-1)$
$y = \dfrac{x^3-3x^2}{x^2-1} = \dfrac{x^2(x-3)}{(x+1)(x-1)}$	none	$x = \pm 1$	none	$y=x-3$	$(0,0)$ $(3,0)$	$(0,0)$

1(a)

Points labeled: $(-5,-1)$, $(-3,-1)$

D: $x \neq -4$
R: $y > -2$
x-int $\left(-4 \pm \frac{\sqrt{2}}{2}, 0\right)$
y-int $\left(0, -3\frac{1}{16}\right)$
VA: $x = -4$
HA: $y = -2$
OA: none

(b)

Points labeled: $(-3,1)$, $(-1,-1)$
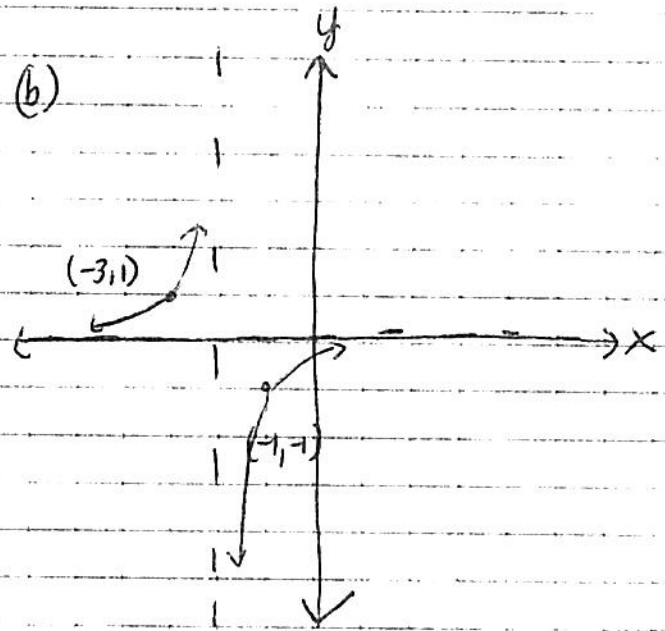
D: $x \neq -2$
R: $y \neq 0$
x-int: none
y-int: $\left(0, -\frac{1}{2}\right)$
VA $x = -2$
HA $y = 0$
OA none

(c)

Points labeled: $(-4,2)$, $(-2,0)$
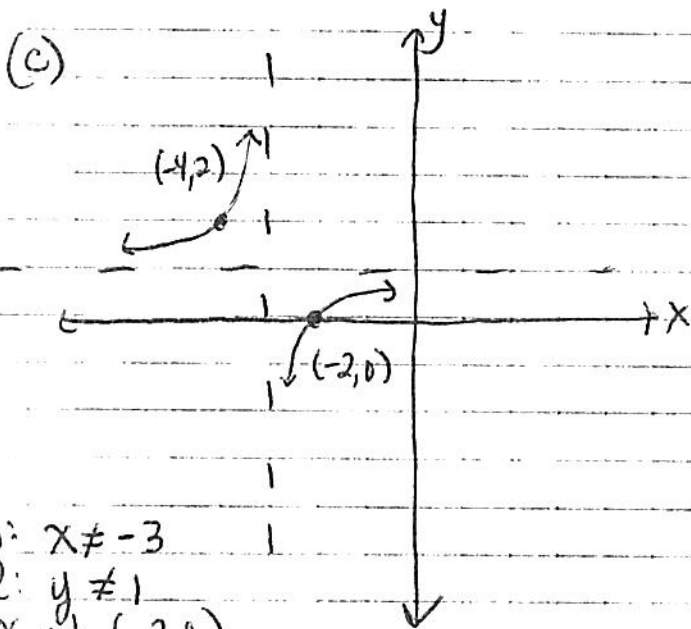
D: $x \neq -3$
R: $y \neq 1$
x-int $(-2, 0)$
y-int $\left(0, \frac{2}{3}\right)$
VA $x = -3$
HA $y = 1$
OA none

(d)

$(1,1) \rightarrow (1,-3)$
$(-1,1) \rightarrow (-1,-3)$

D: $x \neq 0$ VA $x = 0$
R: $y < 0$ HA $y = 0$
x-int: none   OA none
y-int: none

2 (a)
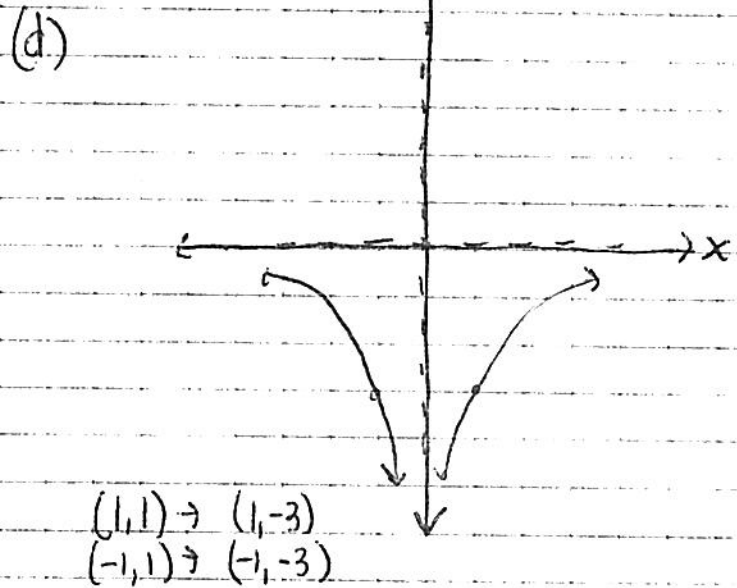
$$y = \frac{x^3-1}{x-1} = \frac{(x-1)(x^2+x+1)}{x-1} = x^2+x+1$$
$$= (x+\tfrac{1}{2})^2 + \tfrac{3}{4}$$

hole: (1,3)
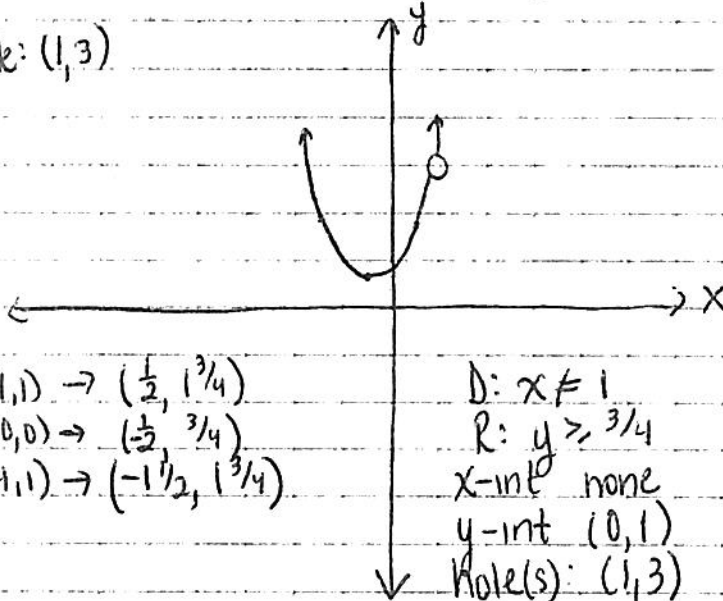
$(1,1) \to (\tfrac{1}{2}, 1\tfrac{3}{4})$
$(0,0) \to (-\tfrac{1}{2}, \tfrac{3}{4})$
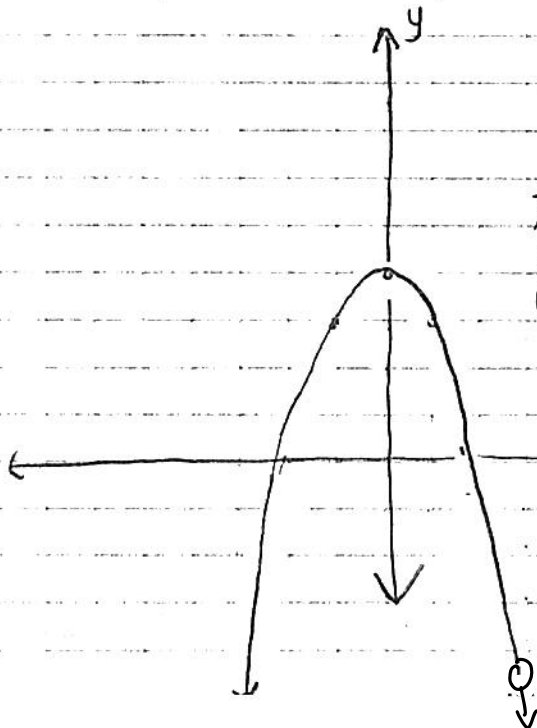$(-1,1) \to (-1\tfrac{1}{2}, 1\tfrac{3}{4})$

D: $x \neq 1$
R: $y \geq 3/4$
x-int none
y-int (0,1)
Hole(s): (1,3)
VA : none
HA : none
OA : none

(b)

$$y = \frac{x-4}{x^2-16} = \frac{x-4}{(x-4)(x+4)} = \frac{1}{x+4}$$

(-3,1)

(-5,-1)
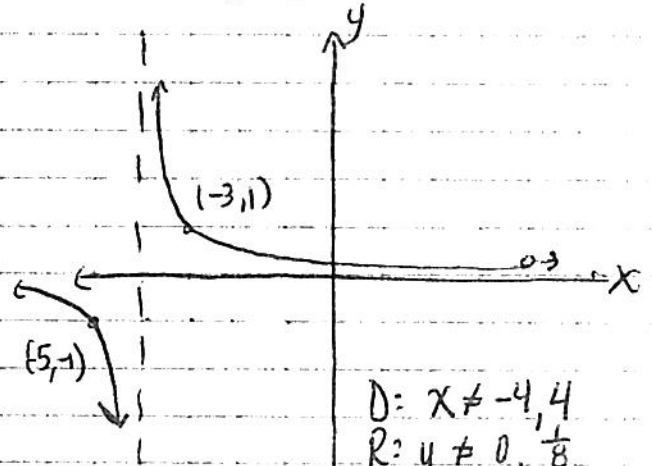
D: $x \neq -4, 4$
R: $y \neq 0, \tfrac{1}{8}$
x-int : none
y-int : $(0, \tfrac{1}{4})$
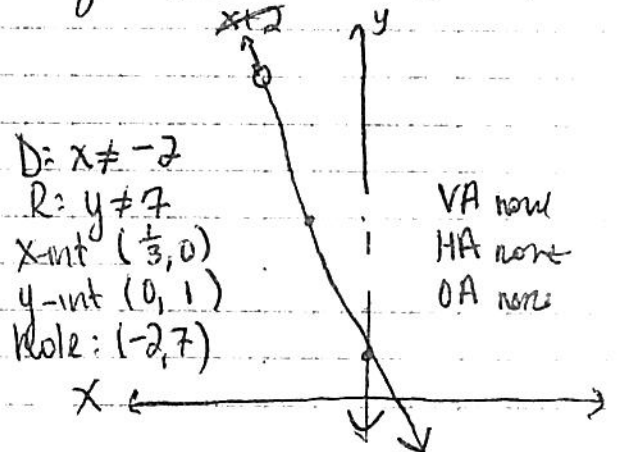hole: $(4, \tfrac{1}{8})$
VA $x = -4$
HA $y = 0$
OA none

(c) $y = \frac{x^3-3x^2-4x+12}{3-x}$

$x^2(x-3) -4(x-3)$

$$= \frac{(x^2-4)(x-3)}{3-x} = -(x^2-4) = -x^2+4$$

D: $x \neq 3$
R: $y \leq 4$
x-int $(\pm 2, 0)$
y-int $(0, 4)$
hole: $(3, -5)$
VA none
HA none
OA none

(d) $y = \frac{-3x^2-5x+2}{2-5x-3x^2}$ ... $y = \frac{2-5x-3x^2}{x+2}$

$$y = \frac{-(3x^2+5x-2)}{x+2}$$
$$y = \frac{-(3x-1)(x+2)}{x+2} = -3x+1$$

D: $x \neq -2$
R: $y \neq 7$
x-int $(\tfrac{1}{3}, 0)$
y-int $(0, 1)$
Hole: $(-2, 7)$

VA none
HA none
OA none