Name: _____ Date:_____

PC Ms. Loughran

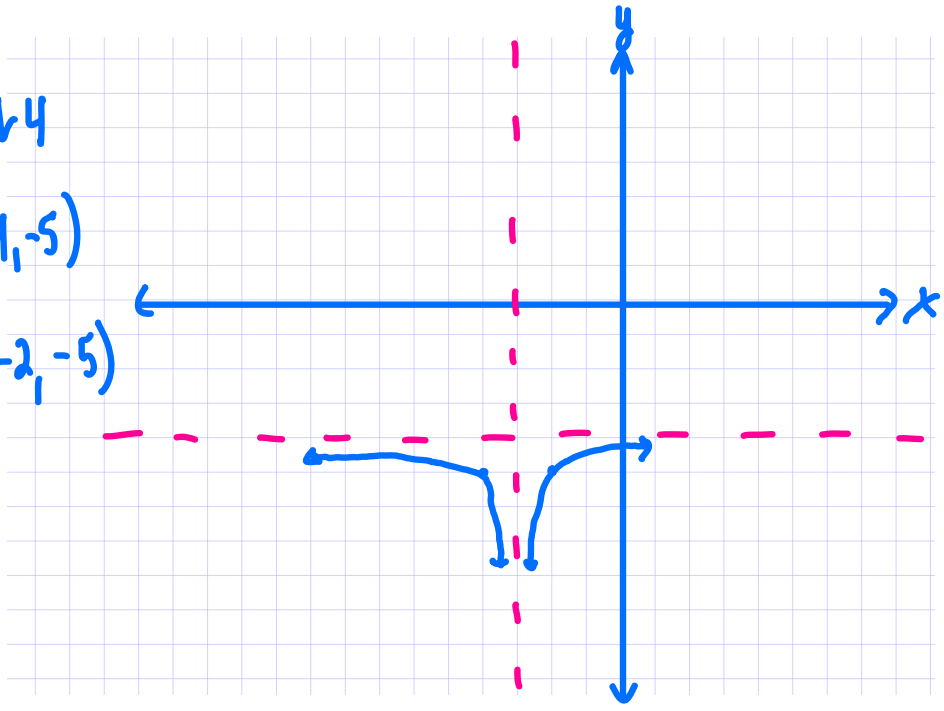
Do Now:

1. Sketch the graph of $y = -\frac{1}{(x+3)^2} - 4$. (Be sure to include a minimum of 2 points and any and all asymptotes.) State the domain, range, intercepts and equations of any asymptotes.

volcano left 3 reflect over x-axis ↓4

(-1, 1) (-4, 1) (-4, -1) (-4, -5)

(1, 1) (-2, 1) (-2, -1) (-2, -5)



VA: $x = -3$

HA: $y = -4$

Cross? $\frac{-1}{(x+3)^2} - 4 = -4$

$\frac{-1}{(x+3)^2} = \frac{0}{1}$ no

$0 \neq -1$

D: $\{x \mid x \neq -3\}$

R: $\{y \mid y < -4\}$ or $(-\infty, -4)$

x-int: no

y-int: $(0, -4\frac{1}{9})$ or $(0, \frac{-37}{9})$

Name:_____ Date:_____

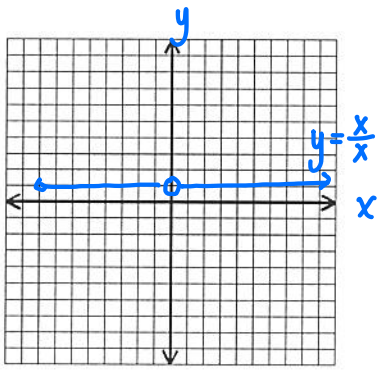
PC: Reducible Functions Ms. Loughran

Undefined: $\frac{a}{b}$ where $b = 0$ and $a \neq 0$

Indeterminate: $\frac{a}{b}$ where $b = 0$ and $a = 0$

A rational function that is indeterminate for a value of x is *reducible*. A "hole" occurs at the value(s) of x which make the given function indeterminate and the reduced fraction defined.

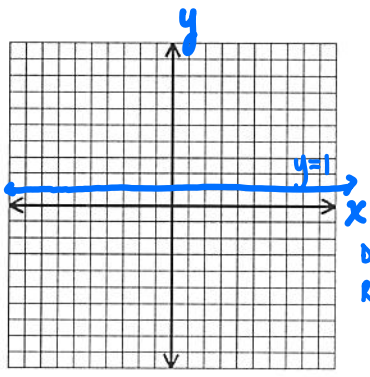
Is the graph of $y = \frac{x}{x}$ the same as the graph of $y = 1$?



$y = \frac{x}{x} = 1 \quad x \neq 0$

D: $\{x | x \neq 0\}$

R: $\{1\}$



D: $(-\infty, \infty)$

R: $\{1\}$

If a function is reducible use the reduced function when finding the intercepts.

Graph each of the following. State the domain, range, and any intercepts and asymptotes.

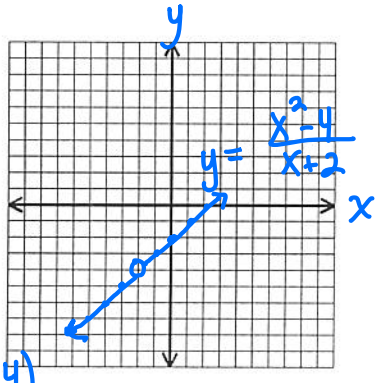
1. $y = \frac{x^2 - 4}{x + 2} = \frac{(x+2)(x-2)}{x+2}$

To find the x-value of the hole: set the factor you canceled out $= 0$ and solve

$x + 2 = 0$

$x = -2$

hole: $(-2, -4)$



D: $\{x | x \neq -2\}$

R: $\{y | y \neq -4\}$

x-int: $(2, 0)$

y-int: $(0, -2)$

asymptotes: no

reduced function: $y = x - 2$ ↑ line with $m = 1$, $b = -2$

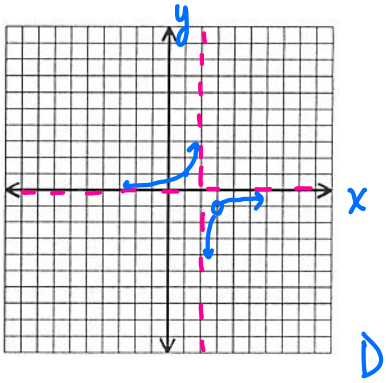
To find the y value of the hole plug the x value of the hole into the reduced function.

2. $y = \dfrac{3-x}{x^2-5x+6}$ $\quad -1$

$(x-3)(x-2)$

hole: $(3, -1)$

reduced function: $y = \dfrac{-1}{x-2}$



* plug $x=0$ into reduced function

D: $\{x \mid x \neq 2, 3\}$

R: $\{y \mid y \neq -1, 0\}$

x-int: none

y-int: $(0, \frac{1}{2})$

VA: $x=2$

HA: $y=0$

VA: $x=2$

HA: $y=0$

Cross? $\dfrac{-1}{x-2} \neq 0$

hyperbola

right 2

reflect over x-axis

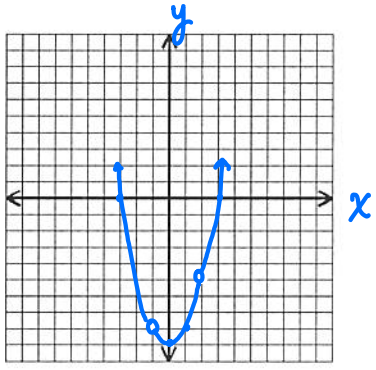
$(-1,-1)$ $(1,-1)$ $(1,1)$

$(1,1)$ $(3,1)$ $(3,-1)$

3. $y = \dfrac{(x+1)(x+3)(x-3)(x-2)}{(x+1)(x-2)}$

holes: $(-1, -8)$

$(2, -5)$



D: $\{x \mid x \neq -1, 2\}$

R: $\{y \mid y \geq -9\}$

x-int: $(\pm 3, 0)$

y-int: $(0, -9)$

asymptotes: none

reduced function $y = (x+3)(x-3)$

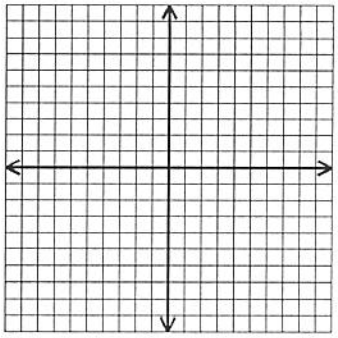
$y = x^2 - 9$

↑ parabola ↓ 9

$0 = x^2 - 9$

$0 = (x+3)(x-3)$

$x = -3 \mid x = 3$



4. $y = \dfrac{x^3 - 1}{x - 1}$ $(x-1)(x^2+x+1)$

hole: $(1, 3)$

reduced function: $y = x^2 + x + 1$

$y = x^2 + x + \frac{1}{4} - \frac{1}{4} + 1$

$y = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4}$

vertex: $\left(-\frac{1}{2}, \frac{3}{4}\right)$

to be continued...

Practice

Graph each of the following. State the domain, range, and any intercepts and asymptotes.

1. $y = \dfrac{x^2 - 9}{x + 3}$

2. $y = \dfrac{x^2 - x - 6}{x - 3}$

3. $y = \dfrac{x^2 - 16}{x + 4}$

4. $y = \dfrac{x + 1}{x^2 - 1}$

5. $y = \dfrac{x - 1}{x^2 + x - 2}$

6. $y = \dfrac{1 + x - 2x^2}{x - 1}$

7. $y = \dfrac{x^3 - 8}{x - 2}$

8. $y = \dfrac{x - 1}{x^2 - 1}$

9. $y = \dfrac{x^3 - 2x^2 - 3x + 6}{2 - x}$

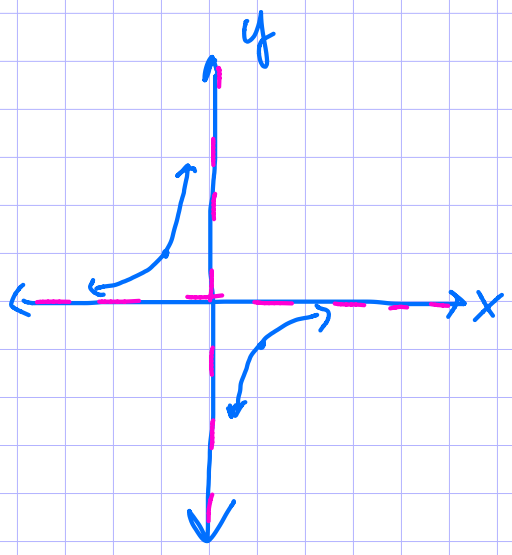
Name:_____ Date:_____

PC: More Hyperbolas and Volcanoes

Sketch each function using a minimum of 2 points and including any and all asymptotes.
For each graph, state the domain, range, intercepts and equations of any asymptotes.

ODDS

1. $y = -\frac{1}{x}$



hyperbola reflected over x-axis

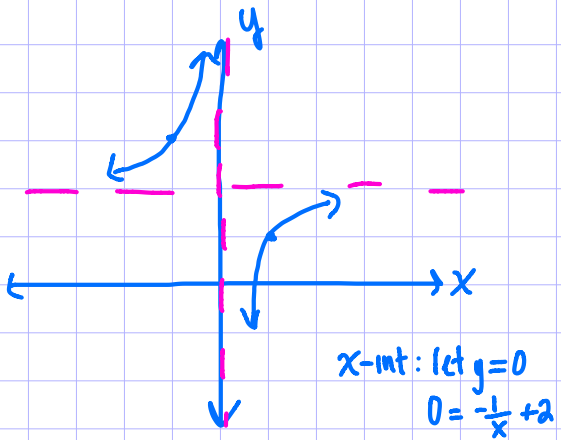
(-1,-1) (-1,1)

(1,1) (1,-1)

D: $x \neq 0$ VA: $x = 0$ x-int: none

R: $y \neq 0$ HA: $y = 0$ y-int: none

3. $y = -\frac{1}{x} + 2$



hyperbola reflected over x-axis ↑2

(-1,-1) (-1,1) (-1,3)

(1,1) (1,-1) (1,1)

D: $x \neq 0$ VA: $x = 0$ x-int $(\frac{1}{2}, 0)$

R: $y \neq 2$ HA $y = 2$ y-int: none

x-int: let $y = 0$

$0 = -\frac{1}{x} + 2$

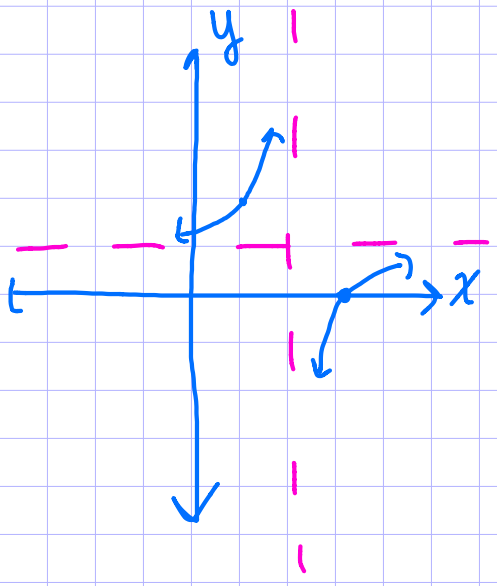
$-2 = -\frac{1}{x}$

$2 = \frac{1}{x}$

$2x = 1$

$x = \frac{1}{2}$

5. $y = -\frac{1}{x-2} + 1$



hyperbola right 2 reflect over x axis ↑1

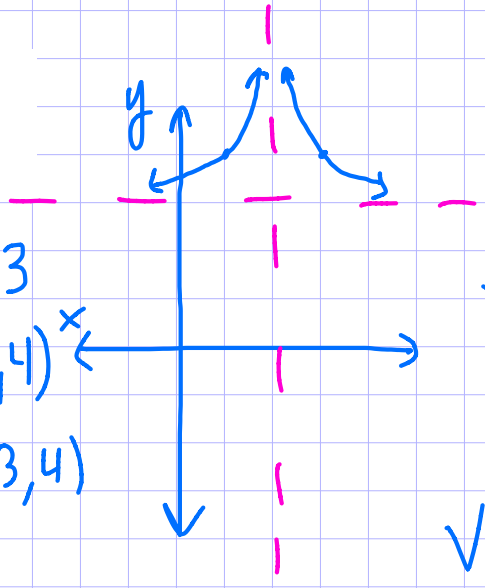
(-1,-1) (1,-1) (1,1) (1,2)

(1,1) (3,1) (3,-1) (3,0)

D: $x \neq 2$ VA: $x = 2$ x-int (3, 0)

R: $y \neq 1$ HA: $y = 1$ y-int: $(0, 1\frac{1}{2})$

7. $y = \frac{1}{(x-2)^2} + 3$



volcano right 2 ↑3

(-1,1) (1,1) (1,4)

(1,1) (3,1) (3,4)

D: $x \neq 2$

R $y > 3$

VA: $x = 2$

HA: $y > 3$

x-int: none

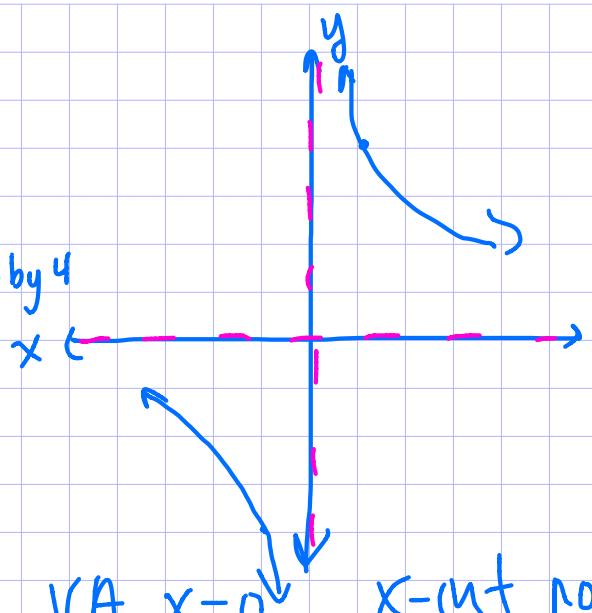
y-int: $(0, 3\frac{1}{4})$

9. $y = \frac{4}{x} = 4 \cdot \frac{1}{x}$

hyperbola mult. y values by 4

$(-1,-1)$ $(-1,-4)$

$(1,1)$ $(+1,4)$



D $x \neq 0$ VA $x=0$ x-int none

R $y \neq 0$ HA $y=0$ y-int none

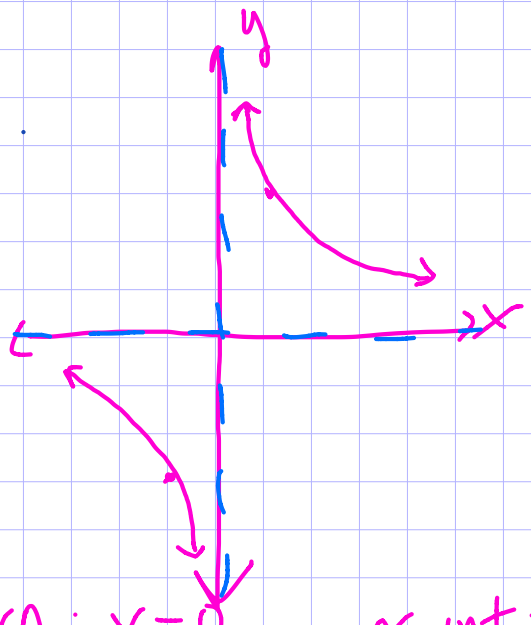
11. $xy = 3$

$y = \frac{3}{x} = 3 \cdot \frac{1}{x}$

hyperbola mult. y's by 3

$(-1,-1) \rightarrow (-1,-3)$

$(1,1) \rightarrow (1,3)$



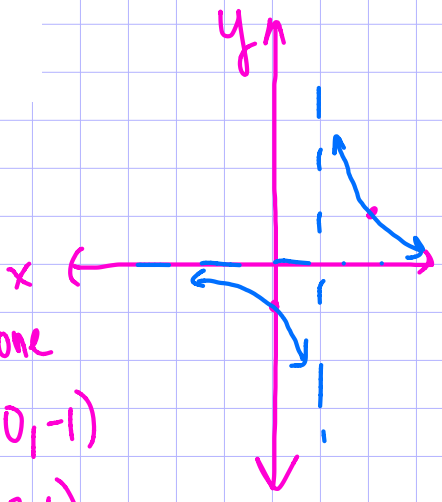
D: $x \neq 0$ VA: $x=0$ x-int: none

R: $y \neq 0$ HA: $y=0$ y-int: none

13. $xy - y = 1$

$y(x-1) = 1$

$y = \frac{1}{x-1}$



hyperbola right one

$(-1,-1)$ $(0,-1)$

$(1,1)$ $(2,1)$

D: $x \neq 1$
R: $y \neq 0$

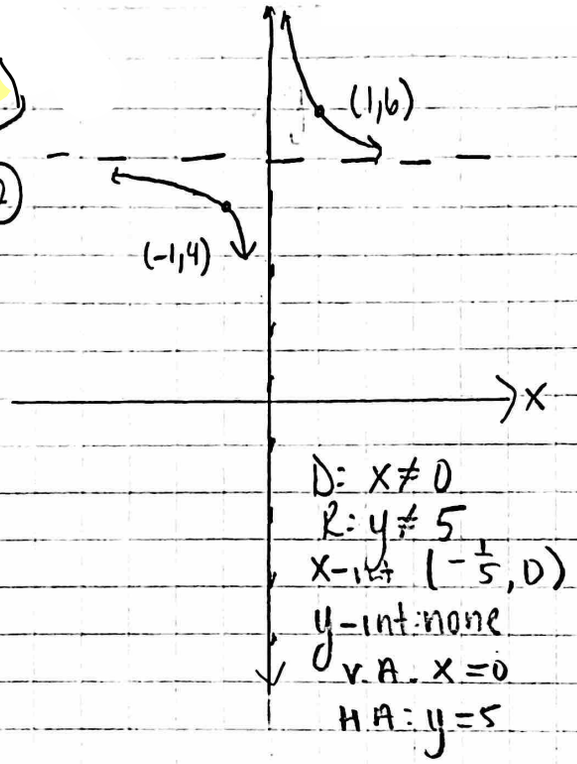
VA: $x = 1$
HA: $y = 0$

x-int: none
y-int: $(0,-1)$

EVENS

② $y = \frac{1}{x} + 5$

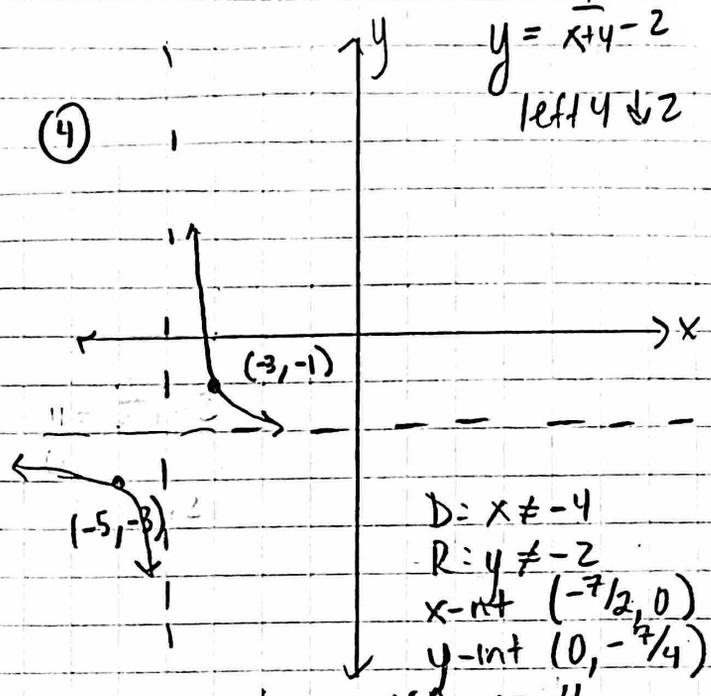
↑5



D: $x \neq 0$
R: $y \neq 5$
x-int $(-\frac{1}{5}, 0)$
y-int: none
V.A. $x = 0$
H.A: $y = 5$

④ $y = \frac{1}{x+4} - 2$

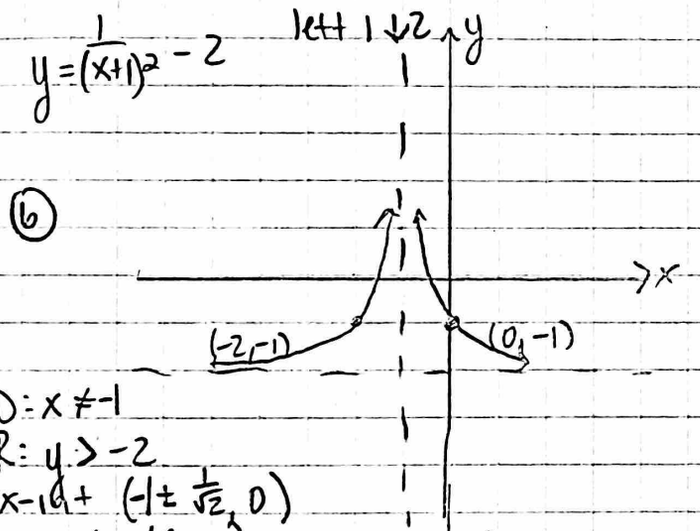
left 4 ↓2



D: $x \neq -4$
R: $y \neq -2$
x-int $(-7/2, 0)$
y-int $(0, -7/4)$
V.A. $x = -4$
H.A $y = -2$

⑥ $y = \frac{1}{(x+1)^2} - 2$

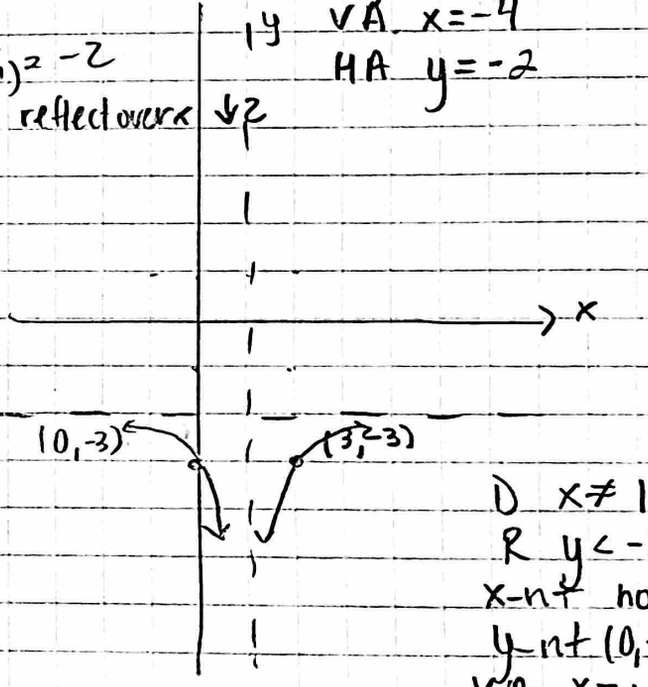
left 1 ↓2



D: $x \neq -1$
R: $y > -2$
x-int $(-1 \pm \frac{1}{\sqrt{2}}, 0)$
y-int $(0,-1)$
V.A $x = -1$
H.A $y = -2$

⑧ $y = \frac{-1}{(x-1)^2} - 2$

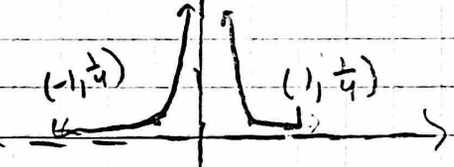
right 1 reflect over x ↓2



D $x \neq 1$
R $y < -2$
x-int none
y-int $(0,-3)$
V.A $x = 1$
H.A $y = -2$

⑩ $y = \frac{1}{4}x^2$

mult. y's by $\frac{1}{4}$

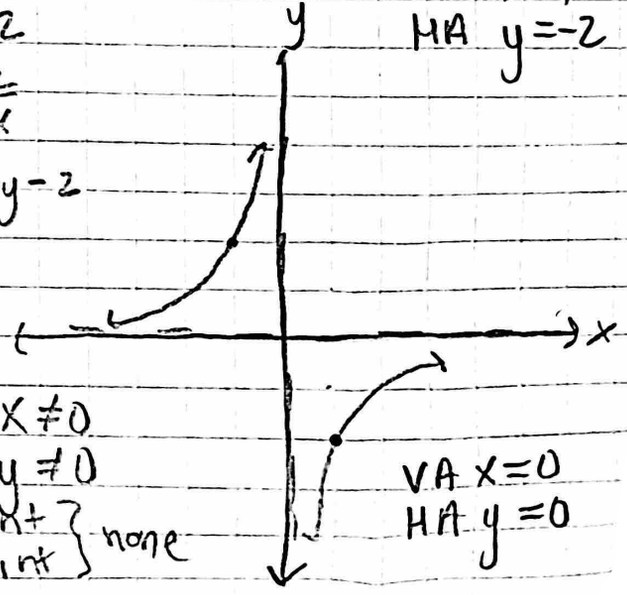


D: $x \neq 0$
R: $y > 0$
x-int none
y-int none
V.A $x = 0$
H.A $y = 0$

⑫ $xy = -2$

$y = \frac{-2}{x}$

mult. y's by -2



D: $x \neq 0$
R: $y \neq 0$
x-int } none
y-int }

V.A $x = 0$
H.A $y = 0$

⑭ $xy-2x=1$

$x(y-2)=1$

$y-2=\frac{1}{x}$

$y=\frac{1}{x}+2$

$\frac{1}{x}$ shifted up 2 units

$(-1,-1) \rightarrow (-1,1)$

$(1,1) \rightarrow (1,3)$

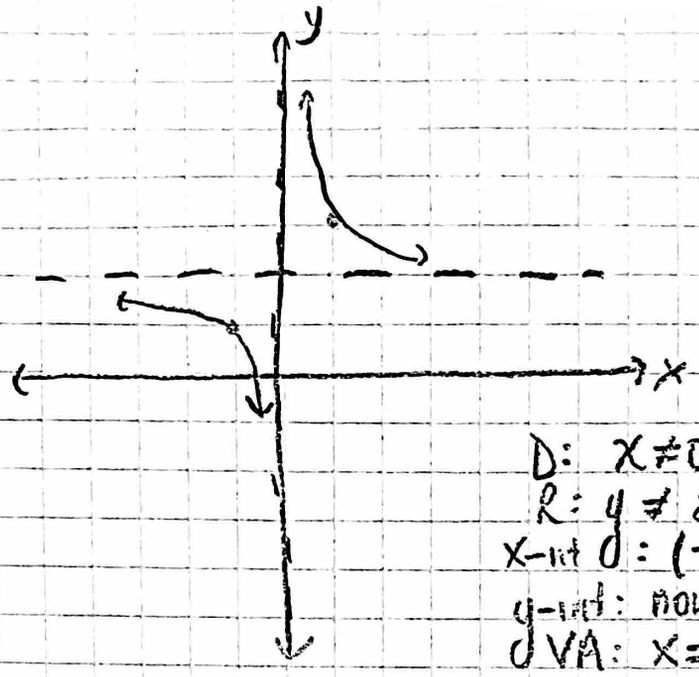
x-int (let $y=0$)

$0=\frac{1}{x}+2$

$-\frac{2}{1}=\frac{1}{x}$

$-2x=1$

$x=-\frac{1}{2}$



D: $x \neq 0$

R: $y \neq 2$

x-int 0: $(-1/2, 0)$

y-int: none

VA: $x=0$

HA: $y=2$