Name:____ Date:__________

PC: Vertical and Horizontal Asymptotes Ms. Loughran

Do Now:

$x^2-1=0$

$(x-1)(x+1)=0$

$x=\pm 1$

1. Graph $y = \dfrac{\overset{(\cancel{x^2-1})(x^2-1)}{x^4-2x^2+1}}{\cancel{x^2-1}}$. State the domain, range coordinates of any hole(s), x- and y-intercepts and the equations of any asymptotes.

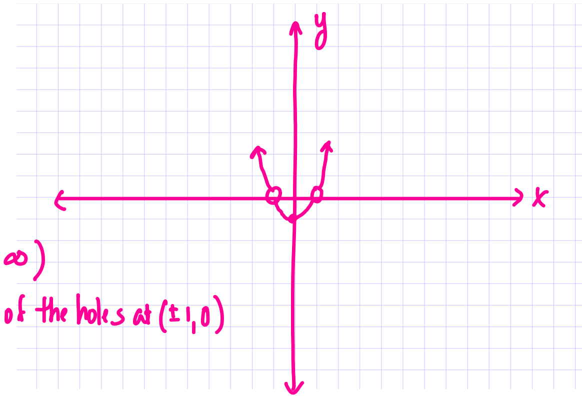
Reduced Function

$y = x^2-1$

parabola ↓ 1

holes $(1,0)$

$(-1,0)$



D: $\{x \mid x \neq \pm 1\}$

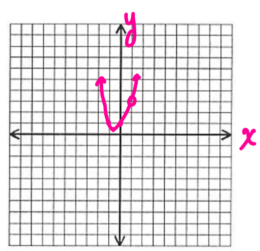
R: $[-1,0) \cup (0,\infty)$

x-int: none b/c of the holes at $(\pm 1, 0)$

y-int: $(0,-1)$

asymptotes: none

Finishing up #4 from yesterday's packet



4. $y = \dfrac{\overset{(\cancel{x-1})(x^2+x+1)}{x^3-1}}{\cancel{x-1}}$

hole: $(1,3)$

reduced function: $y = x^2+x+1$

$y = \underbrace{x^2+x+\tfrac{1}{4}}-\tfrac{1}{4}+1$

$y = (x+\tfrac{1}{2})^2+\tfrac{3}{4}$

vertex: $(-\tfrac{1}{2}, \tfrac{3}{4})$

b/c the transformation involves moving $\tfrac{1}{2}$ box to the left and $\tfrac{3}{4}$ ↑ I'm going to do a plug in instead

plug in $x=0, y=1$ $(0,1)$

$x=-1, y=1$ $(-1,1)$

D: $\{x \mid x \neq 1\}$

R: $\{y \mid y \geq \tfrac{3}{4}\}$

x-int: none

y-int: $(0,1)$

asymptotes: none

Let r be the **REDUCED** rational function

$$r(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \ldots + b_1 x + b_0}$$

1. The vertical asymptotes of r are the lines $x = a$, where a is a zero of the denominator.

   In other words: set the denominator of the REDUCED function $= 0$ to find the vertical asymptotes.

2. (a) If $n < m$, then r has a horizontal asymptote of $y = 0$.

   In other words: If the degree of the numerator is less then the degree of the denominator, you have a horizontal asymptote at $y = 0$.

   (b) If $n = m$, then r has a horizontal asymptote of $y = \frac{a_n}{b_m}$.

   In other words: If the degree of the numerator is = the degree of the denominator, you have a horizontal asymptote of $y =$ ratio of the leading coefficients

   (c) If $n > m$, then r has no horizontal asymptote.

   In other words: If the degree of the numerator is greater than the degree of the denominator, you don't have a horizontal asymptote.

* To find the x- and y- intercepts use the reduced function *

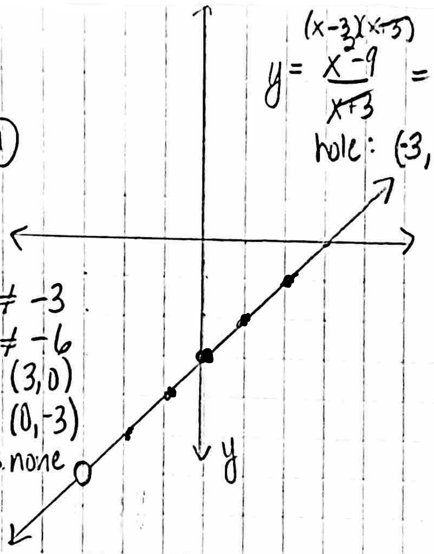
Reduced function	Function	Hole(s) (set the factor you cancelled = 0 to find x, then plug that x into RF to find y value of hole)	Vertical Asymptote(s) (set the den. of RF = 0)	Horizontal Asymptote (look at the degrees)	x-intercept(s) (let y=0 in RF)	y-intercept (let x=0 in RF)
	1. $y=\frac{1-x}{x+3}$	no	$x+3=0$ $x=-3$	$y=\frac{-1}{1}$ $y=-1$	$\frac{1-x}{x+3}=\frac{0}{1}$ $1-x=0$ $1=x$ $(1,0)$	$y=\frac{1-0}{0+3}=\frac{1}{3}$ $(0,\frac{1}{3})$
$y=\frac{1}{x+2}$	2. $y=\frac{\cancel{x-2}}{x^2-4}$ $(x+2)\cancel{(x-2)}$	$x-2=0$ $x=2$ $(2,\frac{1}{4})$	$x+2=0$ $x=-2$	$y=0$	$\frac{1}{x+2}=0$ $0\neq 1$ none	$y=\frac{1}{0+2}=\frac{1}{2}$ $(0,\frac{1}{2})$
$y=x-5$	3. $\cancel{(x+4)}(x-5)$ $y=\frac{x^2-x-20}{\cancel{x+4}}$	$x+4=0$ $x=-4$ $(-4,-9)$	no	no	$x-5=0$ $x=5$ $(5,0)$	$y=0-5=-5$ $(0,-5)$
	4. $(x+4)(x-5)$ $y=\frac{x^2-x-20}{x+1}$	no	$x+1=0$ $x=-1$	no	$\frac{(x+4)(x-5)}{x+1}=0$ $(x+4)(x-5)=0$ $(-4,0),(5,0)$	$y=\frac{0^2-0-20}{0+1}=\frac{-20}{1}$ $(0,-20)$
$y=\frac{2x^2}{x^2+1}$	5. $y=\frac{2x^{\cancel{3}\,2}}{x^3+x}$ $\cancel{x}(x^2+1)$	$x=0$ $(0,0)$	$x^2+1=0$ $x^2=-1$ $x=\pm\sqrt{-1}=\pm i$ none	$y=\frac{2}{1}$ $y=2$	$\frac{2x^2}{x^2+1}=0$ $2x^2=0$ $(0,0)$ none	also none b/c of hole
	6. $y=\frac{x-1}{x^2-4}$					

Homework 01-02

①

$y = \frac{x^2-9}{x+3} = \frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} = x-3$

hole: $(-3,-6)$



D: $x \neq -3$

R: $y \neq -6$

x-int: $(3,0)$

y-int: $(0,-3)$

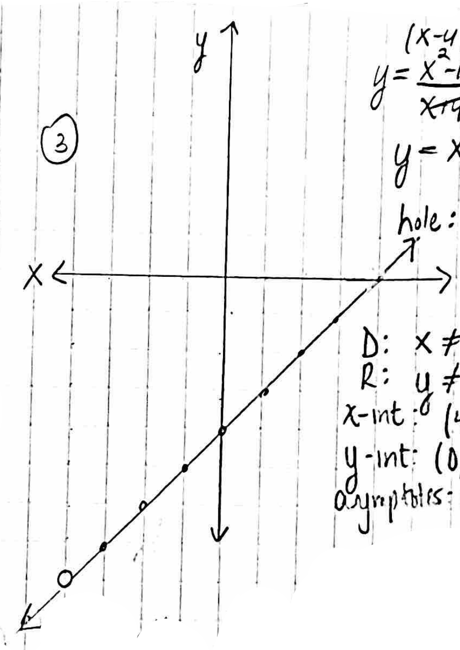
asymptotes: none

③

$y = \frac{x^2-16}{x+4} = \frac{(x-4)\cancel{(x+4)}}{\cancel{x+4}}$

$y = x-4$

hole: $(-4,-8)$



D: $x \neq -4$

R: $y \neq -8$

x-int: $(4,0)$

y-int: $(0,-4)$

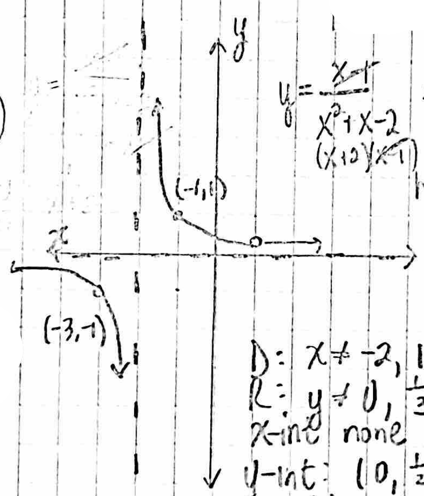
asymptotes: none

⑤

$y = \frac{\cancel{x-1}}{x^2+x-2} = \frac{1}{x+2}$, $x^2+x-2 = (x+2)\cancel{(x-1)}$

hole: $(1, \frac{1}{3})$

hyperbola moved left 2



D: $x \neq -2, 1$

R: $y \neq 0, \frac{1}{3}$

x-int: none

y-int: $(0, \frac{1}{2})$

asymptotes: V.A. $x = -2$

HA $y = 0$

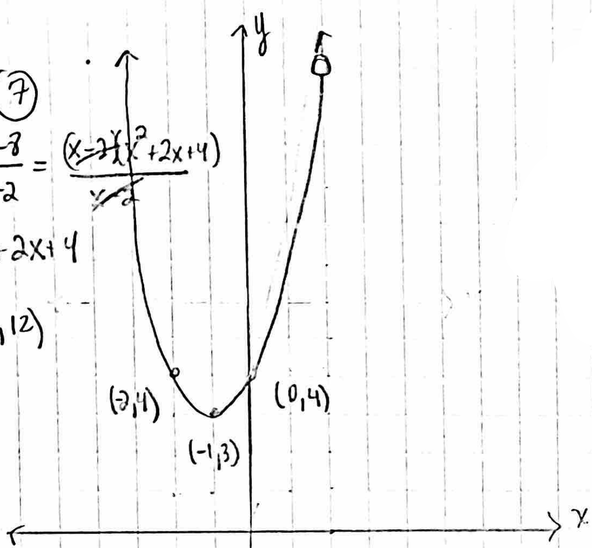
Reducible Functions Key

(7)

$y = \frac{x^3-8}{x-2} = \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$

$y = x^2+2x+4$

hole: (2, 12)



$y = x^2+2x+1-1+4$

$y = (x+1)^2+3$

x^2 left 1 ↑3

D: $x \neq 2$
R: $y \geq 3$
x-int: none
y-int (0,4)
asymptotes: none

(9)

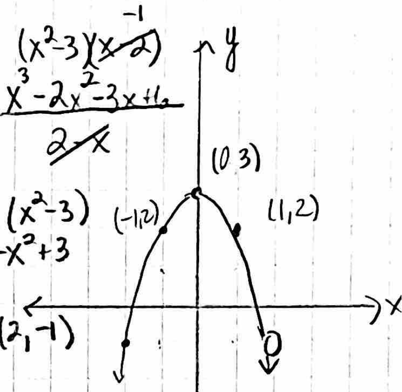
$(x^2-3)\cancel{(x-2)}$ -1

$y = \frac{x^3-2x^2-3x+6}{\cancel{2-x}}$

$y = -(x^2-3)$
$y = -x^2+3$

hole: (2, -1)

parabola reflected over x-axis ↑3



D: $x \neq 2$
R: $y \leq 3$
x-int: $(\pm\sqrt{3}, 0)$
y-int: (0, 3)
asymptotes: none

x^3-2x^2-3x+6
$x^2(x-2)-3(x-2)$
$(x^2-3)(x-2)$

x-int: let $y=0$ $\quad 0 = -x^2+3$
$x^2 = 3$
$x = \pm\sqrt{3}$