

Do Now: #8 from the Practice section of the Reducible Functions packet

Sketch the graph and state the domain, range, coordinates of any hole(s), intercepts and the equations of any asymptotes.

$$8. \ y = \frac{x-1}{x^2-1}$$

$$y = \frac{x-1}{(x+1)(x-1)}$$

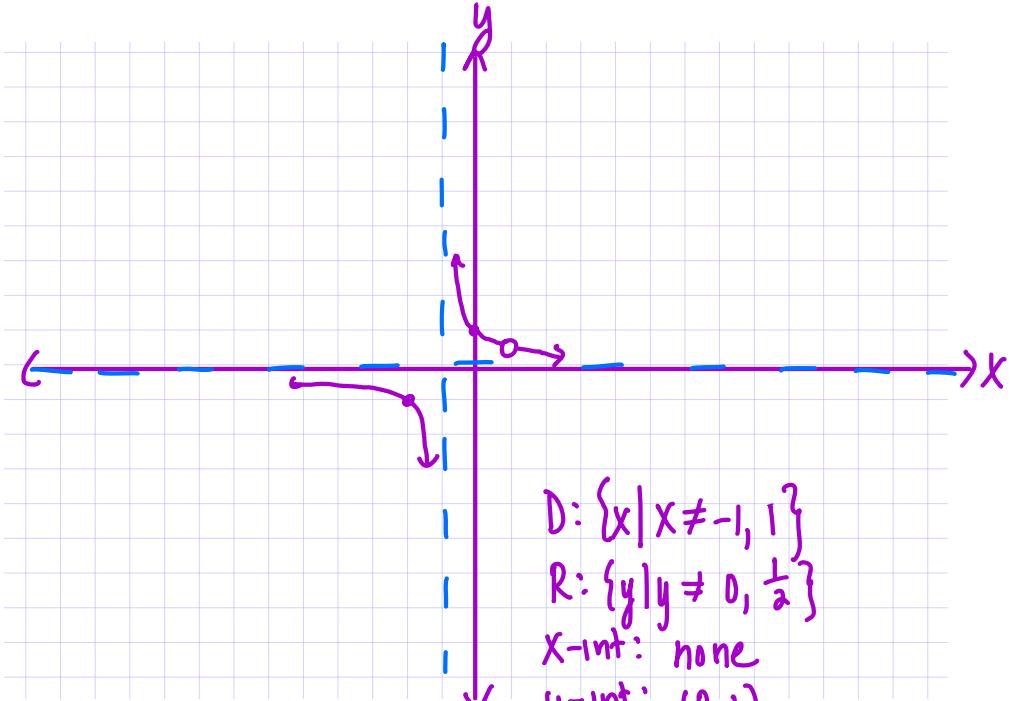
$$\text{RF: } y = \frac{1}{x+1}$$

$$\text{hole: } (1, \frac{1}{2})$$

hyperbola
left + 1

$$(-1, -1) \quad (-2, -1)$$

$$(1, 1) \quad (0, 1)$$



$$D: \{x | x \neq -1, 1\}$$

$$R: \{y | y \neq 0, \frac{1}{2}\}$$

X-int: none

$$y-\text{int}: (0, 1)$$

$$\text{VA: } x = -1$$

$$\text{HA: } y = 0$$

Name: _____
 PC: Oblique Asymptotes

Date: _____
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Do Now:

1. Find the vertical asymptote(s) of the function $y = \frac{x+6}{x^2 - 36}$

$$X=6$$

$$\begin{array}{l} 1 \\ (x+6)(x-6) \end{array}$$

2. Find the horizontal asymptote of the function $y = \frac{x^2 + 2x + 1}{x + 1}$

none

$$\begin{array}{l} RF: y = \frac{1}{x-6} \quad X-6=0 \\ \quad \quad \quad x=6 \\ = \frac{(x+1)(x+1)}{x+1} \quad RF: y = x+1 \end{array}$$

3. Is there a hole in the graph of $y = \frac{x^2 + 9}{x + 3}$?

no b/c it is not reducible

4. What is the domain of the function $y = \frac{x^2 - x - 12}{x - 4}$?

$$y = \frac{(x-4)(x+3)}{x-4} = x+3$$

hole (4, 7) $\{x | x \neq 4\}$

5. Are there any x- or y- intercepts for the graph of $y = \frac{3x^2 + x - 2}{x + 1}$? If so, state them.

$$y = \frac{(x+1)(3x-2)}{x+1} = 3x-2$$

hole: (-1, -5)

$$\begin{array}{l} x\text{-int: } (\frac{2}{3}, 0) \\ 0 = 3x-2 \\ 2 = 3x \\ \frac{2}{3} = x \\ y\text{-int: } (0, -2) \end{array}$$

When the end behavior of a rational function is not horizontal (meaning there is no horizontal asymptote), it is oblique.

Recall: In what situation is there no horizontal asymptote for a rational function?

-if the degree of the numerator > the degree of the denominator.

To find oblique asymptotes:

1. reduce the function if possible
2. divide the numerator by the denominator using long or synthetic division
3. the oblique asymptote is $y =$ the quotient (ignore the remainder)

- OA
 1. Find the oblique asymptote of $y = \frac{x^2 - 3x + 5}{x + 2}$

$$\begin{array}{r} -2 | 1 \ -3 \ 5 \\ \quad -2 \ 10 \\ \hline 1 \ -5 \ 15 \text{ remainder} \end{array}$$

$$OA: y = x - 5$$

2. Find the oblique asymptote of $y = \frac{x^2}{x+1}$.

$$\begin{array}{r} \underline{-1} \ 1 \ 0 \ 0 \\ \quad -1 \ 1 \\ \hline 1 \ -1 \ (\textcircled{1}) \text{ remainder} \end{array}$$

OA: $y = x - 1$

3. Find the oblique asymptote for $y = \frac{x^2 - 4}{x}$

Since you can not \div by 0, you have to use LD

$$\begin{array}{r} x \\ x \overline{) x^2 + 0x - 4} \\ \underline{-x^2} \\ \hline -4 \text{ remainder} \end{array}$$

OA: $y = x$

4. Find the oblique asymptote of $y = \frac{x^2 - 1}{-x + 3}$

$$\begin{aligned} -x + 3 &= 0 \\ 3 &= x \end{aligned}$$

$$\begin{array}{r} \underline{3} \ 1 \ 0 \ -1 \\ \quad 3 \ 9 \\ \hline 1 \ 3 \ (\textcircled{8}) \text{ remainder} \\ \quad \quad \quad \div -1 \end{array}$$

OA: $y = -x - 3$

Name: _____
 PC: Vertical, Horizontal and Oblique Asymptotes

Date: _____
 Ms. Loughran

Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	Oblique Asymptote	x-intercept(s)	y-intercept
① $y = \frac{x+2}{x^2 - 16}$ $(x+4)(x-4)$	no	$x-4=0 \quad x+4=0$ $x=4 \quad x=-4$ $x=\pm 4$	$y=0$	none	$\frac{x+2}{x^2 - 16} = 0$ $x+2=0$ $x=-2$ $(-2, 0)$	$y = \frac{0+2}{0-16} = \frac{2}{-16}$ $(0, -\frac{1}{8})$
② $y = \frac{(x-4)(x+4)}{x+2}$	no	$x=-2$	none	$\frac{-2 \ 1 \ 0 \ -16}{1 \ -2 \ -12}$ $(x-4)(x+4)=0$ $y=x-2$	$x=\pm 4$ $(\pm 4, 0)$	$(0, -8)$
③ $y = \frac{2x^2}{x^2 + 4}$						
④ $y = \frac{x(2x+3)}{x}$	$(0, 3)$	none	none	none	$0=2x+3$ $-3=2x$ $-\frac{3}{2}=x$ $(-\frac{3}{2}, 0)$	$y = 2(0)+3$ $(0, 3)$ none b/c of the hole at $(0, 3)$
⑤ $y = \frac{3x+21}{9-x}$						
⑥ $y = \frac{1}{(x+6)(x-1)}$						

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$y = \frac{x-1}{x^2-4}$ $(x-2)(x+2)$	none	$(x-2)(x+2)=0$ $x=\pm 2$	$y=0$	$\frac{x-1}{(x-2)(x+2)}=0$ $0=x-1$ $1=x$ $(1, 0)$	$y = \frac{0-1}{(0-2)(0+2)}$ $y = -\frac{1}{4}$ $(0, -\frac{1}{4})$
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Function	Hole(s)	Vertical Asymptote(s)	Horizontal Asymptote	x -intercept(s)	y -intercept
7 $y = \frac{4x}{x-3}$	none	$x=3$	$y=4$	$(0, 0)$	$(0, 0)$
8 $y = \frac{5x^2}{3+x}$ $x \neq -3$	none	$x=-3$	none	$(0, 0)$	$(0, 0)$
9 $y = \frac{-4x^2}{(x-2)(x+4)}$	none	$x=2$, $x=-4$	$y=0$	$(0, 0)$	$(0, 0)$
10 $y = \frac{(x-2)(x+2)}{-4+x^2}$ $y = \frac{-4+x^2}{(x-2)(x+4)}$	$(2, \frac{2}{3})$	$x=-4$	$y=1$	$(-2, 0)$	$(0, \frac{1}{2})$
11 $y = \frac{3x(x-1)}{2x^2-5x+3}$ $y = \frac{(2x-3)(x-1)}{x^2-1}$	$(1, -3)$	$x=\frac{3}{2}$	$y=\frac{3}{2}$	$(0, 0)$	$(0, 0)$
12 $y = \frac{x}{x^4-1}$	none	$x=\pm 1$	$y=0$	$(0, 0)$	$(0, 0)$

