Do Now: #s 17-20 from the Midterm Review Packet

17. Determine algebraically if the following functions are even, odd, or neither

a. $f(x) = -x^4 + 4x^2$

$f(-x) = -(-x)^4 + 4(-x)^2$

$f(-x) = -x^4 + 4x^2$

even

b. $f(x) = \dfrac{x^3}{x^2 - 4}$

$f(-x) = \dfrac{(-x)^3}{(-x)^2 - 4}$

$f(-x) = \dfrac{-x^3}{x^2 - 4} = -1 \cdot \dfrac{x^3}{x^2-4}$

odd

18. Use polynomial long division to find the quotient of $x^4 - 5x^2 + 6x - 7$ divided by $x^2 + 2$

$x^2 - 7$

$$\begin{array}{r|l} & x^2 - 7 \\ \hline x^2+2 & x^4 + 0x^3 - 5x^2 + 6x - 7 \\ & x^4 \quad\;\; + 2x^2 \\ \hline & -7x^2 + 6x - 7 \\ & -7x^2 \qquad - 14 \\ \hline & 6x + 7 \end{array}$$

$6x + 7$ ← remainder

19. Use synthetic division to find the quotient of $(x^4 - 5x + 10) \div (x - 3)$

$$\begin{array}{r|rrrrr} 3 & 1 & 0 & 0 & -5 & 10 \\ & & 3 & 9 & 27 & 66 \\ \hline & 1 & 3 & 9 & 22 & 76 \end{array}$$

76 remainder

$x^3 + 3x^2 + 9x + 22$

20. Show that (x-3) is a factor of $P(x) = x^3 - 7x - 6$, and find the other factors.

$$
\begin{array}{r|rrrr}
3 & 1 & 0 & -7 & -6 \\
 & & 3 & 9 & 6 \\
\hline
 & 1 & 3 & 2 & 0
\end{array}
$$

B/c the remainder is 0
X-3 is a factor

$$x^2 + 3x + 2$$

$$(x+2)(x+1)$$

if the question has asked for the complete factorization:

$$(x-3)(x+2)(x+1)$$

Midterm Review Pocket Key

Homework 01-12 and 01-16

① $f(x-3) = (x-3)^3 + 3(x-3) - 2$

$= x^3 - 9x^2 + 27x - 27 + 3x - 9 - 2$

$= x^3 - 9x^2 + 30x - 38$

$(x-3)(x-3)(x-3)$

$(x^2 - 6x + 9)(x-3)$

$x^3 - 6x^2 + 9x - 3x^2 + 18x - 27$

$x^3 - 9x^2 + 27x - 27$

② $(f \circ h \circ g)(x)$

$g(x) = 2x^2$

$h(2x^2) = \sqrt{2x^2 - 9}$

$f(\sqrt{2x^2-9}) = \sqrt{2x^2-9} + 3$

③ $\dfrac{(x+h)^2 - 2(x+h) + 5 - (x^2 - 2x + 5)}{h}$ $h \neq 0$

$\dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h + \cancel{5} - \cancel{x^2} + \cancel{2x} - \cancel{5}}{h} = \dfrac{2xh + h^2 - 2h}{h} = 2x + h - 2,\ h \neq 0$

④ $f(x) = \sqrt{2x+3}$

$y = \sqrt{2x+3}$

$(x)^2 = (\sqrt{2y+3})^2$

$x^2 = 2y + 3$

$\dfrac{x^2-3}{2} = \dfrac{2y}{2}$

$\dfrac{x^2-3}{2} = y = f^{-1}(x)$

⑤ $m = \dfrac{4-2}{4-(-7)} = \dfrac{2}{11}$

point slope $y - 4 = \frac{2}{11}(x-4)$ or $y - 2 = \frac{2}{11}(x+7)$

$y - 4 = \frac{2}{11}x - \frac{8}{11}$

$y = \frac{2}{11}x - \frac{8}{11} + 4$

slope intercept $y = \frac{2}{11}x + \frac{36}{11}$

$11\left(\frac{2}{11}x - y = -\frac{36}{11}\right)$

standard form $2x - 11y = -36$

⑥ $f(x) = x + 12$

$g(x) = \sqrt{x}$

$h(x) = \dfrac{12}{x}$ as $h(g(f(x)))$

answers can vary

(7) $f(g(x)) = g(f(x)) = x$

$f(x^2-3)$ $\quad$ $g(\sqrt{x+3})$

$\sqrt{x^2 \cancel{-3} \cancel{+3}}$ $\quad$ $(\sqrt{x+3})^2 - 3$

$\sqrt{x^2}$ $\quad$ $x \cancel{+3} \cancel{-3}$

$x = x$ $\quad$ ∴ f and g are inverses

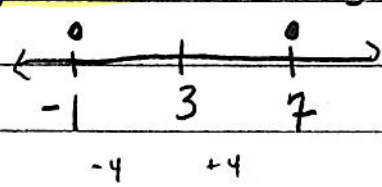
(8) $\dfrac{2x^4}{x^3-x^2} = \dfrac{2x^{\cancel{4}\,2}}{\cancel{x^2}(x-1)} = \dfrac{2x^2}{x-1}$ $\quad x \neq 0, 1$

(9) $|3-x| = 4$

Remember $|3-x| = |x-3|$

(a) $|x-3| = 4$

x's distance from 3 is 4



$\{-1, 7\}$

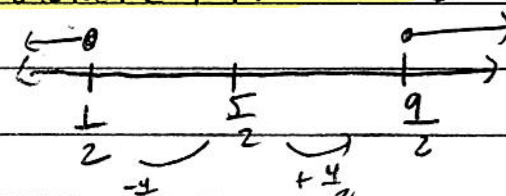
$|5-2x| \geq 4$ $\quad$ * $|5-2x| = |2x-5|$

(b) $|2x-5| \geq 4$

$2\left|x - \frac{5}{2}\right| \geq 4$

$\left|x - \frac{5}{2}\right| \geq \frac{4}{2}$

x's distance from $5/2 \geq \frac{4}{2}$



$\{x \mid x \leq 1/2 \text{ or } x \geq 9/2\}$ set builder

$(-\infty, \frac{1}{2}] \cup [\frac{9}{2}, \infty)$ interval

(10) (a) $\dfrac{4-x^{-2}}{2x^{-1}-x^{-2}} = \dfrac{4 - \frac{1}{x^2}}{\frac{2}{x} - \frac{1}{x^2}}$ $\quad x^2$

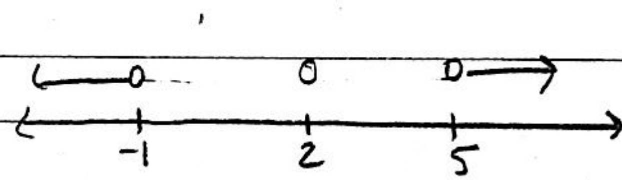
$\dfrac{4x^2-1}{2x-1} = \dfrac{(2x+1)\cancel{(2x-1)}}{\cancel{(2x-1)}} = 2x+1$ $\quad x \neq 0, \frac{1}{2}$

(10)(b) $\dfrac{x^2-xy}{xy+2y^3} \div \dfrac{x^2+xy}{xy+y^2}$

$\dfrac{\cancel{x}(x-y)}{\cancel{y}(x+2y^2)} \cdot \dfrac{\cancel{y}\cancel{(x+y)}}{\cancel{x}\cancel{(x+y)}} = \dfrac{x-y}{x+2y^2}$

$y \neq 0,\ x \neq 0$
$x \neq -2y^2, -y$

(11) $\dfrac{\overset{(x-5)(x+1)}{x^2-4x-5}}{(x-2)^2} > 0$



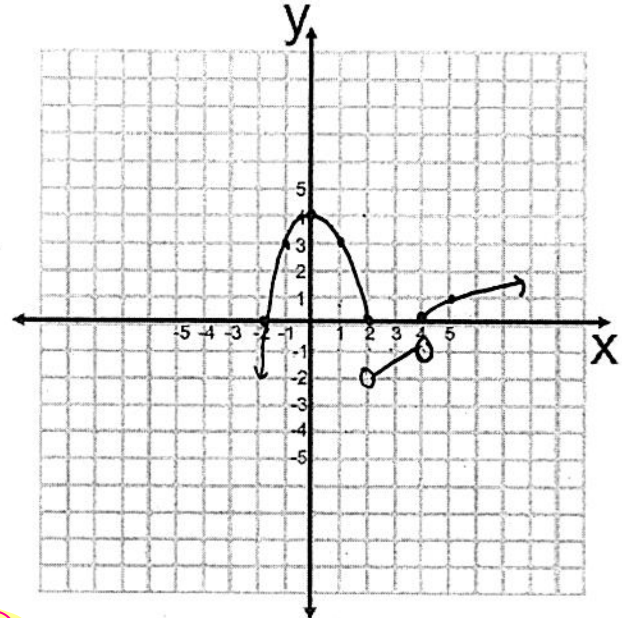
$+ \quad - \quad - \quad +$

SB (a) $\{x \mid x < -1 \vee x > 5\}$
IN (b) $(-\infty, -1) \cup (5, \infty)$

(12) $f(x) = -x^2 + 4x + 6$
$f(x) = -(x^2 - 4x + 4 - 4 - 6)$
$f(x) = -(x-2)^2 - (-10)$
$f(x) = -(x-2)^2 + 10$

13. Sketch the function **without** using a graphing calculator. Find the domain and range of each function.

a. $f(x)=\begin{cases} -x^2+4, & x\le 2 \\ \frac{1}{2}x-3, & 2<x<4 \\ \sqrt{x-4}, & x\ge 4 \end{cases}$



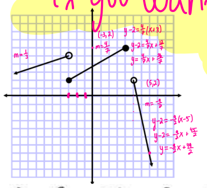
D: $(-\infty,\infty)$

R: $(-\infty,\infty)$

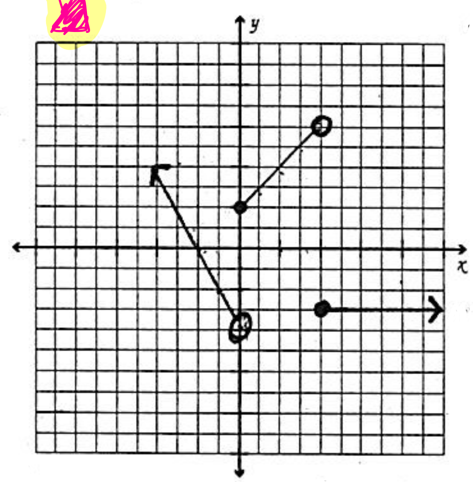
Original #14 but some pts were hard to see, so if you want try this new one

14. Write a piecewise function for the graph

$f(x)=\begin{cases} \frac{1}{3}x+6 & x<-3 \\ \frac{4}{7}x+\frac{26}{7} & -3\le x\le 4 \\ -\frac{9}{2}x+\frac{49}{2} & x>5 \end{cases}$



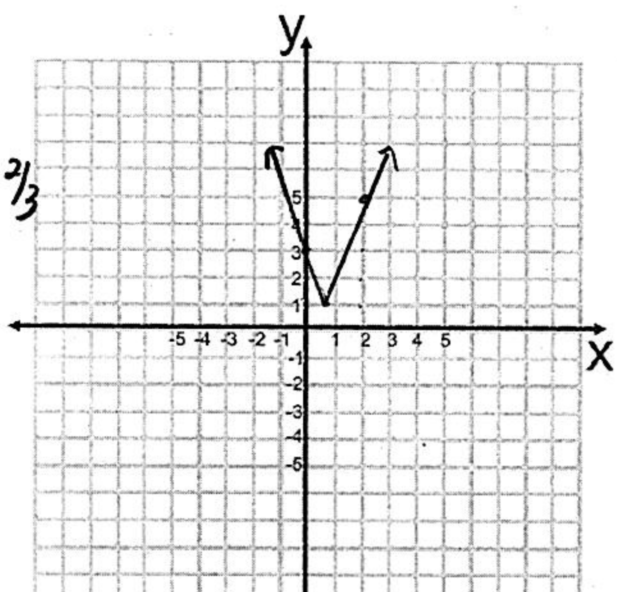
14. Write a piecewise function for the graph



$f(x)=\begin{cases} -2x-4 & x<0 \\ x+2 & 0\le x<4 \\ -3 & x\ge 4 \end{cases}$

15. Use the algebraic definition of absolute value to rewrite $f(x)=|3x-2|+1$ as a piecewise function and then sketch each graph.

$f(x)=\begin{cases} 3x-2+1=3x-1 & \text{if } 3x-2\ge 0,\ x\ge 2/3 \\ -3x+2+1=-3x+3 & \text{if } x<2/3 \end{cases}$



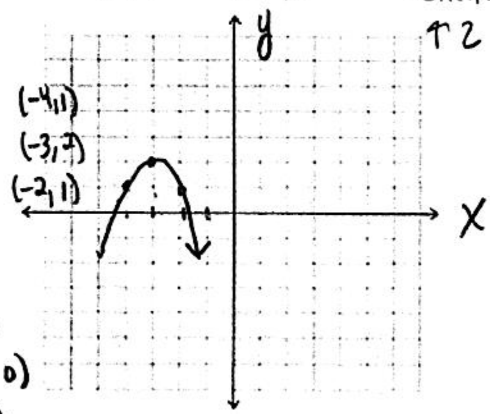
(a) x-int
$0 = 2-(x+3)^2$
$-2 = -(x+3)^2$
$2 = (x+3)^2$
$\pm\sqrt{2} = x+3$
$-3\pm\sqrt{2} = x$

16. Describe each transformation in terms of the parent function and then graph the function. State the domain, range, and any x- or y- intercepts.

a. $f(x) = 2-(x+3)^2$

left 3
reflect over x
↑2

(-1,1) (-4,1) (-4,-1) (-4,1)
(0,0) (-3,0) (-3,0) (-3,2)
(1,1) (-2,1) (-2,-1) (-2,1)

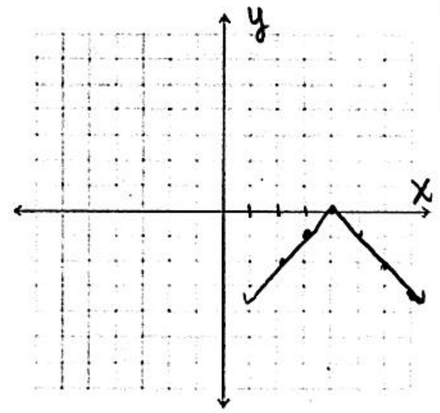


D: $(-\infty,\infty)$
R: $(-\infty, 2]$
x-int $(-3\pm\sqrt{2}, 0)$
y-int $(0,-7)$

b. $f(x) = -|x-4|$

right 4
reflect over x

(-1,1) → (3,1) → (3,-1)
(0,0) (4,0) (4,0)
(1,1) (5,1) (5,-1)

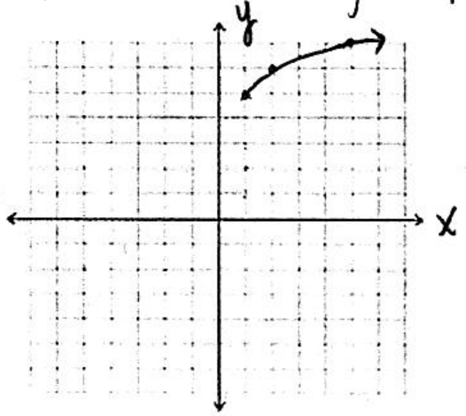


D: $(-\infty,\infty)$
R: $(-\infty, 0]$
x-int (4,0)
y-int (0,-4)

c. $f(x) = \sqrt{x-1}+5$

right 1 ↑5

(0,0) → (1,0) → (1,5)
(1,1) (2,1) (2,6)
(4,2) (5,2) (5,7)

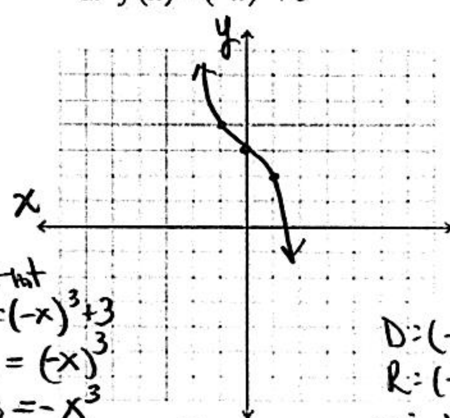


D: $[1,\infty)$
R: $[5,\infty)$
x-int: none
y-int: none

d. $f(x) = (-x)^3+3$

reflect over y
↑3

(-1,-1) → (1,-1)
(0,0) (0,0)
(1,1) (-1,1)



x-int
$0 = (-x)^3+3$
$-3 = (-x)^3$
$-3 = -x^3$
$3 = x^3$ $x = \sqrt[3]{3}$

D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
x-int $(\sqrt[3]{3}, 0)$
y-int (0,3)

(1,2)
(0,3)
(-1,4)