Name:_____ Date:_____

PC : Solving Multivariable Linear Systems Ms. Loughran

Do Now

Solve each of the following systems algebraically:

1. $8x-4y=4$
   $4x-2y=2$

   $-2(4x-2y=2)$

2. $3x-6y=9$
   $-2x+4y=1$

   $2(3x-6y=9)$
   $3(-2x+4y=1)$

$8x-4y=4$
$-8x+4y=-4$
$0=0$

infinitely many solutions

general solution: $(x, 2x-1)$

$6x-12y=18$
$-6x+12y=3$
$0=21$

no solution

$8x-4y=4$
$2x-y=1$
$2x-1=y$

3. Solve algebraically:
   A $x-2y+3z=9$
   B $-x+3y=-4$
   C $2x-5y+5z=17$

5A + -3C to eliminate z

$5x-10y+15z=45$
$-6x+15y-15z=-51$
D $-x+5y=-6$

-B + D to eliminate x

$x-3y=4$
$-x+5y=-6$
$2y=-2$
$y=-1$

Plug $y=-1$ into B

$-x+3(-1)=-4$
$-x-3=-4$
$-x=-1$
$x=1$

Plug $y=-1$ and $x=1$ into A

$1-2(-1)+3z=9$
$3+3z=9$
$3z=6$
$z=2$

(x, y, z)

Answer is an ordered triple $(1, -1, 2)$

Remember for a system of linear equations, exactly one is true:

1. There is exactly one solution
2. There are infinitely many solutions.
3. There is no solution.

1-5, solve the system of linear equations.

A $3x-2y+4z=1$

1. B $x+y-2z=3$

C $2x-3y+6z=8$

A+2B to eliminate y

$3x-2y+4z=1$

$2x+2y-4z=6$

$5x=7$

$x=\frac{7}{5}$

-2B+C to eliminate x

$-2x-2y+4z=-6$

$2x-3y+6z=8$

(D) $-5y+10z=2$

-3B+A to eliminate x

$-3x-3y+6z=-9$

$3x-2y+4z=1$

(E) $-5y+10z=-8$

impossible

-D+E to eliminate y

$5y-10z=-2$

$-5y+10z=-8$

$0=-10$

2.

A $4x + y - 3z = 11$

B $2x - 3y + 2z = 9$

C $x + y + z = -3$

-A + C to eliminate y

$-4x - y + 3z = -11$

$x + y + z = -3$

D $-3x + 4z = -14$

3A + B to eliminate y

$12x + 3y - 9z = 33$

$2x - 3y + 2z = 9$

$\frac{14x}{7} - \frac{7z}{7} = \frac{42}{7}$

E $2x - z = 6$

D + 4E to eliminate z

$-3x + 4z = -14$

$8x - 4z = 24$

$5x = 10$

$x = 2$

Plug $x = 2$ into E

$2(2) - z = 6$

$4 - z = 6$

$-z = 2$

$z = -2$

Plug $x = 2$ and $z = -2$ into C

$2 + y - 2 = -3$

$y = -3$

$(2, -3, -2)$

$x + y - 5z = 3$

5. $x - 2z = 1$

   $2x - y - z = 0$

Homework 02/01

① $y = -x + 2$
$x - y = 0$

$y = -x + 2$
$y = x$
$2y = 2$
$y = 1$
$\therefore x = 1$ (1,1)

② $\begin{matrix} x + 2y = 1 \\ 5x + 3y = -23 \end{matrix}$ $\times -5$

$-5x - 10y = -5$
$5x + 3y = -23$
$-7y = -28$
$y = 4$
$\therefore x = -7$

③ $x - y = 0$
$7x + y = 0$
$8x = 0$
$x = 0$ (0,0)
$\therefore y = 0$

④ $\times -3$ then $+y = -16$
$8x + y = -16$
$+3x - y = +5$
$11x = -11$
$x = -1$
$y = -8$

⑤ $\times -2$
$2x + y = 5$
$4x + 2y = 10$

$2x + y = 5$
$y - 5 = -2x$
$\frac{y-5}{-2} = x$
general solution: $(x, -2x+5)$ or $\left(\frac{y-5}{-2}, y\right)$

$-4x - 2y = -10$
$4x + 2y = 10$
$0 = 0$
infinitely many solutions

⑥ $x - y = 2$
$-2x + 2y = 5$

$2x - 2y = 4$
$-2x + 2y = 5$
$0 \neq 9$
no solution

⑦ $\times 2$ then $3x + 8 = -1$, $3x = -9$
$\begin{matrix} 3x + 4y = -1 \\ -3(2x + 5y = 4) \end{matrix}$

$6x + 8y = -2$
$-6x - 15y = -12$
$-7y = -14$
$y = 2$
$\therefore x = -3$ (-3,2)

⑧ $4x - 3y = 25$
$-3x + 8y = 10$

$12x - 9y = 75$
$-12x + 32y = 40$
$23y = 115$
$y = 5$
$x = 10$

$4x - 15 = 25$
$4x = 40$
$x = 10$

⑨ $\times 3$
$5x + 4y = -30$
$\times -5$
$3x - 9y = -18$

$15x + 12y = -90$
$-15x + 45y = 90$
$57y = 0$
$y = 0$
$\therefore x = -6$
(-6,0)

$2x+8y=6$
$x+4y=3$

(10) $2x+8y=6$; $-5x-20y=-15$ (×2)

$10x+40y=30$
$-10x-40y=-30$
$0=0$

infinitely many solutions

general solution: $(x, \frac{3-x}{4})$ or $(3-4y, y)$

(11) $5x+4y=-14$; $3x+6y=6$ (×−5)

$15x+12y=-42$
$-15x-30y=-30$
$-18y=-72$
$y=4$
$x=-6$

$5x+16=14$
$5x=-30$
$x=-6$

$(-6,4)$

(12) $-4x-15y=-17$; $-x+5y=-13$ (×−4)

$-4x-15y=-17$
$4x-20y=52$
$-35y=35$
$y=-1$
$\therefore -x-5=-13$
$-x=-8$
$x=8$

(13) $8x+14y=4$; $-6x-7y=-10$ (×2)

$8x+14y=4$
$-12x-14y=-20$
$-4x=-16$
$x=+4$

$+32+14y=4$
$14y=-28$
$y=-2$

$(4,-2)$

$2x-y=1$
$2x-1=y$

(14) $2x-y=1$; $4x-2y=2$

$4x-2y=2$
$4x-2y=2$
$0=0$

infinitely many

general solution: $(x, 2x-1)$ or $(\frac{1+y}{2}, y)$

(15) $\frac{1}{5}x+\frac{1}{2}y=8$; $x+y=20$ (×$-\frac{1}{2}$)

$\frac{2}{10}\ \frac{1}{5}x+\frac{1}{2}y=8$
$-\frac{5}{10}\ -\frac{1}{2}x-\frac{1}{2}y=-10$
$-\frac{3}{10}x=-2$ (×$-\frac{10}{3}$)
$x=+\frac{20}{3}$

$+\frac{20}{3}+y=20$
$y=20-\frac{20}{3}$
$y=\frac{40}{3}$

$(\frac{20}{3}, \frac{40}{3})$

(16) $\frac{1}{5}x-\frac{1}{3}y=1$ (×15) ; $-3x+5y=9$

$3x-5y=1$
$-3x+5y=9$
$0=10$

no solution

(17) $2.5x-3y=1.5$ (×−4) ; $10x-12y=6$

$-10x+12y=-6$
$10x-12y=6$
$0=0$

infinitely many

general solution: $(x, \frac{5x-3}{6})$ or $(\frac{6y+3}{5}, y)$

(18) $-7x-8y=9$ (×4) ; $-4x+9y=-22$ (×−7)

$-28x-32y=36$
$28x-63y=154$
$-95y=190$
$y=-2$

$x=1$

$\frac{22}{7}$
154
36
190

$10x-12y=6$
$5x-6y=3$
$5x-3=6y$

$5x=6y+3$
$x=\frac{6y+3}{5}$

$\frac{5x-3}{6} = y$

(19)

(a) $4x+3y=-8$ $\xrightarrow{\div 4}$ $x+\frac{3}{4}y=-2$

$x+ky=-2$ $\quad x+ky=-2$

$\therefore k=3/4$

$4x+3y=-8$

$-4x-4ky=+8$

$0 \qquad 0$

$3y-4ky=0$

$3-4k=0$

$k=\frac{3}{4}$

(b) $3x-12y=9$ $\xrightarrow{\div 3}$ $x-4y=3$

$\rightarrow (x-4y=k)$ $\quad x-4y=k$

$3x-12y=9$

$-3x+12y=-3k$

$9-3k=0$

$-3k=-9$

$\therefore k=3$

No solution

may not have same slope

(a) $4x+3y=-8$

$x+ky=-2 \rightarrow \cdot -4$

$4x+3y=-8$

$-4x-4ky=8$

$3-4k=0$

$k \neq 3/4$

any value of k other than 3/4 the lines will intersect so no solution is not possible

do have same slope

(b) $\begin{cases} 3x-12y=9 \\ (x-4y=k)\cdot -3 \end{cases}$

$3x-12y=9$

$-3x+12y=-3k$

$0=9-3k$

$3k=9$

$k \neq 3$

no solution for all k, $k \neq 3$