

Name: \_\_\_\_\_  
 PC : Solving Multivariable Linear Systems

Date: \_\_\_\_\_  
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Do Now

Solve each of the following systems algebraically:

$$1. \begin{cases} 8x - 4y = 4 \\ 4x - 2y = 2 \end{cases}$$

$$2. \begin{cases} 3x - 6y = 9 \\ -2x + 4y = 1 \end{cases}$$

$$\begin{array}{r} 8x - 4y = 4 \\ -8x + 4y = -4 \\ \hline 0 = 0 \end{array}$$

infinitely many  
solutions

$$\begin{array}{r} 6x - 12y = 18 \\ -6x + 12y = 3 \\ \hline 0 = 21 \end{array}$$

$$\begin{array}{l} 8x - 4y = 4 \\ 2x - y = 1 \\ 2x - 1 = y \end{array}$$

general solution:  $(x, 2x-1)$

no  
solution

$$A \quad x - 2y + 3z = 9$$

3. Solve algebraically:  
 B  $-x + 3y = -4$   
 C  $2x - 5y + 5z = 17$

5A + -3C to eliminate z

$$\begin{array}{r} 5x - 10y + 15z = 45 \\ -6x + 15y - 15z = -51 \\ \hline -x + 5y = -6 \end{array}$$

$(x, y, z)$

Answer is an ordered triple  $(1, -1, 2)$

-B + D to eliminate x

$$\begin{array}{r} x - 3y = 4 \\ -x + 5y = -6 \\ \hline 2y = -2 \\ y = -1 \end{array}$$

Plug  $y = -1$  into B

$$\begin{array}{r} -x + 3(-1) = -4 \\ -x - 3 = -4 \\ -x = -1 \\ x = 1 \end{array}$$

Plug  $y = -1$  and  $x = 1$  into A

$$\begin{array}{r} 1 - 2(-1) + 3z = 9 \\ 3 + 3z = 9 \\ 3z = 6 \end{array}$$

$$z = 2$$

Remember for a system of linear equations, exactly one is true:

1. There is exactly one solution
2. There are infinitely many solutions.
3. There is no solution.

1-5, solve the system of linear equations.

A

$$3x - 2y + 4z = 1$$

B

$$x + y - 2z = 3$$

C

$$2x - 3y + 6z = 8$$

~~A + 2B to eliminate y~~

~~$$3x - 2y + 4z = 1$$~~

~~$$2x + 2y - 4z = 6$$~~

~~$$\begin{array}{l} 5x = 7 \\ x = \frac{7}{5} \end{array}$$~~

-2B+C to eliminate x

$$-2x - 2y + 4z = -6$$

$$\underline{2x - 3y + 6z = 8}$$

$$\textcircled{D} \quad -5y + 10z = 2$$

-3B+A to eliminate x

$$-3x - 3y + 6z = -9$$

$$\underline{3x - 2y + 4z = 1}$$

$$\textcircled{E} \quad -5y + 10z = -8$$

impossible

-D+E to eliminate y

$$5y - 10z = -2$$

$$\underline{-5y + 10z = -8}$$

$$0 = -10$$

$$A \quad 4x + y - 3z = 11$$

$$2. \quad 2x - 3y + 2z = 9$$

$$C \quad x + y + z = -3$$

-A + C to eliminate y

$$\begin{array}{r} -4x - y + 3z = -11 \\ x + y + z = -3 \\ \hline -3x + 4z = -14 \end{array}$$

$$D \quad -3x + 4z = -14$$

3A + B to eliminate y

$$\begin{array}{r} 12x + 3y - 9z = 33 \\ 2x - 3y + 2z = 9 \\ \hline 14x - 7z = 42 \end{array}$$

$$E \quad 2x - z = 6$$

D + 4E to eliminate z

$$\begin{array}{r} -3x + 4z = -14 \\ 8x - 4z = 24 \\ \hline 5x = 10 \\ x = 2 \end{array}$$

$$(2, -3, -2)$$

Plug  $x=2$  into E

$$\begin{array}{r} 2(2) - z = 6 \\ 4 - z = 6 \\ -z = 2 \\ z = -2 \end{array}$$

Plug  $x=2$  and  $z=-2$   
into C

$$2 + y - 2 = -3$$

$$y = -3$$

$$x + y - 5z = 3$$

$$5. \quad x - 2z = 1$$

$$2x - y - z = 0$$

# Homework 02/01

$$\textcircled{1} \quad \begin{aligned} y &= -x + 2 \\ x - y &= 0 \end{aligned}$$

$$\textcircled{2} \quad \begin{aligned} x + 2y &= 1 \\ 5x + 3y &= -23 \end{aligned}$$

$$\begin{aligned} -5x - 10y &= -5 \\ 5x + 3y &= -23 \\ -7y &= -28 \end{aligned}$$

$$y = -x + 2$$

$$y = x$$

$$2y = 2$$

$$y = 1$$

$$\therefore x = 1 \quad (\textcircled{1}, \textcircled{1})$$

$$y = 4$$

$$\therefore x = -7$$

$$\textcircled{3} \quad x - y = 0$$

$$\begin{array}{r} 7x + y = 0 \\ \hline 8x = 0 \end{array}$$

$$x = 0$$

$$(0, 0)$$

$$\therefore y = 0$$

$$\textcircled{4} \quad \begin{array}{r} -3x + y = -16 \\ +3x + y = +5 \\ \hline 11x = -11 \end{array}$$

$$x = -1$$

$$y = -8$$

$$\textcircled{5} \quad \begin{array}{r} 2x + y = 5 \\ 4x + 2y = 10 \\ \hline 2x + y = 5 \end{array}$$

$$\begin{array}{l} \text{general} \\ \text{solution:} \end{array}$$

$$\begin{array}{l} (x, -2x+5) \quad \text{or} \\ \left( \frac{y-5}{2}, y \right) \end{array}$$

$$-4x - 2y = -10$$

$$\begin{array}{r} 4x + 2y = 10 \\ \hline 0 = 0 \end{array}$$

infinitely many  
solutions: 0

$$\textcircled{6} \quad x - y = 2$$

$$\begin{array}{r} 2x - 2y = 4 \\ -2x + 2y = 5 \\ \hline 0 \neq 9 \end{array}$$

no solution

$$2x - 2y = 4$$

$$\textcircled{7} \quad \begin{array}{r} 3x + 4y = -1 \\ -3(2x + 5y = 4) \\ \hline 3x + 8y = -2 \end{array}$$

$$\begin{array}{r} 6x + 8y = -2 \\ -6x - 15y = -12 \\ \hline -7y = -14 \end{array}$$

$$y = 2$$

$$(-3, 2)$$

$$\therefore x = -3$$

$$\textcircled{8} \quad 4x - 3y = 25$$

$$\begin{array}{r} -3x + 8y = 10 \\ \hline 11x = 115 \end{array}$$

$$\begin{array}{l} 4x - 15 = 25 \\ 4x = 40 \\ x = 10 \end{array}$$

$$y = 5$$

$$x = 10$$

$$\textcircled{9} \quad \begin{array}{r} 3(5x + 4y = -30) \\ -5(3x - 9y = -18) \\ \hline 15x + 12y = -90 \end{array}$$

$$15x + 12y = -90$$

$$-15x + 45y = 90$$

$$\therefore x = -6$$

$$51y = 0$$

$$y = 0$$

$$(-6, 0)$$

$$\begin{aligned} 2x + 8y &= b \\ x + 4y &= 3 \end{aligned}$$

$$\begin{array}{l} \textcircled{10} \quad \begin{array}{l} (2x + 8y = b) \\ 2 - (5x - 20y = -15) \end{array} \quad \begin{array}{l} 10x + 40y = 30 \\ -10x - 40y = -30 \end{array} \quad \begin{array}{l} 5x + 4y = -14 \\ -5(3x + 6y = b) \end{array} \quad \begin{array}{l} 15x + 12y = -42 \\ -15x - 30y = -30 \\ -18y = -72 \end{array} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \end{array}$$

$$\begin{array}{l} \text{general solution: } (x, \frac{3-x}{4}) \quad \text{or} \quad \begin{array}{l} \text{infinitely many} \\ \text{solutions} \end{array} \quad \begin{array}{l} 5x + 10 = 14 \\ 5x = -30 \\ x = -6 \end{array} \quad \begin{array}{l} y = 4 \\ x = -6 \\ (-6, 4) \end{array} \end{array}$$

$$\begin{array}{l} \textcircled{12} \quad \begin{array}{l} -4x - 15y = -17 \\ -4(-x + 5y = -13) \end{array} \quad \begin{array}{l} -4x - 15y = -17 \\ 4x - 20y = 52 \\ -35y = 35 \end{array} \quad \begin{array}{l} 8x + 14y = 4 \\ 2(-6x - 7y = -10) \end{array} \quad \begin{array}{l} 8x + 14y = 4 \\ -12x - 14y = -20 \\ -4x = -16 \end{array} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \end{array}$$

$y = -1$

$$\therefore \begin{array}{l} -x - 5 = -13 \\ -x = -8 \\ x = 8 \end{array}$$

$x = +4$

$y = -2$

$(4, -2)$

$$\begin{array}{l} \textcircled{14} \quad \begin{array}{l} 2x - y = 1 \\ 2x - 1 = y \end{array} \quad \begin{array}{l} 4x - 2y = 2 \\ 4x - 2(1) = 2 \\ 0 = 0 \end{array} \quad \text{general solution: } (x, 2x-1) \quad \text{or} \quad \begin{array}{l} \text{infinitely many} \\ \text{solutions} \end{array} \quad \begin{array}{l} \frac{1}{3}x + \frac{1}{2}y = 8 \\ -\frac{1}{3}(x + y = 20) \end{array} \quad \begin{array}{l} \frac{1}{3}x + \frac{1}{2}y = 8 \\ \frac{5}{10}x - \frac{1}{2}x - \frac{1}{2}y = -10 \\ \frac{1}{10}x = -2 \end{array} \\ \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \qquad \end{array}$$

$\frac{1}{3}x + y = 20$

$$y = 20 - \frac{1}{3}x$$

$x = +\frac{20}{3}$

$y = \frac{40}{3}$

$$\begin{array}{l} \textcircled{15} \quad \begin{array}{l} \frac{1}{5}x - \frac{1}{3}y = 1 \\ -3x + 5y = 9 \end{array} \quad \begin{array}{l} 3x - 5y = 1 \\ -3x + 5y = 9 \\ 8 = 10 \end{array} \quad \text{no solution} \quad \begin{array}{l} \left(\frac{20}{3}, \frac{40}{3}\right) \end{array} \quad \begin{array}{l} \frac{22}{7} \\ \frac{15}{4} \end{array} \\ \textcircled{18} \quad \begin{array}{l} -7x - 8y = 9 \\ -4x + 9y = -22 \end{array} \quad \begin{array}{l} -10x + 12y = -6 \\ -10x - 12y = 6 \\ 0 = 0 \end{array} \quad \begin{array}{l} -28x - 32y = 36 \\ 28x - 63y = 154 \\ -95y = 190 \end{array} \quad \begin{array}{l} x = 1 \\ y = -2 \end{array} \end{array}$$

$$\begin{array}{l} \textcircled{17} \quad \begin{array}{l} 2.5x - 3y = 1.5 \\ -10x + 12y = 6 \end{array} \quad \begin{array}{l} -10x + 12y = -6 \\ -10x - 12y = 6 \\ 0 = 0 \end{array} \quad \begin{array}{l} -28x - 32y = 36 \\ 28x - 63y = 154 \\ -95y = 190 \end{array} \quad \begin{array}{l} x = 1 \\ y = -2 \end{array} \\ \text{general solution: } (x, \frac{5x-3}{6}) \quad \text{or} \quad \begin{array}{l} \text{infinitely many} \\ \text{solutions} \end{array} \end{array}$$

$$\begin{array}{l} \begin{array}{l} 10x - 12y = 6 \\ 5x - 6y = 3 \\ 5x - 3 = 6y \end{array} \quad \begin{array}{l} 5x = 6y + 3 \\ x = \frac{6y+3}{5} \end{array} \end{array}$$

$$\frac{5x-3}{6} = y$$

(19)

$$(a) \begin{array}{l} 4x+3y = -8 \\ x+ky = -2 \end{array} \rightarrow \begin{array}{l} \div 4 \\ x + \frac{3}{4}y = -2 \\ x+ky = -2 \end{array}$$

$$\left\{ \begin{array}{l} 4x+3y = -8 \\ -4x+4ky = +8 \end{array} \right.$$

$$0 \qquad \qquad 0$$

$$\therefore k = \frac{3}{4}$$

$$3y - 4ky = 0$$

$$3 - 4k = 0$$

$$k = \frac{3}{4}$$

$$(b) \begin{array}{l} 3x-12y = 9 \\ \rightarrow (x-4y = k) \end{array} \rightarrow \begin{array}{l} \div 3 \\ x-4y = 3 \\ x-4y = k \end{array}$$

$$\begin{array}{l} 3x-12y = 9 \\ -3x+12y = -3k \end{array} \rightarrow \begin{array}{l} 9-3k=0 \\ -3k=-9 \end{array} \therefore k=3$$

No solution

may not have  
same slope

$$(a) \begin{array}{l} 4x+3y = -8 \\ x+ky = -2 \rightarrow \cdot -4 \end{array} \rightarrow \begin{array}{l} 4x+3y = -8 \\ -4x-4ky = 8 \end{array} \rightarrow \begin{array}{l} 3-4k = 0 \end{array}$$

$$k \neq \frac{3}{4}$$

any value of  $k$   
other than  $\frac{3}{4}$

the lines will intersect  
so no solution is  
not possible

$$(b) \begin{array}{l} 3x-12y = 9 \\ (x-4y = k) \cdot 3 \end{array} \rightarrow \begin{array}{l} 3x-12y = 9 \\ -3x+12y = -3k \end{array} \rightarrow \begin{array}{l} 0 = 9-3k \\ 3k = 9 \\ k \neq 3 \end{array}$$

no solution  
for all  $k, k \neq 3$