Do Now: #16 from the Solving Multivariable Linear Systems packet

A
16) $-6x - 2y - z = -17$
B $5x + y - 6z = 19$
$\frac{-4x}{2} - \frac{6y}{2} - \frac{6z}{2} = \frac{-20}{2}$

C $-2x - 3y - 3z = -10$

3B + C to eliminate y

$$15x + 3y - 18z = 57$$
$$-2x - 3y - 3z = -10$$

D $13x - 21z = 47$

A + 2B to eliminate y

$$-6x - 2y - z = -17$$
$$10x + 2y - 12z = 38$$

E $4x - 13z = 21$

4D + -13E to eliminate x

$$52x - 84z = 188$$
$$-52x + 169z = -273$$
$$85z = -85$$
$$z = -1$$

Plug z = -1 into E

$$4x + 13 = 21$$
$$4x = 8$$
$$x = 2$$

(2, 3, -1)

Plug z = -1 and x = 2 into A

$$-12 - 2y + 1 = -17$$
$$-2y - 11 = -17$$
$$-2y = -6$$
$$= 3$$

$\underline{\underline{OR}}$

Do Now: #16 from the Solving Multivariable Linear Systems packet

16)
A $-6x - 2y - z = -17$
B $5x + y - 6z = 19$
C $-4x - 6y - 6z = -20$

$-B + C$ to eliminate z

$-5x - y + 6z = -19$
$-4x - 6y - 6z = -20$

D $-9x - 7y = -39$

$-6A + B$ to eliminate z

$36x + 12y + 6z = 102$
$5x + y - 6z = 19$

E $41x + 13y = 121$

$13D + 7E$ to eliminate y

$-117x - 91y = -507$
$287x + 91y = 847$

$170x = 340$
$x = 2$

Plug $x=2$ into D

$-18 - 7y = -39$
$-7y = -21$
$y = 3$

Plug $x=2$ and $y=3$ into C

$-8 - 18 - 6z = -20$
$-26 - 6z = -20$
$-6z = 6$
$z = -1$

$(2, 3, -1)$

or
you could have eliminated x to start

Name: ____ Date: __________

PC : Matrices Intro Ms. Loughran

A **matrix** is a rectangular array of numbers

An $m \times n$ matrix has m rows (across) and n columns (down). Each number is an entry.

Examples:

mxn ← dimensions of matrix
order of matrix

2×2 matrix $\begin{bmatrix} 5 & 2 \\ 0 & -1 \end{bmatrix}$ 3×1 matrix $\begin{bmatrix} 3 \\ 8 \\ -1 \end{bmatrix}$ 1×4 matrix $\begin{bmatrix} 1 & -3 & 0 & 4 \end{bmatrix}$

If $m = n$ then it is called a square matrix.

We can perform elementary operations on matrices. We can:

1. interchange two rows
2. multiply a row by a nonzero constant
3. add a multiple of one row to another row

Notice that these are the same operations that we used when we solved systems of equations.

Two matrices are row equivalent if one can be obtained from the other by a sequence of elementary row operations.

Let's practice some row operations.

1. $\begin{bmatrix} 0 & 1 & 3 & 4 \\ -1 & 2 & 0 & 3 \\ 2 & -3 & 4 & 1 \end{bmatrix}$ Switch 1st row (R_1) and 2nd row (R_2)

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} -1 & 2 & 0 & 3 \\ 0 & 1 & 3 & 4 \\ 2 & -3 & 4 & 1 \end{bmatrix}$$

2. $\begin{bmatrix} 2 & -4 & 6 & -2 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$ Multiply R_1 by $\frac{1}{2}$

$\frac{1}{2}R_1 \to$ row 1

$$\begin{bmatrix} 1 & -2 & 3 & -1 \\ 1 & 3 & -3 & 0 \\ 5 & -2 & 1 & 2 \end{bmatrix}$$

3. $\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 2 & 1 & 5 & -2 \end{bmatrix}$ Add $-2R_1$ to R_3

$-2R_1 + R_3$

$-2R_1: -2 \quad -4 \quad 8 \quad -6$

$R_3 \quad 2 \quad 1 \quad 5 \quad -2$

$$\begin{bmatrix} 1 & 2 & -4 & 3 \\ 0 & 3 & -2 & -1 \\ 0 & -3 & 13 & -8 \end{bmatrix}$$

4. Given: $\begin{bmatrix} 1 & 0 & -5 & 3 \\ 3 & 2 & -1 & 7 \\ 4 & -2 & -3 & 1 \end{bmatrix}$

(a) Interchange R_1 and R_3. Label the new matrix as B.
(b) Multiply R_3 of matrix B by 2. Label the new matrix as C.
(c) In matrix C, add $-3R_2$ to R_1. Label the new matrix as D.

(a) $B = \begin{bmatrix} 4 & -2 & -3 & 1 \\ 3 & 2 & -1 & 7 \\ 1 & 0 & -5 & 3 \end{bmatrix}$ (b) $2R_3$ $C = \begin{bmatrix} 4 & -2 & -3 & 1 \\ 3 & 2 & -1 & 7 \\ 2 & 0 & -10 & 6 \end{bmatrix}$

(c) $D = \begin{bmatrix} -5 & -8 & 0 & -20 \\ 3 & 2 & -1 & 7 \\ 2 & 0 & -10 & 6 \end{bmatrix}$

$$\begin{array}{lrrrr} -3R_2 & -9 & -6 & 3 & -21 \\ R_1 & 4 & -2 & -3 & 1 \\ \hline & -5 & -8 & 0 & -20 \end{array}$$

Practice

1. Given: $\begin{bmatrix} 3 & -2 & 4 \\ 1 & 1 & -2 \\ 2 & -3 & 6 \end{bmatrix}$

(a) Multiply R_2 by -1. Label the new matrix as G.
(b) Using G add 2 times R_1 to R_3. Label the new matrix as H.
(c) Interchange R_2 and R_3 of H. Label the new matrix as J.
(d) Using J, add R_1 to R_2. Label the new matrix as K.

$$\begin{array}{lrrr} 2R_1 & 6 & -4 & 8 \\ R_3 & 2 & -3 & 6 \end{array}$$

(a) $-R_2$ $\quad G = \begin{bmatrix} 3 & -2 & 4 \\ -1 & -1 & 2 \\ 2 & -3 & 6 \end{bmatrix}$

(b) $2R_1 + R_3$ $\quad H = \begin{bmatrix} 3 & -2 & 4 \\ -1 & -1 & 2 \\ 8 & -7 & 14 \end{bmatrix}$

(c) $R_2 \leftrightarrow R_3$ $\quad J = \begin{bmatrix} 3 & -2 & 4 \\ 8 & -7 & 14 \\ -1 & -1 & 2 \end{bmatrix}$

(d) $R_1 + R_2$ $\quad K = \begin{bmatrix} 3 & -2 & 4 \\ 11 & -9 & 18 \\ -1 & -1 & 2 \end{bmatrix}$

$$x - 2y + 3z = 9$$
$$-x + 3y = -4$$
$$2x - 5y + 5z = 17$$

2. Given: $\begin{bmatrix} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{bmatrix}$

(a) Add R_1 to R_2. Label the new matrix as L.
(b) Using L, add $-2R_1$ to R_3. Label the new matrix as P.
(c) Using P, add R_2 to R_3. Label the new matrix as S.
(d) Using S, multiply R_3 by $\frac{1}{2}$. Label the new matrix as T.

(a) $R_1 + R_2$ $L = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{bmatrix}$

(b) $-2R_1 + R_3$ $P = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{bmatrix}$

(c) $R_2 + R_3$ $S = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{bmatrix}$

(d) $\frac{1}{2}R_3$ $T = \begin{bmatrix} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$z = 2$

$y + 3z = 5$
$y + 3(2) = 5$
$y + 6 = 5$
$y = -1$

$(1, -1, 2)$

$x - 2y + 3z = 9$
$x - 2(-1) + 3(2) = 9$
$x + 2 + 6 = 9$
$x = 1$

Homework 02-05

5. (r, s, t) $(1, 3, 1)$

6. $(0, 0, -3)$

7. $(-5, -5, 3)$

8. no solution