Name:______________________________ Date:__________

PC: Using Matrices to Solve Systems of Linear Equations Ms. Loughran

We can use matrices as a streamlined technique for solving systems of linear equations.

Model:

1. Given: $\begin{aligned} x-2y+3z &= 9 \\ -x+3y &= -4 \\ 2x-5y+5z &= 17 \end{aligned}$

Coefficient Matrix

$$\begin{bmatrix} 1 & -2 & 3 \\ -1 & 3 & 0 \\ 2 & -5 & 5 \end{bmatrix}$$

***constant terms are not included**

* **Augmented Matrix**

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

***constant terms are included**

To solve a linear system of equations we will use an augmented matrix.

To solve a matrix we use the elementary row operations that we discussed. Remember the 3 elementary row operations are the same three operations that we used to solve the linear systems of equations by elimination.

Goal : $\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$ Row echelon form

Let's get back to solving the system.

$$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ -1 & 3 & 0 & -4 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

$R_1 + R_2$ $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 2 & -5 & 5 & 17 \end{array}\right]$$

$-2R_1 + R_3$ $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & -1 & -1 & -1 \end{array}\right]$$

$-2R_1$ $-2\ 4\ -6\ -18$
$2\ -5\ 5\ 17$

$R_2 + R_3$ $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 2 & 4 \end{array}\right]$$

$\frac{1}{2}R_3$ $$\left[\begin{array}{ccc|c} 1 & -2 & 3 & 9 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$(1, -1, 2)$

$z = 2$

$y + 3z = 5$
$y + 3(2) = 5$
$y + 6 = 5$
$y = -1$

$x - 2y + 3z = 9$
$x - 2(-1) + 3(2) = 9$
$x + 2 + 6 = 9$
$x + 8 = 9$
$x = 1$

This last matrix is said to be in row-echelon form. The term echelon refers to the stair step pattern formed by the nonzero elements of the matrix. To be in row echelon form, a matrix must have these properties:

1. **All rows consisting entirely of zeros occur at the bottom of the matrix.**
2. **For each row that does not consist entirely of zeros, the first nonzero entry is 1 (called a leading 1).**
3. **For two successive (nonzero) rows, the leading 1 in the higher row is father to the left than the leading 1 in the lower row.**

A matrix in row-echelon form is in *reduced row-echelon form* if every column that has a leading 1 has zeros in every position above and below its leading one.

Goal: $\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$

2. Solve the following system using matrices:

$$\begin{aligned} x + y - 5z &= 3 \\ x \qquad - 2z &= 1 \\ 2x - y - z &= 0 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 1 & 0 & -2 & 1 \\ 2 & -1 & -1 & 0 \end{array}\right] \quad -R_1 + R_2 \quad \begin{array}{cccc} -1 & -1 & 5 & -3 \\ 1 & 0 & -2 & 1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 0 & -1 & 3 & -2 \\ 2 & -1 & -1 & 0 \end{array}\right]$$

$$-R_2 \left[\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 2 & -1 & -1 & 0 \end{array}\right] \quad -2R_1 + R_3 \quad \begin{array}{cccc} -2 & -2 & 10 & -6 \\ 2 & -1 & -1 & 0 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & -3 & 9 & -6 \end{array}\right]$$

$$3R_2 + R_3 \quad \begin{array}{cccc} 0 & 3 & -9 & 6 \\ 0 & -3 & 9 & -6 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & -5 & 3 \\ 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

infinitely many solutions

3. Solve the following system using matrices:

$$
\begin{aligned}
x - 2y + z &= 7 \\
3x + y - z &= 2 \\
2x + 3y + 2z &= 7
\end{aligned}
$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{array}\right] \xrightarrow{-3R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 2 & 3 & 2 & 7 \end{array}\right]$$

$-3R_1 + R_2$

$-3 \;\; 6 \;\; -3 \;\; -21$

$3 \;\; 1 \;\; -1 \;\; 2$

*time out on R_2, let's work on R_3

$-2R_1 + R_3$

$-2 \;\; 4 \;\; -2 \;\; -14$

$2 \;\; 3 \;\; 2 \;\; 7$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 7 & 0 & -7 \end{array}\right] \xrightarrow{\frac{1}{7}R_3 \leftrightarrow R_2} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 7 & -4 & -19 \end{array}\right]$$

$0 \;\; 1 \;\; 0 \;\; -1$

$(2, -1, 3)$

$-7R_2 + R_3$

$0 \;\; -7 \;\; 0 \;\; 7$

$0 \;\; 7 \;\; -4 \;\; -19$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -4 & -12 \end{array}\right] \xrightarrow{-\frac{1}{4}R_3} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$z = 3$

$y = -1$

$x - 2y + z = 7$

$x - 2(-1) + 3 = 7$

$x + 2 + 3 = 7$

$x + 5 = 7$

$x = 2$

Steps:

1. convert syst. of eqs. into an augmented matrix
2. work to put the matrix in row echelon form $\begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$
3. find the value of z from last row and then back substitute to find y and then x

Homework 02-06

① 3×2 ③ 3×1 ⑤ 2×2
② 1×4 ④ 3×4 ⑥ 6×1

23. (a) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 3 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & -5 & -10 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -10 \\ 0 & 0 & 0 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

24. (a) $\begin{bmatrix} 7 & 1 \\ 0 & 2 \\ -3 & 4 \\ 1 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ -3 & 4 \\ 7 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ 0 & 19 \\ 7 & 1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 5 \\ 0 & 2 \\ 0 & 19 \\ 0 & -34 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 5 \\ 0 & 1 \\ 0 & 19 \\ 0 & -34 \end{bmatrix}$ ~~(f)~~ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

26. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix}$ **28.** $\begin{bmatrix} 1 & -3 & 0 & -7 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

30. $\begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ **32.** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ **34.** $\begin{aligned} x + 5y &= 0 \\ y &= -1 \end{aligned}$
$(5, -1)$

36. $\begin{aligned} x + 2y - 2z &= -1 \\ y + z &= 9 \\ z &= -3 \end{aligned}$ **38.** $(-2, 4)$ **40.** $(3, -1, 0)$