

## Do Now: #2 from Friday's sheet

$$3x - 2y + z - 1 = 0$$

$$2. \quad x - y - z - 2 = 0$$

$$6x - 4y + 2z - 3 = 0$$

$$\left[ \begin{array}{ccc|c} 3 & -2 & 1 & 1 \\ 1 & -1 & -1 & 2 \\ 6 & -4 & 2 & 3 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[ \begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 3 & -2 & 1 & 1 \\ 6 & -4 & 2 & 3 \end{array} \right]$$

$$\begin{array}{l} -3R_1 + R_2 \\ -3 \quad 3 \quad 3 \quad -6 \\ 3 \quad -2 \quad 1 \quad 1 \end{array} \left[ \begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 4 & -5 \\ 6 & -4 & 2 & 3 \end{array} \right] \begin{array}{l} -6R_1 + R_3 \\ -6 \quad 6 \quad 6 \quad -12 \\ 6 \quad -4 \quad 2 \quad 3 \end{array} \left[ \begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 4 & -5 \\ 0 & 2 & 8 & -9 \end{array} \right]$$

$$\begin{array}{l} -2R_2 + R_3 \\ 0 \quad -2 \quad -8 \quad 10 \\ 0 \quad 2 \quad 8 \quad -9 \end{array} \left[ \begin{array}{ccc|c} 1 & -1 & -1 & 2 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 1 \end{array} \right] \quad \text{no solution}$$

# Continuing on Friday's sheet:

$$x + y + z = 6$$

4.  $2x - y + 3z = 9$

$$-x + 2y + 2z = 9$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 2 & -1 & 3 & 9 \\ -1 & 2 & 2 & 9 \end{array} \right] \xrightarrow{\substack{-2R_1 + R_2 \\ 2R_1 + R_3}} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & -3 \\ -1 & 2 & 2 & 9 \end{array} \right]$$

$$\xrightarrow{R_1 + R_3} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & -3 \\ 0 & 3 & 3 & 15 \end{array} \right] \xrightarrow{\frac{1}{3}R_3} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & -3 & 1 & -3 \\ 0 & 1 & 1 & 5 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & -3 & 1 & -3 \end{array} \right] \xrightarrow{\substack{3R_2 + R_3 \\ 0 \ 3 \ 3 \ 15 \\ 0 \ -3 \ 1 \ -3}} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 4 & 12 \end{array} \right]$$

$$\xrightarrow{\frac{1}{4}R_3} \left[ \begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$$z = 3$$

$$y + z = 5$$

$$y + 3 = 5$$

$$y = 2$$

$$x + y + z = 6$$

$$x + 2 + 3 = 6$$

$$x + 5 = 6$$

$$x = 1$$

$(1, 2, 3)$

$$2x - y - 3z = -1$$

$$5. \quad 2x - y + z = -9$$

$$x + 2y - 4z = 17$$

$$(-1, 5, -2)$$

$$\left[ \begin{array}{ccc|c} 2 & -1 & -3 & -1 \\ 2 & -1 & 1 & -9 \\ 1 & 2 & -4 & 17 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_3} \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 2 & -1 & 1 & -9 \\ 2 & -1 & -3 & -1 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_2 \\ -2 \quad -4 \quad 8 \quad -34 \\ 2 \quad -1 \quad 1 \quad -9 \end{array} \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & -5 & 9 & -43 \\ 2 & -1 & -3 & -1 \end{array} \right] \xrightarrow{-2R_1 + R_3} \begin{array}{l} -2R_1 + R_3 \\ -2 \quad -4 \quad 8 \quad -34 \\ 2 \quad -1 \quad -3 \quad -1 \end{array} \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & -5 & 9 & -43 \\ 0 & -5 & 5 & -35 \end{array} \right]$$

$$-\frac{1}{5}R_3 \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & -5 & 9 & -43 \\ 0 & 1 & -1 & 7 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_3} \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & 1 & -1 & 7 \\ 0 & -5 & 9 & -43 \end{array} \right]$$

$$5R_2 + R_3 \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 4 & -8 \end{array} \right] \xrightarrow{\frac{1}{4}R_3} \left[ \begin{array}{ccc|c} 1 & 2 & -4 & 17 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

$$z = -2$$

$$y - z = 7$$

$$y - (-2) = 7$$

$$y + 2 = 7$$

$$y = 5$$

$$x + 2y - 4z = 17$$

$$x + 2(5) - 4(-2) = 17$$

$$x + 10 + 8 = 17$$

$$x + 18 = 17$$

$$x = -1$$

$$2x - 3y + z = 5$$

$$6. \quad x + 3y + 8z = 22$$

$$3x - y + 2z = 12$$

$$(3, 1, 2)$$

$$\left[ \begin{array}{ccc|c} 2 & -3 & 1 & 5 \\ 1 & 3 & 8 & 22 \\ 3 & -1 & 2 & 12 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 2 & -3 & 1 & 5 \\ 3 & -1 & 2 & 12 \end{array} \right]$$

$$\begin{array}{l} -2R_1 + R_2 \\ -2 \quad -6 \quad -16 \quad -44 \\ 2 \quad -3 \quad 1 \quad 5 \end{array} \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & -9 & -15 & -39 \\ 3 & -1 & 2 & 12 \end{array} \right] \xrightarrow{-3R_1 + R_3} \begin{array}{l} -3 \quad -9 \quad -24 \quad -66 \\ 3 \quad -1 \quad 2 \quad 12 \end{array}$$

$$\left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & -9 & -15 & -39 \\ 0 & -10 & -22 & -54 \end{array} \right] \xrightarrow{-\frac{1}{3}R_2} \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & 3 & 5 & 13 \\ 0 & -10 & -22 & -54 \end{array} \right]$$

$$\begin{array}{l} 10R_2 + 3R_3 \\ 0 \quad 30 \quad 50 \quad 130 \\ 0 \quad -30 \quad -66 \quad -162 \end{array} \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & 3 & 5 & 13 \\ 0 & 0 & -16 & -32 \end{array} \right] \xrightarrow{-\frac{1}{16}R_3} \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & 3 & 5 & 13 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\frac{1}{3}R_2 \left[ \begin{array}{ccc|c} 1 & 3 & 8 & 22 \\ 0 & 1 & \frac{5}{3} & \frac{13}{3} \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$\begin{aligned} z &= 2 \\ y + \frac{5}{3}(2) &= \frac{13}{3} \\ y + \frac{10}{3} &= \frac{13}{3} \\ y &= \frac{3}{3} = 1 \end{aligned}$$

$$\begin{aligned} x + 3y + 8z &= 22 \\ x + 3(1) + 8(2) &= 22 \\ x + 3 + 16 &= 22 \\ x + 19 &= 22 \\ x &= 3 \end{aligned}$$