Name:____ Date:______________
PC : Operations on Matrices Ms. Loughran

Two matrices are equal if they have the same order $m \times n$ and their corresponding entries are equal.

1. Solve for $a_{11}, a_{12}, a_{21},$ and a_{22} in the following matrix equation.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$$

$a_{11} = 2$ $a_{12} = -1$

$a_{21} = -3$ $a_{22} = 0$

Matrix Addition

You can add two matrices (of the same order) by adding their corresponding entries. The sum of two matrices of different orders is undefined.

2. $\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} =$ $\begin{bmatrix} 0 & 5 \\ -1 & 3 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} =$ $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 2 & 3 \end{bmatrix}$

4. $\begin{bmatrix} 1 \\ -3 \\ -2 \end{bmatrix} + \begin{bmatrix} -1 \\ 3 \\ 2 \end{bmatrix} =$ $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

5. The sum of $A = \overset{3 \times 3}{\begin{bmatrix} 2 & 1 & 0 \\ 4 & 0 & -1 \\ 3 & -2 & 2 \end{bmatrix}}$ and $B \overset{3 \times 2}{\begin{bmatrix} 0 & 1 \\ -1 & 3 \\ 2 & 4 \end{bmatrix}}$ is undefined b/c the dimensions are not the same

Scalar Multiplication

In work with matrices, numbers are usually referred to as scalars. For our purposes, scalars will always be real numbers. You can multiply a matrix *A* by a scalar *c* by multiplying each entry in *A* by *c*.

The symbol $-A$ represents the scalar product $(-1)A$. Moreover, if *A* and *B* are of the same order, $A-B$ represents the sum of *A* and $(-1)B$. That is,

$$A-B = A+(-1)B \qquad \text{(Subtraction of matrices)}$$

6. For the following matrices, find (a) $3A$
   (b) $-B$
   (c) $3A-B$

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 0 & 0 \\ 1 & -4 & 3 \\ -1 & 3 & 2 \end{bmatrix}$$

a) $3A = \begin{bmatrix} 6 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix}$

b) $-B = \begin{bmatrix} -2 & 0 & 0 \\ -1 & 4 & -3 \\ 1 & -3 & -2 \end{bmatrix}$

c) $3A-B = \begin{bmatrix} 4 & 6 & 12 \\ -10 & 4 & -6 \\ 7 & 0 & 4 \end{bmatrix}$

Properties of Matrix Addition and Scalar Multiplication

Let A, B, and C be $m \times n$ and let c and d be scalars.

1. $A + B = B + A$ (commutative prop of matrix addition)

2. $A + (B + C) = (A + B) + C$ (associative prop. of matrix addition)

3. $(cd)A = c(dA)$ (associative prop. of scalar multiplication)

4. $IA = A$ (identity matrix)

5. $c(A + B) = cA + cB$ (distributive prop.)

6. $(c + d)A = cA + dA$ (distributive prop)

7. $$\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ -3 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

8. Solve for X in the equation $3X + A = B$, where

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 4 \\ 2 & 1 \end{bmatrix}.$$

$3X + A = B$

$3X = B - A$

$X = \dfrac{B-A}{3}$

$X = \frac{1}{3}(B - A)$

$-A = \begin{bmatrix} -1 & 2 \\ 0 & -3 \end{bmatrix}$

$B - A = \begin{bmatrix} -4 & 6 \\ 2 & -2 \end{bmatrix}$

$\frac{1}{3}(B-A) = \begin{bmatrix} -\frac{4}{3} & 2 \\ \frac{2}{3} & -\frac{2}{3} \end{bmatrix} = X$

Matrix Multiplication

To find the entries of the product, multiply each row of A by each column of B. Note that the number of columns of A must be equal to the of rows of B.

have to match

$3 \times 2 \qquad 2 \times 2$

dimensions of product

3×2

9. Find the product AB where

$$A = \begin{bmatrix} -1 & 3 \\ 4 & -2 \\ 5 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -3 & 2 \\ -4 & 1 \end{bmatrix}.$$

$$AB = \begin{bmatrix} (-1)(-3) + 3(-4) & (-1)(2) + 3(1) \\ 4(-3) + (-2)(-4) & 4(2) + (-2)(1) \\ 5(-3) + 0(-4) & 5(2) + 0(1) \end{bmatrix} \qquad AB = \begin{bmatrix} -9 & 1 \\ -4 & 6 \\ -15 & 10 \end{bmatrix}$$

$2 \times 3 \qquad 3 \times 3$

10. $$\begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 4 & 2 \\ 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} =$$

2×3

$$\begin{bmatrix} -5 & 7 & -1 \\ -3 & 6 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1(-2) + 0(1) + 3(-1) & 1(4) + 0(0) + 3(1) & 1(2) + 0(0) + 3(-1) \\ 2(-2) + (-1)(1) + (-2)(-1) & 2(4) + (-1)(0) + (-2)(1) & 2(2) + (-1)(0) + (-2)(-1) \end{bmatrix}$$

$2 \times 2 \qquad 2 \times 2$

11. $$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

2×2

$$\begin{bmatrix} 3 & 4 \\ -2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3 + 0 & 0 + 4 \\ -2 + 0 & 0 + 5 \end{bmatrix}$$

12. $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix} =$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1(-1)+2(1) & 1(2)+2(-1) \\ 1(-1)+1(1) & 1(2)+1(-1) \end{bmatrix}$$

13. $\underset{1\times 3}{\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}}\underset{3\times 1}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}} =$

1×1

$$\begin{bmatrix} 1(2)+(-2)(-1)+(-3)(1) \end{bmatrix}$$

$$\begin{bmatrix} 1 \end{bmatrix}$$

14. $\underset{3\times 1}{\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}}\underset{1\times 3}{\begin{bmatrix} 1 & -2 & -3 \end{bmatrix}} =$

3×3

$$\begin{bmatrix} 2 & -4 & -6 \\ -1 & 2 & 3 \\ 1 & -2 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 2(1) & 2(-2) & 2(-3) \\ -1(1) & -1(-2) & -1(-3) \\ 1(1) & 1(-2) & 1(-3) \end{bmatrix}$$

$\neq$

$3 \times 2 \qquad 3 \times 4$

15. Find the product of AB. If $A = \begin{bmatrix} -2 & 1 \\ 1 & -3 \\ 1 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 & 1 & 4 \\ 0 & 1 & -1 & 2 \\ 2 & -1 & 0 & 1 \end{bmatrix}$.

undefined

$A \to C \to B \to D$

Classwork 02-16

A

$$x + y + z = 0$$
$$x - y + 3z = -12$$
$$4x + y + z = -12$$

(-1,-4,2)

B

$$x - y + 3z = 1$$
$$5x + z = 0$$
$$x + 3y + z = -3$$

No solution

C

$$2x + 4y + 10z = 0$$
$$x + 2y + 5z = -38$$
$$x + y + z = -6$$

$(-4, 5, -1)$

D

$$x - y + z = 5$$
$$x + y + z = -3$$
$$x + y - z = -7$$

$(0, -1, 0)$