Name:________________________________ Date:________

PC: Inverses of Matrices Ms. Loughran

Do Now:

Find AB if possible.

3x3 3x2 3x2

1. $A = \begin{bmatrix} 0 & -1 & 0 \\ 4 & 0 & 2 \\ 8 & -1 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -3 & 4 \\ 1 & 6 \end{bmatrix}$ $\begin{bmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{bmatrix}$

2x1 1x4 2x4

2. $A = \begin{bmatrix} 10 \\ 12 \end{bmatrix}$ $B = \begin{bmatrix} 6 & -2 & 1 & 6 \end{bmatrix}$ $\begin{bmatrix} 60 & -20 & 10 & 60 \\ 72 & -24 & 12 & 72 \end{bmatrix}$

The identity matrix of a square matrix has entries of 1 on its main diagonal and 0's as all other entries.

I_2 means the identity matrix of a 2×2 matrix, I_3 means the identity matrix of a 3×3 matrix and so on.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Let A be an $n \times n$ matrix. If there exists a matrix A^{-1} such that $AA^{-1} = I_n = A^{-1}A$, A^{-1} is called the **inverse** of A.

1. Show that B is the inverse of A, where

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -2 \\ 1 & -1 \end{bmatrix}$$

$-1+2$

$2-2$

$-1+1$

$2-1$

Plan: $AB = BA = I_2$

$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$

2. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$$

Plan: $AB = BA = I_2$

$$AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

3. Show that B is the inverse of A, where

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & -1 \\ 6 & -2 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -2 & -3 & 1 \\ -3 & -3 & 1 \\ -2 & -4 & 1 \end{bmatrix}$$

Plan: $AB = BA = I_3 \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

To find the inverse of a 2×2 matrix we are going to use the determinant.

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A is $ad - bc$, and $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

4. Find A^{-1} and verify that $AA^{-1} = A^{-1}A = I_2$

$$A = \begin{bmatrix} 4 & 5 \\ 2 & 3 \end{bmatrix}$$

$\det(A) = |A| = 4(3) - 5(2) = 2$

$6 - 5$
$-10 + 10$
$3 - 3$
$-5 + 6$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & -\frac{5}{2} \\ -1 & 2 \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

5. Find the inverse of A.

$$A = \begin{bmatrix} 7 & -4 \\ 8 & 0 \end{bmatrix}$$

$\det(A) = |A| = 7(0) - 8(-4) = 32$

$$A^{-1} = \frac{1}{32}\begin{bmatrix} 0 & 4 \\ -8 & 7 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{8} \\ -\frac{1}{4} & \frac{7}{32} \end{bmatrix}$$

6. Find the inverse of B, if it exists.

$$B = \begin{bmatrix} 8 & 4 \\ -4 & -2 \end{bmatrix}$$

$|B| = 8(-2) - (-4)(4)$
$= -16 + 16 = 0$

B is not invertible, meaning it does not have an inverse.

We can use inverses to solve systems of linear equations.

If A is an invertible matrix (if A has an inverse), the system of linear equations represented by $AX = B$ has a unique solution:

$$AX = B$$

$$\underbrace{A^{-1} \cdot A}X = A^{-1} \cdot B$$

$$IX = A^{-1} \cdot B$$

$$\boxed{X = A^{-1} \cdot B}$$

must be in this order b/c multiplication of matrices is not commutative

7. Solve the system using the inverse, if possible.

Plan: $X = A^{-1} \cdot B$

$$2x - 5y = 15$$
$$3x - 6y = 36$$

$$\overset{A}{\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 15 \\ 36 \end{bmatrix}}$$

need A^{-1}

$\det(A) = -12 - (-15) = 3$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix}$$

$-30 + 60$

$-15 + 24$

$$X = A^{-1} \cdot B = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

$x = 30 \quad y = 9$

8. Solve the system using the inverse, if possible.

$$3x + 4y = -2$$
$$5x + 3y = 4$$

$$\overset{A}{\begin{bmatrix} 3 & 4 \\ 5 & 3 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} -2 \\ 4 \end{bmatrix}}$$

$\frac{-10}{11} \quad -\frac{12}{11}$

$|A| = 3(3) - 4(5) = -11$

$$A^{-1} = \frac{1}{-11}\begin{bmatrix} 3 & -4 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{-11} & \frac{4}{11} \\ \frac{5}{11} & -\frac{3}{11} \end{bmatrix}$$

$$X = A^{-1} \cdot B = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$x = 2$

$y = -2$

Homework: Textbook pp. 625-626 #2-16 even, 38, 40, 46, 48

1. $x=-4, y=22$
3. $x=2, y=3$
5. a) $\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 0 \\ 3 & -9 \end{bmatrix}$ c) $\begin{bmatrix} 3 & -3 \\ 6 & -3 \end{bmatrix}$ d) $\begin{bmatrix} -1 & -1 \\ -8 & -19 \end{bmatrix}$
7. a) $\begin{bmatrix} 7 & 3 \\ 1 & 9 \\ -2 & 15 \end{bmatrix}$ b) $\begin{bmatrix} 5 & -5 \\ 3 & -1 \\ -4 & -5 \end{bmatrix}$ c) $\begin{bmatrix} 18 & -3 \\ 6 & 12 \\ -9 & 15 \end{bmatrix}$ d) $\begin{bmatrix} 16 & -11 \\ 8 & 2 \\ -11 & -5 \end{bmatrix}$
9. a) $\begin{bmatrix} 3 & 3 & -2 & 1 & 1 \\ -2 & 5 & 7 & -6 & -8 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 & 0 & -1 & 1 \\ 4 & -3 & -11 & 6 & 6 \end{bmatrix}$
c) $\begin{bmatrix} 6 & 6 & -3 & 0 & 3 \\ 3 & 3 & -6 & 0 & -3 \end{bmatrix}$ d) $\begin{bmatrix} 4 & 4 & -1 & -2 & 3 \\ 9 & -5 & -24 & 12 & 11 \end{bmatrix}$
11. $\begin{bmatrix} -6 & -9 \\ -1 & 0 \\ 17 & -10 \end{bmatrix}$
13. $\begin{bmatrix} 3 & 3 \\ -\frac{1}{2} & 0 \\ -\frac{13}{2} & \frac{11}{2} \end{bmatrix}$

2. $x = 13, y = 12$ **4.** $x = -4, y = 9$

6. (a) $\begin{bmatrix} -2 & 0 \\ 6 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 4 \\ -2 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} 3 & 6 \\ 6 & 3 \end{bmatrix}$ (d) $\begin{bmatrix} 9 & 10 \\ -2 & -1 \end{bmatrix}$

8. (a) $\begin{bmatrix} 4 & -2 & 5 \\ -4 & 0 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 4 & -3 \\ 2 & -2 & 6 \end{bmatrix}$

(c) $\begin{bmatrix} 6 & 3 & 3 \\ -3 & -3 & 12 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & 9 & -5 \\ 3 & -5 & 16 \end{bmatrix}$

10. (a) $\begin{bmatrix} -1 \\ 8 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 7 \\ -4 \\ -3 \end{bmatrix}$ (c) $\begin{bmatrix} 9 \\ 6 \\ -3 \end{bmatrix}$ (d) $\begin{bmatrix} 17 \\ -6 \\ -7 \end{bmatrix}$

12. $\begin{bmatrix} -2 & -\frac{5}{2} \\ 0 & 0 \\ 5 & -\frac{7}{2} \end{bmatrix}$ **14.** $\begin{bmatrix} 2 & -5 \\ -5 & 0 \\ 5 & 6 \end{bmatrix}$

21 not possible

22 $\begin{bmatrix} 3 & -4 \\ 10 & 16 \\ 26 & 46 \end{bmatrix}$ 23 $\begin{bmatrix} -1 & 19 \\ 4 & -27 \\ 0 & 14 \end{bmatrix}$ 24 $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -10 \end{bmatrix}$

25. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{7}{2} \end{bmatrix}$

26. $\begin{bmatrix} 60 & -20 & 10 & 60 \\ 72 & -24 & 12 & 72 \end{bmatrix}$

27. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

28. Not possible