Name:______________________________________ Date:_____________

PC: Determinants of Square Matrices Ms. Loughran

$AB = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

Do Now:

1. Determine if B is the inverse of A.

yes they are inverses

$$A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

Plan:

Check if $AB = BA = I_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$BA = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$

Determinants help us to see if a matrix is invertible. If $d \neq 0$ then the matrix is invertible (has an inverse).

Remember: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then the determinant of A det (A) = $|A|$ is $ad - bc$.

Find the determinant of each matrix.

1. $A = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$ $\det(A) = 2(2) - (-3)(1) = 7$

2. $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$ $|B| = 2(2) - (4)(1) = 0$

3. $C = \begin{bmatrix} 0 & \frac{3}{2} \\ 2 & 4 \end{bmatrix}$ $\det(C) = 0(4) - 2(\frac{3}{2}) = -3$

The determinant of a matrix of order 1×1 is defined simply as the entry of the matrix.

4. $A = [-2]$ $\qquad$ $\det(A) = -2$

Finding the determinant of a 3×3 matrix

1. Expand matrix by rewriting the matrix with first and second column repeated at the end.

2. Multiply along the diagonals running left to right, and add up numbers.

3. Multiply along the diagonals running right to left, and add up numbers.

4. Subtract what you got in step 3 from what you got in step 2.

$$\overbrace{-4+0+6}^{2}$$

5. $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & -1 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{matrix} 0 & 2 \\ 3 & -1 \\ 4 & 0 \end{matrix}$

$$\underbrace{0+16+0}_{16}$$

$\det(A) = 16 - 2 = 14$

$$\overbrace{0+0+0}^{0}$$

6. $B = \begin{bmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{bmatrix} \begin{matrix} 6 & 3 \\ 0 & 0 \\ 4 & -6 \end{matrix}$

$$\underbrace{0+0+0}_{0}$$

$\det(B) = 0 - 0 = 0$

$$\overbrace{0+90+0}^{90}$$

7. $$C=\begin{bmatrix}5&-1&2\\4&0&6\\-2&3&0\end{bmatrix}\begin{matrix}5&-1\\4&0\\-2&3\end{matrix}$$

$$\underbrace{0+12+24}_{36}$$

$$|C| = 36 - 90 = -54$$

$$\overbrace{-16-30+0}^{-46}$$

8. $$D=\begin{bmatrix}-3&8&4\\0&1&2\\-4&5&2\end{bmatrix}\begin{matrix}-3&8\\0&1\\-4&5\end{matrix}$$

$$\underbrace{-6\ -64+0}_{-70}$$

$$\det(D) = -70-(-46) = -24$$

$$\overbrace{0+0+0}^{0}$$

9. $$E=\begin{bmatrix}1&0&9\\0&5&4\\0&0&-2\end{bmatrix}\begin{matrix}1&0\\0&5\\0&0\end{matrix}$$

$$\underbrace{-10+0+0}_{-10}$$

$$|E| = -10-0 = -10$$

10. $F = \begin{bmatrix} 1 & a & b \\ 0 & c & d \\ 2 & x & y \end{bmatrix}$

$\overbrace{2bc + dx + 0}^{2bc+dx}$

$\begin{matrix} 1 & a & b & 1 & a \\ 0 & c & d & 0 & c \\ 2 & x & y & 2 & x \end{matrix}$

$cy + 2ad + 0$

$\det(F) = cy + 2ad - (2bc + dx)$

$cy + 2ad - 2bc - dx$

11. What value of x makes the determinant -4?

$$\begin{bmatrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{bmatrix}$$

$\overbrace{0 + 0 + 0}^{0}$

$\left[\begin{matrix} -2 & 0 & 0 \\ -6 & x & 1 \\ -4 & 0 & -1 \end{matrix}\right] \begin{matrix} -2 & 0 \\ -6 & x \\ -4 & 0 \end{matrix}$

$\underbrace{2x + 0 + 0}_{2x}$

$\det = 2x - 0 = 2x$

$2x = -4$

$x = -2$

Homework: Textbook p. 634 #s 1-15 odd

In Exercises 15, 17, 18, 22, and 31, the inverse matrix does not exist.
In Exercises 47, 48, 53, and 54, the system has no solution.

8.3 /// EXERCISES Homework 02-28

In Exercises 1–8, show that B is the inverse of A.

1. $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$

* For 2-8 even, show that $AB = BA = I_n$

2. $A = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

3. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$

4. $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

5. $A = \begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$, $B = \frac{1}{3}\begin{bmatrix} -4 & -5 & 3 \\ -4 & -8 & 3 \\ 1 & 2 & 0 \end{bmatrix}$

6. $A = \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$

7. $A = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 3 & 0 & 0 & 1 \\ -1 & 1 & -2 & 1 \\ 4 & -1 & 1 & 0 \end{bmatrix}$,

$B = \begin{bmatrix} -1 & 2 & -1 & -1 \\ -4 & 9 & -5 & -6 \\ 0 & 1 & -1 & -1 \\ 3 & -5 & 3 & 3 \end{bmatrix}$

8. $A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$,

$B = \frac{1}{3}\begin{bmatrix} -3 & 1 & 1 & -3 \\ -3 & -1 & 2 & -3 \\ 0 & 1 & 1 & 0 \\ -3 & -2 & 1 & 0 \end{bmatrix}$

In Exercises 9–24, find the inverse of the matrix (if it exists).

9. $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

$\begin{bmatrix} 7 & -2 \\ -3 & 1 \end{bmatrix}$

11. $\begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}$

12. $\begin{bmatrix} -7 & 33 \\ 4 & -19 \end{bmatrix}$

$\begin{bmatrix} -19 & -33 \\ -4 & -7 \end{bmatrix}$

13. $\begin{bmatrix} -1 & 1 \\ -2 & 1 \end{bmatrix}$

14. $\begin{bmatrix} 11 & 1 \\ -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 \\ 1 & 11 \end{bmatrix}$

15. $\begin{bmatrix} 2 & 4 \\ 4 & 8 \end{bmatrix}$

16. $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

$\begin{bmatrix} \frac{4}{5} & -\frac{3}{5} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

17. $\begin{bmatrix} 2 & 7 & 1 \\ -3 & -9 & 2 \end{bmatrix}$

18. $\begin{bmatrix} -2 & 5 \\ 6 & -15 \\ 0 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 5 & 4 \\ 3 & 6 & 5 \end{bmatrix}$

20. $\begin{bmatrix} 1 & 2 & 2 \\ 3 & 7 & 9 \\ -1 & -4 & -7 \end{bmatrix}$

21. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 4 & 0 \\ 2 & 5 & 5 \end{bmatrix}$

22. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 5 & 5 \end{bmatrix}$

23. $\begin{bmatrix} -8 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & -5 \end{bmatrix}$

24. $\begin{bmatrix} 1 & 3 & -2 & 0 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

In Exercises 25–34, use the matrix capabilities of a graphing utility to find the inverse of the matrix (if it exists).

25. $\begin{bmatrix} 1 & 2 & -1 \\ 3 & 7 & -10 \\ -5 & -7 & -15 \end{bmatrix}$

26. $\begin{bmatrix} 10 & 5 & -7 \\ -5 & 1 & 4 \\ 3 & 2 & -2 \end{bmatrix}$

27. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

28. $\begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 2 \\ -4 & 4 & 3 \end{bmatrix}$

29. $\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$

30. $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

31. $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \\ 1 & 0 & 3 & 0 \\ 0 & 2 & 0 & 4 \end{bmatrix}$

32. $\begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & -1 \\ 2 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

33. $\begin{bmatrix} 1 & -2 & -1 & -2 \\ 3 & -5 & -2 & -3 \\ 2 & -5 & -2 & -5 \\ -1 & 4 & 4 & 11 \end{bmatrix}$

34. $\begin{bmatrix} 4 & 8 & -7 & 14 \\ 2 & 5 & -4 & 6 \\ 0 & 2 & 1 & -7 \\ 3 & 6 & -5 & 10 \end{bmatrix}$

35. If A is a 2×2 matrix given by

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

then A is invertible if and only if $ad - bc \neq 0$. If $ad - bc \neq 0$, verify that the inverse is given by

$$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

36. Use the result of Exercise 35 to find the inverse of each matrix.

(a) $\begin{bmatrix} 5 & -2 \\ 2 & 3 \end{bmatrix}$

(b) $\begin{bmatrix} 7 & 12 \\ -8 & -5 \end{bmatrix}$

In Exercises 37–40, use an inverse matrix to solve the system of linear equations. (Use the inverse matrix found in Exercise 11.)

37. $x - 2y = 5$
$2x - 3y = 10$

38. $x - 2y = 0$
$2x - 3y = 3$

$x = 6$
$y = 3$

39. $x - 2y = 4$
$2x - 3y = 2$

40. $x - 2y = 1$
$2x - 3y = -2$

$x = -7$
$y = -4$

In Exercises 41 and 42, use an inverse matrix to solve the system of linear equations. (Use the inverse matrix found in Exercise 19.)

41. $x + y + z = 0$
$3x + 5y + 4z = 5$
$3x + 6y + 5z = 2$

42. $x + y + z = -1$
$3x + 5y + 4z = 2$
$3x + 6y + 5z = 0$

In Exercises 43 and 44, use an inverse matrix and the matrix capabilities of a graphing utility to solve the system of linear equations. (Use the inverse matrix found in Exercise 33.)

43. $x_1 - 2x_2 - x_3 - 2x_4 = 0$
$3x_1 - 5x_2 - 2x_3 - 3x_4 = 1$
$2x_1 - 5x_2 - 2x_3 - 5x_4 = -1$
$-x_1 + 4x_2 + 4x_3 + 11x_4 = 2$

44. $x_1 - 2x_2 - x_3 - 2x_4 = 1$
$3x_1 - 5x_2 - 2x_3 - 3x_4 = -2$
$2x_1 - 5x_2 - 2x_3 - 5x_4 = 0$
$-x_1 + 4x_2 + 4x_3 + 11x_4 = -3$

In Exercises 45–52, use an inverse matrix to solve (if possible) the system of linear equations.

45. $3x + 4y = -2$
$5x + 3y = 4$

46. $18x + 12y = 13$
$30x + 24y = 23$

$x = \frac{1}{2}$
$y = \frac{1}{3}$

47. $-0.4x + 0.8y = 1.6$
$2x - 4y = 5$

48. $13x - 6y = 17$
$26x - 12y = 8$

not possible to solve using an inverse matrix as $\det(A) = 0$ so the inverse of A doesn't exist

49. $3x + 6y = 6$
$6x + 14y = 11$

50. $3x + 2y = 1$
$2x + 10y = 6$

51. $4x - y + z = -5$
$2x + 2y + 3z = 10$
$5x - 2y + 6z = 1$

52. $4x - 2y + 3z = -2$
$2x + 2y + 5z = 16$
$8x - 5y - 2z = 4$