Name:___________________________________ Date:________

PC: Applications of Matrices and Determinants Ms. Loughran

Do Now:

1. Given: $A = \begin{bmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{bmatrix}$

(a) Find $AB$.

$$\begin{bmatrix} 50 & -55 & 22 \\ 360 & 40 & 124 \\ 190 & 25 & 68 \end{bmatrix}$$

(b) Find det($A$)

$$\underbrace{-36 + 120 + 0}_{84}$$

$$\begin{bmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{bmatrix} \begin{matrix} 5 & 0 \\ 0 & 12 \\ 1 & 6 \end{matrix}$$

$$\underbrace{180 + 0 + 0}_{180}$$

$\det(A) = 180 - 84$

$= 96$

(c) Find det($AB$)

$$\begin{bmatrix} 50 & -55 & 22 \\ 360 & 40 & 124 \\ 190 & 25 & 68 \end{bmatrix} \begin{matrix} 50 & -55 \\ 360 & 40 \\ 190 & 25 \end{matrix}$$

$\det(AB) = 62,400$

The **area of a triangle** with vertices $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ is given by

$$\text{Area} = \pm\frac{1}{2}\text{determinant of}\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

where the symbol $\pm$ indicates that the appropriate sign should be chosen to yield a positive area.

you have to chose $+\frac{1}{2}$ or $-\frac{1}{2}$ based on the sign of the determinant
if det is ⊕ use $+\frac{1}{2}$
if det is ⊖ use $-\frac{1}{2}$

1. Find the area of a triangle whose vertices are $(1,0), (2,2)$ and $(4,3)$.

$$\overbrace{8+3+0}^{11}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 2 & 1 \\ 4 & 3 & 1 \end{bmatrix}\begin{matrix} 1 & 0 \\ 2 & 2 \\ 4 & 3 \end{matrix}$$

$$\underbrace{2+0+6}_{8}$$

$\det = 8 - 11 = -3$

$A = -\frac{1}{2}(-3) = \frac{3}{2}$

2. Find the area of a triangle whose vertices are $(-3,5),(2,6)$ and $(3,-5)$.

$$\overbrace{18+15+10}^{43}$$

$$\left[\begin{array}{ccc|cc} -3 & 5 & 1 & -3 & 5 \\ 2 & 6 & 1 & 2 & 6 \\ 3 & -5 & 1 & 3 & -5 \end{array}\right]$$

$$\underbrace{-18+15-10}_{-13}$$

$\det = -13 - 43 = -56$

$A = -\frac{1}{2}(-56) = 28$

**Test for Collinear Points:** Three points $(x_1, y_1), (x_2, y_2)$ and $(x_3, y_3)$ are collinear (lie on the same line) if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 \quad \text{(the determinant = 0)}$$

3. Determine whether the points $(-2,-2),(1,1)$ and $(7,5)$ lie on the same line.

$$\overbrace{7-10-2}^{-5}$$

$$\left[\begin{array}{ccc|cc} -2 & -2 & 1 & -2 & -2 \\ 1 & 1 & 1 & 1 & 1 \\ 7 & 5 & 1 & 7 & 5 \end{array}\right]$$

$$\underbrace{-2-14+5}_{-11}$$

$\det = -11 - (-5) \neq 0$

they are not collinear b/c the $\det \neq 0$

4. Determine whether the points $(3,-1),(0,-3)$ and $(12,5)$ are collinear.

$$\overbrace{-36+15+0}^{-21}$$

$$\left[\begin{array}{ccc|cc} 3 & -1 & 1 & 3 & -1 \\ 0 & -3 & 1 & 0 & -3 \\ 12 & 5 & 1 & 12 & 5 \end{array}\right]$$

$$\underbrace{-9-12+0}_{-21}$$

$\det = -21 - (-21) = 0$

yes they are collinear.

The test for collinear points can be adapted to another use. If you have two points on a rectangular coordinate system, you can find the equation of the line passing through the two points.

Steps ① set up matrix $\begin{bmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{bmatrix}$

② find determinant

③ set determinant $= 0$

5. Find an equation of a line that passes through $(2,4)$ and $(-1,3)$.

$-4 + 3x + 2y$

$$\left[\begin{array}{ccc} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{array}\right] \begin{array}{cc} x & y \\ 2 & 4 \\ -1 & 3 \end{array}$$

$4x - y + 6$

eq. of line:

$x - 3y + 10 = 0$

standard form: $x - 3y = -10$

slope intercept form $x + 10 = 3y \rightarrow \frac{x}{3} + \frac{10}{3} = y$

$\frac{1}{3}x + \frac{10}{3} = y$

$$\det = 4x - y + 6 - (-4 + 3x + 2y)$$

$$\det = 4x - y + 6 + 4 - 3x - 2y = x - 3y + 10$$

6. Find an equation of a line that passes through $(4,3)$ and $(2,2)$.

$6 + 2x + 4y$

$$\left[\begin{array}{ccc} x & y & 1 \\ 4 & 3 & 1 \\ 2 & 2 & 1 \end{array}\right] \begin{array}{cc} x & y \\ 4 & 3 \\ 2 & 2 \end{array}$$

$3x + 2y + 8$

$$\det = 3x + 2y + 8 - (6 + 2x + 4y)$$

$$\det = 3x + 2y + 8 - 6 - 2x - 4y$$

$$\det = x - 2y + 2$$

eq. of line

$0 = x - 2y + 2$

Homework: Textbook pp.646-647 #s 5, 7, 23, 25, 27, 29 (For 23 and 25, determine if those points are collinear without your calculator.)

Homework 03-01

## 8.4 /// EXERCISES

**In Exercises 1–16, find the determinant of the matrix.**

1. $[5]$ det=5

2. $[-8]$

3. $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ det= 5

4. $\begin{bmatrix} -3 & 1 \\ 5 & 2 \end{bmatrix}$

5. $\begin{bmatrix} 5 & 2 \\ -6 & 3 \end{bmatrix}$ det = 27

6. $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

7. $\begin{bmatrix} -7 & 6 \\ \frac{1}{2} & 3 \end{bmatrix}$ det = -24

8. $\begin{bmatrix} 4 & -3 \\ 0 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 2 & 6 \\ 0 & 3 \end{bmatrix}$ det= 6

10. $\begin{bmatrix} 2 & -3 \\ -6 & 9 \end{bmatrix}$

11. $\begin{bmatrix} 2 & -1 & 0 \\ 4 & 2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ det=0

12. $\begin{bmatrix} -2 & 2 & 3 \\ 1 & -1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$

13. $\begin{bmatrix} 6 & 3 & -7 \\ 0 & 0 & 0 \\ 4 & -6 & 3 \end{bmatrix}$ det = 0

14. $\begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 0 \\ -2 & 0 & 3 \end{bmatrix}$

15. $\begin{bmatrix} -1 & 2 & -5 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$ det=-9

16. $\begin{bmatrix} 1 & 0 & 0 \\ -4 & -1 & 0 \\ 5 & 1 & 5 \end{bmatrix}$

**In Exercises 17–20, use the matrix capabilities of a graphing utility to find the determinant of the matrix.**

17. $\begin{bmatrix} 0.3 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 \\ -0.4 & 0.4 & 0.3 \end{bmatrix}$

18. $\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ -0.3 & 0.2 & 0.2 \\ 0.5 & 0.4 & 0.4 \end{bmatrix}$

19. $\begin{bmatrix} 1 & 4 & -2 \\ 3 & 6 & -6 \\ -2 & 1 & 4 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 5 & -2 \\ 0 & 0 & -2 \end{bmatrix}$

**In Exercises 21–24, find all (a) minors and (b) cofactors of the matrix.**

21. $\begin{bmatrix} 3 & 4 \\ 2 & -5 \end{bmatrix}$

22. $\begin{bmatrix} 11 & 0 \\ -3 & 2 \end{bmatrix}$

23. $\begin{bmatrix} 3 & -2 & 8 \\ 3 & 2 & -6 \\ -1 & 3 & 6 \end{bmatrix}$

24. $\begin{bmatrix} -2 & 9 & 4 \\ 7 & -6 & 0 \\ 6 & 7 & -6 \end{bmatrix}$

**In Exercises 25–30, find the determinant of the matrix by the method of expansion by cofactors. Expand using the indicated row or column.**

25. $\begin{bmatrix} -3 & 2 & 1 \\ 4 & 5 & 6 \\ 2 & -3 & 1 \end{bmatrix}$ (a) Row 1 (b) Column 2

26. $\begin{bmatrix} -3 & 4 & 2 \\ 6 & 3 & 1 \\ 4 & -7 & -8 \end{bmatrix}$ (a) Row 2 (b) Column 3

27. $\begin{bmatrix} 5 & 0 & -3 \\ 0 & 12 & 4 \\ 1 & 6 & 3 \end{bmatrix}$ (a) Row 2 (b) Column 2

28. $\begin{bmatrix} 10 & -5 & 5 \\ 30 & 0 & 10 \\ 0 & 10 & 1 \end{bmatrix}$ (a) Row 3 (b) Column 1

29. $\begin{bmatrix} 6 & 0 & -3 & 5 \\ 4 & 13 & 6 & -8 \\ -1 & 0 & 7 & 4 \\ 8 & 6 & 0 & 2 \end{bmatrix}$ (a) Row 2 (b) Column 2

30. $\begin{bmatrix} 10 & 8 & 3 & -7 \\ 4 & 0 & 5 & -6 \\ 0 & 3 & 2 & 7 \\ 1 & 0 & -3 & 2 \end{bmatrix}$ (a) Row 3 (b) Column 1

**In Exercises 31–40, find the determinant of the matrix. Expand by cofactors on the row or column that appears to make the computations easiest.**

31. $\begin{bmatrix} 1 & 4 & -2 \\ 3 & 2 & 0 \\ -1 & 4 & 3 \end{bmatrix}$

32. $\begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & 4 \\ 1 & 0 & 2 \end{bmatrix}$

33. $\begin{bmatrix} 2 & 4 & 6 \\ 0 & 3 & 1 \\ 0 & 0 & -5 \end{bmatrix}$

34. $\begin{bmatrix} -3 & 0 & 0 \\ 7 & 11 & 0 \\ 1 & 2 & 2 \end{bmatrix}$

**47–50** ■ Solve for $x$.

47. $\begin{vmatrix} x & 12 & 13 \\ 0 & x-1 & 23 \\ 0 & 0 & x-2 \end{vmatrix} = 0$

48. $\begin{vmatrix} x & 1 & 1 \\ 1 & 1 & x \\ x & 1 & x \end{vmatrix} = 0$

49. $\begin{vmatrix} 1 & 0 & x \\ x^2 & 1 & 0 \\ x & 0 & 1 \end{vmatrix} = 0$

50. $\begin{vmatrix} a & b & x-a \\ x & x+b & x \\ 0 & 1 & 1 \end{vmatrix} = 0$