Name:______________________________ Date:___________

PC: Cramer's Rule Ms. Loughran

Do Now:

1. Solve using an inverse matrix: $\begin{array}{l} 4x-2y=10 \\ 3x-5y=11 \end{array}$

need A^{-1}

$det = -20-(-6) = -14$

$A^{-1} = \frac{1}{-14}\begin{bmatrix} -5 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} \frac{5}{14} & \frac{-1}{7} \\ \frac{3}{14} & \frac{-2}{7} \end{bmatrix}$

$\frac{50}{14} - \frac{11}{7}$

$\frac{25}{7} - \frac{11}{7} = \frac{14}{7}$

$\frac{30}{14} - \frac{22}{7}$

$\frac{15}{7} - \frac{22}{7}$

Plan: $X = A^{-1} \cdot B$

$\underset{A}{\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}} \cdot \overset{X}{\begin{bmatrix} x \\ y \end{bmatrix}} = \overset{B}{\begin{bmatrix} 10 \\ 11 \end{bmatrix}}$

$X = A^{-1} \cdot B = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ $x=2$, $y=-1$

We can also use Cramer's rule to solve systems of linear equations.

Steps:

1. Set up a coefficient matrix.
2. Find the determinant of the coefficient matrix. If the determinant $\neq 0$ you can use Cramer's Rule.
3. To find *x* value, replace first column (*x* column) with the answer column and find determinant. Now divide this determinant by the original matrix's determinant, this quotient is your *x* value.
4. To solve for *y* value, replace second column (*y* column) with the answer column and find the determinant. Now divide this determinant by the original matrix's determinant, this quotient is your *y* value.

Let's go back to the Do Now and solve the system using Cramer's Rule.

$4x-2y=10$

$3x-5y=11$

$\begin{bmatrix} 4 & -2 \\ 3 & -5 \end{bmatrix}$ $det = -20-(-6) = -14$

To find x: $\begin{bmatrix} 10 & -2 \\ 11 & -5 \end{bmatrix}$ $det = -50-(-22) = -28$

$x\text{-value} = \frac{-28}{-14} = 2$

To find y: $\begin{bmatrix} 4 & 10 \\ 3 & 11 \end{bmatrix}$ $det = 44-30 = 14$

$y\text{-value} = \frac{14}{-14} = -1$

Solve each of the following systems using Cramer's Rule, if possible.

2. $5x+4y=2$
$-x+y=-22$

$$\begin{bmatrix} 5 & 4 \\ -1 & 1 \end{bmatrix} \qquad \det = 5-(-4) = 9$$

To find x:

$$\begin{bmatrix} 2 & 4 \\ -22 & 1 \end{bmatrix} \qquad \det = 2-(-88) = 90$$

$$x\text{-value} = \frac{90}{9} = 10 \qquad (10, -12)$$

To find y:

$$\begin{bmatrix} 5 & 2 \\ -1 & -22 \end{bmatrix} \qquad \det = -110-(-2) = -108$$

$$y\text{-value} = \frac{-108}{9} = -12$$

3. $2x-5y=2$
$3x-7y=1$

$$\begin{bmatrix} 2 & -5 \\ 3 & -7 \end{bmatrix} \qquad \det = -14-(-15) = 1$$

To find x:

$$\begin{bmatrix} 2 & -5 \\ 1 & -7 \end{bmatrix} \qquad \det = -14-(-5) = -9$$

$$x\text{-value} = \frac{-9}{1} = -9$$

To find y:

$$\begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix} \qquad \det = 2-6 = -4$$

$$y\text{-value} = \frac{-4}{1} = -4$$

4. $-2x+8y=1$
$x-4y=5$

$$\begin{bmatrix} -2 & 8 \\ 1 & -4 \end{bmatrix} \qquad \det = 8-8 = 0$$

Using Cramer's Rule is not possible here

You would have to solve it using another method.

Practice

Solve each of the following systems using Cramer's Rule, if possible.

1. $3x - 10y = 15$
   $5x + 4y = 22$

$\begin{bmatrix} 3 & -10 \\ 5 & 4 \end{bmatrix}$ det = 12 - (-50) = 62

To find x: $\begin{bmatrix} 15 & -10 \\ 22 & 4 \end{bmatrix}$ det = 60 - (-220)

x value = $\frac{280}{62} = \frac{140}{31}$

To find y: $\begin{bmatrix} 3 & 15 \\ 5 & 22 \end{bmatrix}$ det = 66 - 75 = -9

y value = $\frac{-9}{62}$

2. $2x + y = 0.3$
   $3x - y = -1.3$

$\begin{bmatrix} 2 & 1 \\ 3 & -1 \end{bmatrix}$ det = -2 - 3 = -5

To find x: $\begin{bmatrix} .3 & 1 \\ -1.3 & -1 \end{bmatrix}$ det = -.3 - (-1.3) = 1.0

x-value = $\frac{1}{-5} = -.2$

To find y: $\begin{bmatrix} 2 & .3 \\ 3 & -1.3 \end{bmatrix}$ -2.6 - (.9) = -3.5

y-value = $\frac{-3.5}{-5} = .7$

3. $x + y - z = 2$
   $2x - y + z = -5$
   $x - 2y + 3z = 4$

4. $2x - 3y + 4z = 10$
   $6x - 9y + 12z = 24$
   $x + 2y - 3z = 5$

$\left[\begin{array}{ccc} 2 & -3 & 4 \\ 6 & -9 & 12 \\ 1 & 2 & -3 \end{array}\right] \begin{array}{cc} 2 & -3 \\ 6 & -9 \\ 1 & 2 \end{array}$

det = 0

Cramer's Rule not possible

Homework: Textbook p. 646 #s 13-16

3. $x + y - z = 2$
   $2x - y + z = -5$
   $x - 2y + 3z = 4$

1 - 2 + 6 = 5

$\left[\begin{array}{ccc} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & -2 & 3 \end{array}\right] \begin{array}{cc} 1 & 1 \\ 2 & -1 \\ 1 & -2 \end{array}$

-3 + 1 + 4 = 2

det = 2 - 5 = -3

To find x:

~~4 - 4~~ -15

$\left[\begin{array}{ccc} 2 & 1 & -1 \\ -5 & -1 & 1 \\ 4 & -2 & 3 \end{array}\right] \begin{array}{cc} 2 & 1 \\ -5 & -1 \\ 4 & -2 \end{array}$

-6 + 4 - 10 = -12

det = -12 - (-15) = 3

x value = $\frac{3}{-3} = -1$

To find y:

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -5 & 1 \\ 1 & 4 & 3 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & -5 \\ 1 & 4 \end{matrix}$$

det = -42

$$y\text{-value} = \frac{-42}{-3} = 14$$

To find z:

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & -5 \\ 1 & -2 & 4 \end{bmatrix} \begin{matrix} 1 & 1 \\ 2 & -1 \\ 1 & -2 \end{matrix}$$

det = 33

$$z\text{-value} = \frac{33}{-3} = -11$$

(-1, 14, 11)

(5) $\frac{33}{8}$ (7) 10 (27) $3x-5y=0$ (29) $x+3y-5=0$ (23) not collin (25) collinear

-66
-7
73

(5) $(0,\frac{1}{2}),(\frac{5}{2},0),(4,3)$

$$A=\pm\frac{1}{2}\begin{vmatrix} 0 & \frac{1}{2} & 1 \\ \frac{5}{2} & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}\begin{matrix} 0 & \frac{1}{2} \\ \frac{5}{2} & 0 \\ 4 & 3 \end{matrix}$$

$0 \quad 0 \quad -\frac{5}{4}$

$0 \quad \frac{1}{2}\cdot 2 \quad \frac{15}{2}$

$38-5$

$$\pm\frac{1}{2}\left(\frac{19}{2}+\left(\frac{-5}{4}\right)\right)$$

$$+\frac{1}{2}\left(\frac{33}{4}\right)=\frac{33}{8}$$

(7)

$-7 \quad -36 \quad -30$

4	5	1	4	5
6	1	1	6	1
7	9	1	7	9

$4 \quad 35 \quad 54$

$54 + 35 = 89$

$93+(-73)=20$

$\frac{1}{2}(20)=10$

(27)

$0 \quad 3y \quad 0$

x	y	1	x	y
0	0	1	0	0
5	3	1	5	3

$0 \quad 5y \quad 0$

$$\begin{array}{r} 5y+0 \\ -3x+0 \quad +0 \\ \hline \end{array}$$

$-3x+5y=0$

or

$3x-5y=0$

$3x-5y=0$

(29)

$6 \quad x \quad -4y$

x	y	1	x	y
-4	3	1	-4	3
2	1	1	2	1

$3x \quad 2y \quad -4$

$$\begin{array}{r} 3x+2y-4 \\ -x+4y-6 \\ \hline \end{array}$$

$2x+6y-10=0$

or

$x+3y-5=0$

$(0, 2)\ (1, 2.4)\ (-1, 1.6)$

$$\begin{array}{ccccc} & & -2.4 & 0 & 2 \\ 0 & 2 & 1 & 0 & 2 \\ 1 & 2.4 & 1 & 1 & 2.4 \\ -1 & 1.6 & 1 & -1 & 1.6 \\ & & 2.4 & -2 & 1.6 \end{array}$$

25

$2 + (2 \quad) = 0$ collinear

23 $(2, -\frac{1}{2})\ (-4, 4)\ (6, -3)$

$$\begin{array}{ccccc} & & 24 & -6 & 2 \\ 2 & -\frac{1}{2} & 1 & 2 & -\frac{1}{2} \\ -4 & 4 & 1 & -4 & 4 \\ 6 & -3 & 1 & 6 & 3 \\ & & 8 & -3 & -12 \end{array}$$

$-7 + (\overset{20}{-24} + 6 - 2) = 13$ not collinear