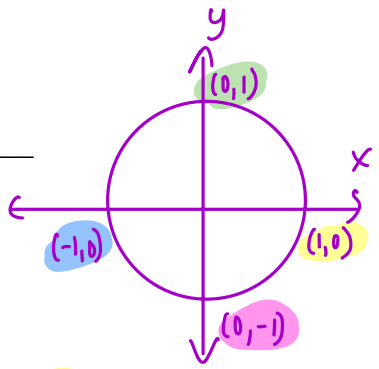
Name: _____ Date:_____

PC:Reference Angles and Special Angles Ms. Loughran

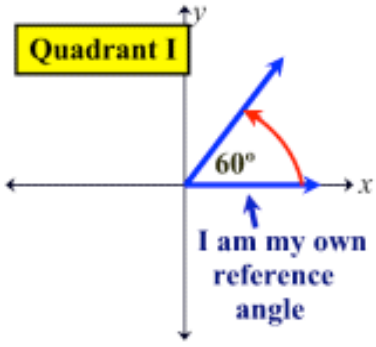


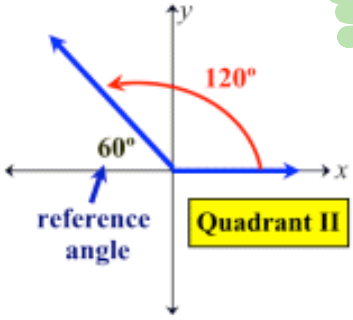
Do Now:

1. Complete the table.

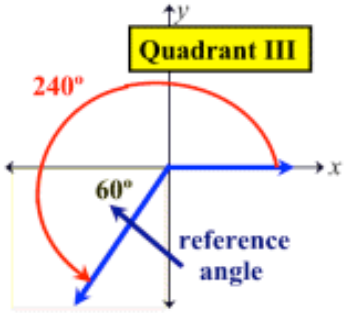
θ	0°	90°	180°	270°	360°
Radians	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Sin θ	0	1	0	-1	0
Cos θ	1	0	-1	0	1
Tan θ	0	undefined	0	undefined	0

Given an angle θ in standard position, the ***reference angle*** of θ, is the positive acute angle formed by the terminal side of θ and the positive or negative portion of the x-axis.

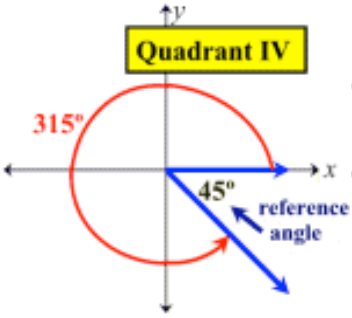




QII
$180^\circ - \theta$



QIII
$\theta - 180^\circ$



QIV
$360^\circ - \theta$

Reference angles will help you to express the sine, cosine or tangent of any angle in terms of the sine, cosine or tangent of a positive acute angle.

Given: $0° \leq \theta \leq 360°$

To find reference $\measuredangle$s

ALL

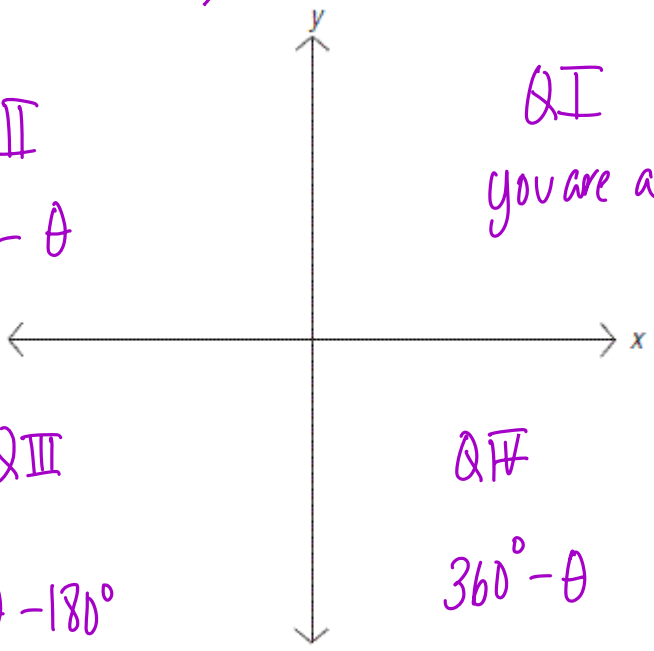
SINE

QII

$180° - \theta$

QI

you are a reference $\measuredangle$



TANGENT

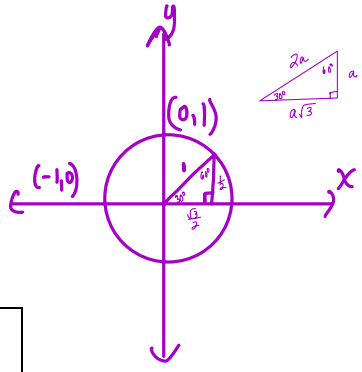
QIII

$\theta - 180°$

QIV

$360° - \theta$

COSINE



You need to memorize the following values.

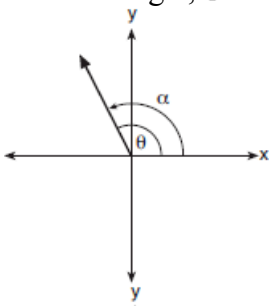
θ	30°	45°	60°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
Sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tan θ	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

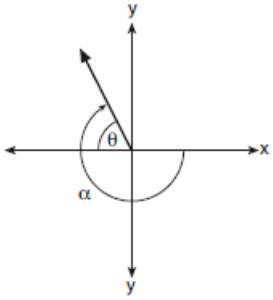
Exercise Set A

1 Which diagram represents an angle, α, measuring $\frac{13\pi}{20}$ radians drawn in standard position, and its reference angle, θ?

1)



2)



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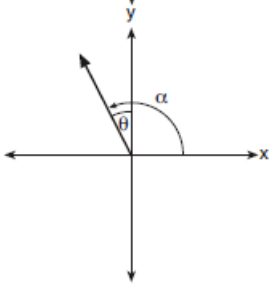

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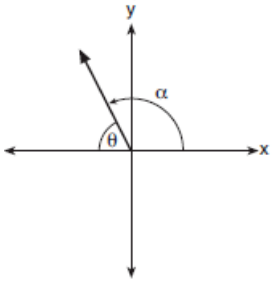


2 Sin 190° is equal to

1) sin 10°
2) cos 10°
3) −sin 10°
4) −cos 10°

Quadrant
Reference Angle
Sign

S | A
T | C

Sin 190°

Q III
R 190° − 180° = 10°
S ⊖

−sin 10°

3 Which expression is equivalent to $\sin(200°)$?

1) $-\sin 20°$
2) $\cos 20°$
3) $\cos 70°$
4) $-\sin 70°$

Q III
R $200° - 180° = 20°$
S $\ominus$
$-\sin 20°$

4 Expressed as a function of a positive acute angle, $\sin 230°$ is equal to

1) $-\sin 40°$
2) $-\sin 50°$
3) $\sin 40°$
4) $\sin 50°$

5 The expression $\sin 240°$ is equivalent to

1) $\sin 60°$
2) $\cos 60°$
3) $-\sin 60°$
4) $-\cos 60°$

$\sin 240°$

6 Which expression is equivalent to $\sin(-120°)$?

1) $\sin 60°$
2) $-\sin 60°$
3) $\cos 30°$
4) $-\cos 60°$

$-120° + 360° = 240°$
Q: III
R: $240° - 180° = 60°$
S: $\ominus$
$-\sin 60°$

7 Expressed as a function of a positive acute angle, $\sin(-230°)$ is equal to

1) $\sin 50°$
2) $-\sin 50°$
3) $\cos 50°$
4) $-\cos 50°$

8 Which expression is *not* equivalent to $\sin 150°$?

1) $\sin 30°$
2) $-\sin 210°$
3) $\cos 60°$
4) $-\cos 60°$

15 Express $\sin(-170°)$ as a function of a positive acute angle.

$-170° + 360° = 190°$ Q: III R: $190° - 180° = 10°$ S: ⊖ $-\sin 10°$

16 Express $\sin(-215°)$ as a function of a positive acute angle.

17 Express $\cos(-155°)$ as a function of a positive acute angle.

$-155° + 360° = 205°$ Q III R $205° - 180° = 25°$ S − $-\cos 25°$ $\cos 205°$

18 Express $\cos(-220°)$ as a function of a positive acute angle.

19 Express $\tan 230°$ as a function of a positive acute angle.

20 Express $\tan(-140°)$ as a function of a positive acute angle.

21 Sketch an angle of 250° in standard position and then express cos 250° as a cosine function of a positive acute angle.

8 The value of $\csc 138°23'$ rounded to four decimal places is
1) -1.3376
2) -1.3408
3) 1.5012
4) 1.5057

9 The value of $\cos 305°$ is
1) 0.5736
2) 0.8192
3) -0.8192
4) -0.5736

10 Find the value of $\sin 135°$ in radical form.

$+\sin 45°$

$\frac{\sqrt{2}}{2}$

Q: II
R $180° - 135° = 45°$
S: ⊕
T

11 Find the value of $\tan 120°$.

$-\tan 60°$

$-\sqrt{3}$

Q: II
R: $180° - 120° = 60°$
S: −
T

12 Find the value of $\tan(-135°)$.

13 Express the product of $\cos 30°$ and $\sin 45°$ in simplest radical form.

14 Find the value of $\tan 31°27'$ to *four decimal places*.

15 Find the value of $\cos 32°32'$ to *four decimal places*.

16 Find the value of $\tan 27°26'$ to *four decimal places*.

17 Find the value of $\sin 37°34'$ to *four decimal places*.

18 Find $\tan 27°13'$ to *four decimal place.*

Homework 03-12

Exercise Set A

For 1 – 19, determine in which quadrant the angle of the given measure lies.

1. 215° QIII
2. -110° QIII
3. 318° QIV
4. 72° QI
5. 95° QII
6. -45° QIV
7. 225° QIII
8. 150° QII
9. 422° -360° / 62° QI
10. -240° QII
11. 680° -360° / 320° QIV
12. 23° QI
13. 812° $812^\circ - 360^\circ = 452^\circ - 360^\circ = 92^\circ$ QII
14. -300° QI
15. 289° QIV
16. $\frac{5\pi}{4} \cdot \frac{180^\circ}{\pi} = 225^\circ$ QIII
17. $-\frac{3\pi}{4} \cdot \frac{180^\circ}{\pi} = -135^\circ$ QIII
18. $\frac{5\pi}{3} \cdot \frac{180^\circ}{\pi} = 300^\circ$ QIV
19. $\frac{7\pi}{6} \cdot \frac{180^\circ}{\pi} = 210^\circ$ QIII

For 20 - 27, name the least possible positive measure and the greatest possible negative measure of an angle that is coterminal with the given angle.

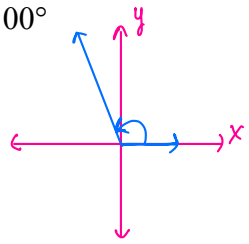
20. 70° $+360^\circ$ → 430°; -360° → -290°
21. -60° $+360^\circ$ → 300°; -360° → -420°
22. 110° $+360^\circ$ → 470°; -360° → -250°
23. 225° $+360^\circ$ → 585°; -360° → -135°
24. 270° $+360^\circ$ → 630°; -360° → -90°
25. -315° $+360^\circ$ → 45°; -360° → -675°
26. -180° $+360^\circ$ → 180°; -360° → -540°
27. -930° $+360^\circ$ → -570°; $+360^\circ$ → -210°; $+360^\circ$ → 150°

For 28- 31, determine whether the following pairs of angles in standard position are coterminal.

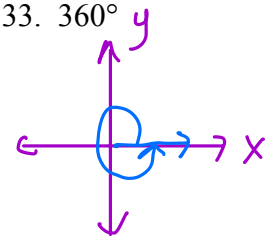
28. 40° and 400° yes $400^\circ - 40^\circ = 360^\circ$ or $40^\circ - 400^\circ = -360^\circ$ ← is a mult. of 360°
29. -120° and 120° No $-120^\circ - 120^\circ = -240^\circ$ not a multiple of 360°
30. 180° and -180° yes $180^\circ - (-180^\circ) = 360^\circ$ or $-180^\circ - 180^\circ = -360^\circ$ } mult. of 360°
31. 90° and -270° Yes $90 - (-270^\circ) = 360^\circ$ ← is a multiple of 360°

For 32 - 41, sketch an angle in standard position with the given measure

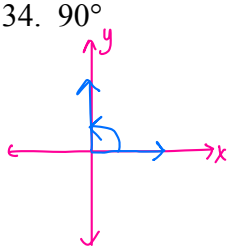
32. 100°



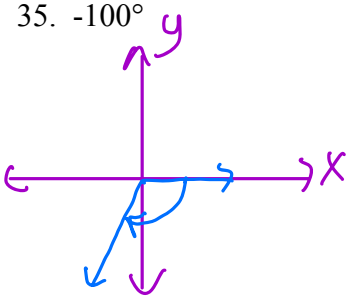
33. 360°



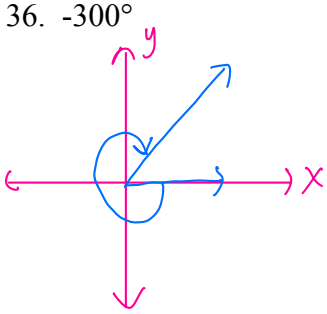
34. 90°



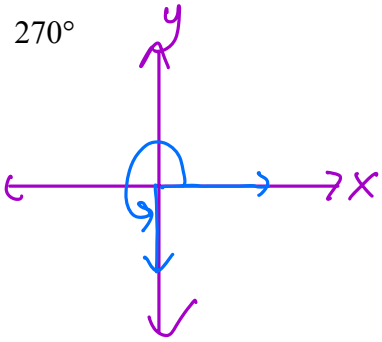
35. -100°



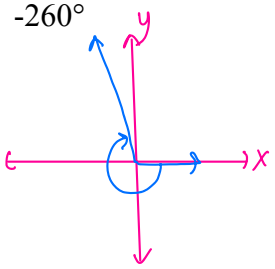
36. -300°



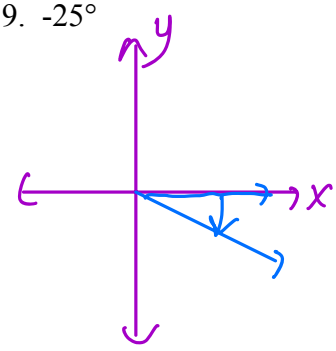
37. 270°



38. -260°

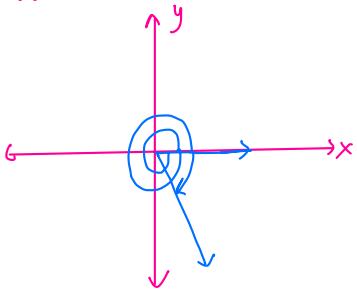


39. -25°



40. -800°

$+360^\circ$
-440°
$+360^\circ$
-80° Q IV



41. 1140°

-360°
780°
-360°
420°
-360°
60°