Name: ____ Date: _____________

PC Ms. Loughran

1. Find $\begin{vmatrix} c & a & t \\ d & o & g \\ e & m & u \end{vmatrix}\begin{matrix} c & a \\ d & o \\ e & m \end{matrix}$

det = cou + age + tdm − (toe + cgm + adu)

2. Find A^{-1}.

$$A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

det = ps − qr

$$A^{-1} = \frac{1}{ps-qr}\begin{bmatrix} s & -q \\ -r & p \end{bmatrix} = \begin{bmatrix} \frac{s}{ps-qr} & \frac{-q}{ps-qr} \\ \frac{-r}{ps-qr} & \frac{p}{ps-qr} \end{bmatrix}$$

3. $\begin{array}{c} 2\times3 \\ \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \end{array} \bullet \begin{array}{c} 3\times2 \\ \begin{bmatrix} g & h \\ i & j \\ k & l \end{bmatrix} \end{array} = \begin{bmatrix} ag+bi+ck & ah+bj+cl \\ dg+ei+fk & dh \quad ej \quad fl \end{bmatrix}$

$\sin^2\theta + \cos^2\theta = 1$
$-\cos^2\theta \quad -\cos^2\theta$
$\sin^2\theta = 1-\cos^2\theta$

Name:______________________________ Date:____________

PC: Trigonometric Identities Ms. Loughran

The Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\tan^2\theta + 1 = \sec^2\theta$$
$$\cot^2\theta + 1 = \csc^2\theta$$

$\sin^2\theta + \cos^2\theta = 1$
$-\sin^2\theta \quad -\sin^2\theta$
$\cos^2\theta = 1-\sin^2\theta$

You are familiar with the following reciprocal identities:

secant — $\sec\theta = \dfrac{1}{\cos\theta}, \cos\theta \neq 0$ cosecant — $\csc\theta = \dfrac{1}{\sin\theta}, \sin\theta \neq 0$ cotangent — $\cot\theta = \dfrac{1}{\tan\theta}, \tan\theta \neq 0$

And the quotient identities:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}, \sin\theta \neq 0$$

An identity is an equation that is true for all permissible replacements of the variable.

Proving an identity:
To prove that a trigonometric statement is an identity, note:

1. **The object is to show that the two sides of the statement are equivalent.**
   ⇒ You may work on only one side and show that it is equivalent to the other.
      ➢ Work on the more complicated side.
   ⇒ You may work on the two sides independently until you arrive at equivalent expressions.
      ➢ You may not perform operations involving the two sides simultaneously. You are not solving an equation. That is, never cross the equal sign for any purpose. As a reminder, use a line between sides.

2. **Use the basic identities to transform one or both sides of the proposed identity.**
   ⇒ A general starting point is to rewrite expressions in terms of sine and cosine, but be alert to situations when a Pythagorean substitution is appropriate.

3. **After replacements have been made, do the algebra suggested by the form of the expression.**
   ⇒ If there is a complex fraction, simplify it.
   ⇒ If there are two fractions, combine them.
   ⇒ Look for possibilities of factoring.

1. $\frac{-1}{\cos A}$ is equivalent to

(1) $\sec A$ (2) $-\sec A$ (3) $\sin A$ (4) $-\sin A$

cosine and secant are reciprocals

2. $\frac{\cot\theta}{\csc\theta}$ is equivalent to

(1) $\sec\theta$ (2) $\sin\theta$ (3) $\cos\theta$ (4) $\csc\theta$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \frac{\cos\theta}{\sin\theta} \div \frac{1}{\sin\theta}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \cos\theta$$

3. $\frac{\sec\theta}{\csc\theta}$ is equivalent to

(1) $\sin\theta$ (2) $\cos\theta$ (3) $\tan\theta$ (4) $\cot\theta$

$$\frac{\frac{1}{\cos\theta}}{\frac{1}{\sin\theta}} = \frac{1}{\cos\theta} \div \frac{1}{\sin\theta}$$

$$\frac{1}{\cos\theta} \cdot \frac{\sin\theta}{1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

4. $\frac{\sin\theta}{\tan\theta}$ is equivalent to

(1) $-\cos\theta$
(2) $\cos\theta$
(3) $1-\cos\theta$
(4) $1+\cos\theta$

$$\frac{\sin\theta}{\frac{\sin\theta}{\cos\theta}} = \sin\theta \div \frac{\sin\theta}{\cos\theta}$$

$$\cancel{\sin\theta}\cdot\frac{\cos\theta}{\cancel{\sin\theta}} = \cos\theta$$

7. The expression $\frac{\tan x}{\sec^2 x}$ is equivalent to

(1) $\sin x$
(2) $\sin x\cos x$
(3) $\frac{\sin^3 x}{\cos x}$
(4) $\frac{\cos^3 x}{\sin x}$

$$\frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{\sin x}{\cancel{\cos x}}\cdot\frac{\cos^{\cancel{2}} x}{1}$$

$$= \sin x\cos x$$

10. $(\cot\theta)(\sec\theta)$ is equivalent to

(1) $\tan\theta$ (2) $\cos\theta$ (3) $\cot\theta$ (4) $\csc\theta$

$$\left(\frac{\cancel{\cos\theta}}{\sin\theta}\right)\left(\frac{1}{\cancel{\cos\theta}}\right) = \frac{1}{\sin\theta} = \csc\theta$$

11. $\tan A \cdot \cos A \cdot \csc A$ is equivalent to

(1) 1 (2) $\frac{1}{2}$ (3) $\sin A$ (4) $\frac{1}{\sin A}$

$$\frac{\cancel{\sin A}}{\cancel{\cos A}} \cdot \cancel{\cos A} \cdot \frac{1}{\cancel{\sin A}} = 1$$

14. $\sin\theta\,(\csc\theta - \sin\theta)$ is equivalent to

(1) 1 (3) $\tan\theta - 1$

(2) $\cos\theta$ (4) $\cos^2\theta$

$$\sin\theta\left(\frac{1}{\sin\theta} - \sin\theta\right)$$

$$\cancel{\sin\theta}\left(\frac{1}{\cancel{\sin\theta}}\right) - (\sin\theta)(\sin\theta)$$

$$1 - \sin^2\theta$$

$$\cos^2\theta$$

17. $\dfrac{\overbrace{\sin^2 x + \cos^2 x}^{=1}}{\cos x}$ is equivalent to

(1) $\sin x \cos x$ (3) $\csc x$

(2) $\tan x \cos x$ (4) $\sec x$

$$\frac{1}{\cos x} = \sec x$$

$(\cos A)$

18. $\underset{(\cos A)\ 1}{\cos A} + \frac{\sin^2 A}{\cos A}$ is equivalent to

(1) 1 (2) $\sec A$ (3) $\csc A$ (4) $\cos A$

$$\frac{\cos^2 A}{\cos A} + \frac{\sin^2 A}{\cos A} = \frac{\cos^2 A + \sin^2 A}{\cos A} = \frac{1}{\cos A} = \sec A$$

$(\sin B)$

26. $\frac{\cos^2 B}{\sin B} + \underset{1\ (\sin B)}{\sin B}$ is equivalent to

(1) 1 (2) $\frac{1}{\csc B}$ (3) $\frac{1}{\sin B}$ (4) $\cos^2 B$

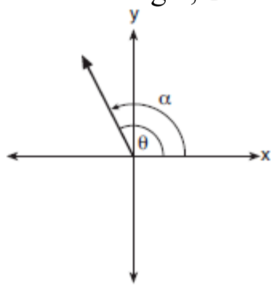
$$\frac{\cos^2 B + \sin^2 B}{\sin B} = \frac{1}{\sin B}$$

Homework 03-13

Exercise Set A

1Which diagram represents an angle, α, measuring $\frac{13\pi}{20}$ radians drawn in standard position, and its reference angle, θ?

1)



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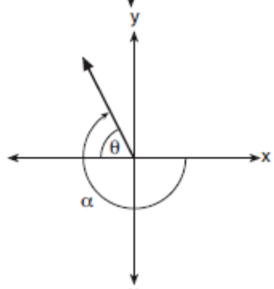

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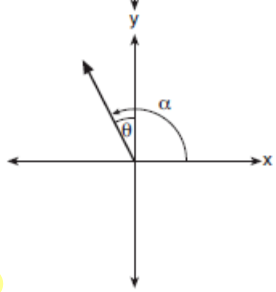
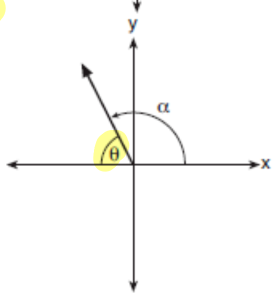


4)



2 Sin 190° is equal to

1) $\sin 10°$
2) $\cos 10°$
3) $-\sin 10°$
4) $-\cos 10°$

3 Which expression is equivalent to $\sin(200°)$?
1) $-\sin 20°$
2) $\cos 20°$
3) $\cos 70°$
4) $-\sin 70°$

4 Expressed as a function of a positive acute angle, $\sin 230°$ is equal to
1) $-\sin 40°$
2) $-\sin 50°$
3) $\sin 40°$
4) $\sin 50°$

5 The expression $\sin 240°$ is equivalent to
1) $\sin 60°$
2) $\cos 60°$
3) $-\sin 60°$
4) $-\cos 60°$

6 Which expression is equivalent to $\sin(-120°)$?
1) $\sin 60°$
2) $-\sin 60°$
3) $\cos 30°$
4) $-\cos 60°$

7 Expressed as a function of a positive acute angle, $\sin(-230°)$ is equal to
1) $\sin 50°$
2) $-\sin 50°$
3) $\cos 50°$
4) $-\cos 50°$

8 Which expression is *not* equivalent to $\sin 150°$?
~~1)~~ $\sin 30°$
2) $-\sin 210°$
3) $\cos 60° = \frac{1}{2}$
4) $-\cos 60°$ $-\frac{1}{2}$

Q II
R $30°$
S +
T

$+\sin 30°$

$\frac{1}{2}$

9 Which expression is equivalent to $\cos 120°$?
1) $\cos 60°$
2) $\cos 30°$
3) $-\sin 60°$
4) $-\sin 30°$

10 Two straight roads intersect at an angle whose measure is 125°. Which expression is equivalent to the cosine of this angle?
1) $\cos 35°$
2) $-\cos 35°$
3) $\cos 55°$
4) $-\cos 55°$

11 Expressed as a function of a positive acute angle, $\cos(-305°)$ is equal to
1) $-\cos 55°$
2) $\cos 55°$
3) $-\sin 55°$
4) $\sin 55°$

12 The expression $\tan(-240°)$ is equivalent to
1) $\tan 60°$
2) $-\tan 30°$
3) $-\tan 60°$
4) $\tan 30°$

13 Expressed as a function of a positive acute angle, $\cot(-120)°$ is equivalent to
1) $-\tan 60°$
2) $\cot 60°$
3) $-\cot 30°$
4) $\cot 30°$

14 The expression $\cot(-200°)$ is equivalent to
1) $-\tan 20°$
2) $\tan 70°$
3) $-\cot 20°$
4) $\cot 70°$

15 Express $\sin(-170°)$ as a function of a positive acute angle.

16 Express $\sin(-215°)$ as a function of a positive acute angle.

$\sin 35°$

17 Express $\cos(-155°)$ as a function of a positive acute angle.

$-\cos 25°$

18 Express $\cos(-220°)$ as a function of a positive acute angle.

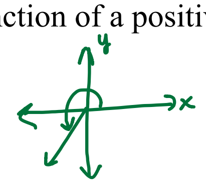
$-\cos 40°$

19 Express $\tan 230°$ as a function of a positive acute angle.

$\tan 50°$

20 Express $\tan(-140°)$ as a function of a positive acute angle.

21 Sketch an angle of 250° in standard position and then express cos 250° as a cosine function of a positive acute angle.



$-\cos 70°$

Exercise Set B

1 Which is the value of $\cos(-240°)$?

1) $-\frac{1}{2}$
2) $\frac{3}{2}$
3) $\frac{1}{2}$
4) $-\frac{3}{2}$

2 What is the value of $\sin(-240°)$?

1) $\frac{1}{2}$
2) $-\frac{1}{2}$
3) $\frac{\sqrt{3}}{2}$
4) $-\frac{\sqrt{3}}{2}$

3 What is the value of $\cos(-120°)$?

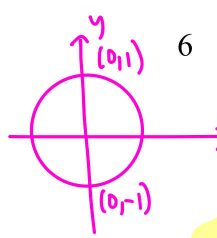
1) $\frac{1}{2}$
2) $-\frac{1}{2}$
3) $\frac{\sqrt{3}}{2}$
4) $-\frac{\sqrt{3}}{2}$

4 The value of $(\sin 60°)(\cos 60°)$ is

1) $\frac{3}{4}$
2) $\frac{\sqrt{2}}{4}$
3) $\frac{\sqrt{3}}{3}$
4) $\frac{\sqrt{3}}{4}$

5 Which is equal in value to $\sin 180°$?

1) $\tan 45°$
2) $\cos 90°$
3) $\cos 0°$
4) $\tan 90°$



6 In the interval $0° \le x < 360°$, $\tan x$ is undefined when x equals

1) 0º and 90º
2) 90º and 180º
3) 180º and 270º
4) 90º and 270º

$\tan 90° = \frac{1}{0} =$ und.

$\tan 270° = \frac{-1}{0}$ und.

7 The value of $\tan 126°43'$ to the *nearest ten-thousandth* is

1) -1.3407
2) -1.3408
3) -1.3548
4) -1.3549

13 Express the product of cos 30° and sin 45° in simplest radical form.

$$\frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4}$$