Do Now: From Exercise Set B #s 21 and 26

21. $\dfrac{\sin\theta\cot\theta + \cos^2\theta}{1+\cos\theta} = \cos\theta$

$$\frac{\cancel{\sin\theta}\cdot\dfrac{\cos\theta}{\cancel{\sin\theta}} + \cos^2\theta}{1+\cos\theta}$$

$$\frac{\cos\theta + \cos^2\theta}{1+\cos\theta}$$

$$\frac{\cos\theta(\cancel{1+\cos\theta})}{\cancel{1+\cos\theta}}$$

$$\cos\theta = \cos\theta$$

26. $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{\sin\theta}{1+\cos\theta}$

$$\frac{(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$\frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$\frac{\sin^{\cancel{2}}\theta}{\cancel{\sin\theta}(1+\cos\theta)}$$

$$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta}{1+\cos\theta}$$

To convert:

From degrees to radians: multiply by $\frac{\pi}{180°}$

From radians to degrees: multiply by $\frac{180°}{\pi}$

The Pythagorean Identities

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$

Sometimes that identity is hidden:

$1 - \sin^2\theta = \cos^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

You are familiar with the following reciprocal identities:

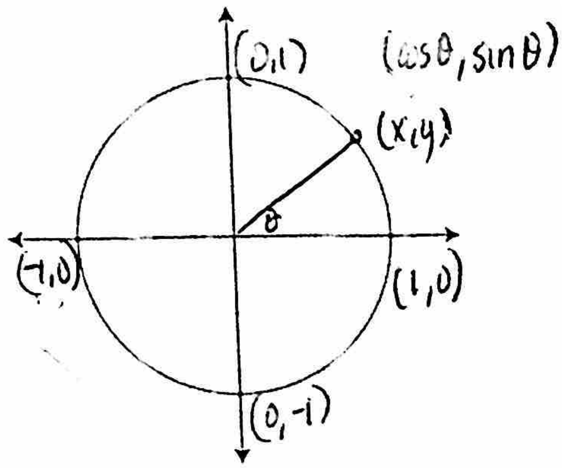
secant: $\sec\theta = \frac{1}{\cos\theta}, \cos\theta \neq 0$ $\quad$ $\cos\theta = \frac{1}{\sec\theta}, \sec\theta \neq 0$

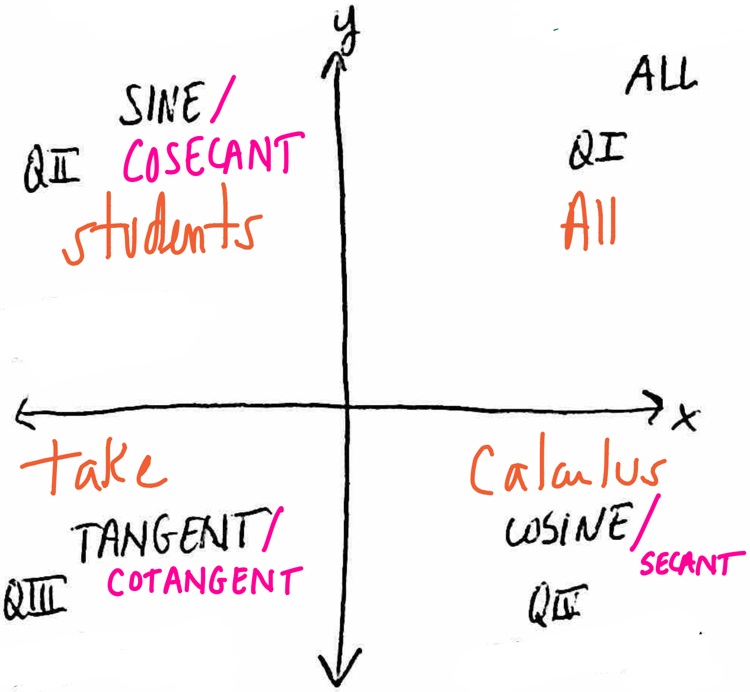
cosecant: $\csc\theta = \frac{1}{\sin\theta}, \sin\theta \neq 0$ $\quad$ $\sin\theta = \frac{1}{\csc\theta}, \csc\theta \neq 0$

cotangent: $\cot\theta = \frac{1}{\tan\theta}, \tan\theta \neq 0$ $\quad$ $\tan\theta = \frac{1}{\cot\theta}, \cot\theta \neq 0$

And the quotient identities:

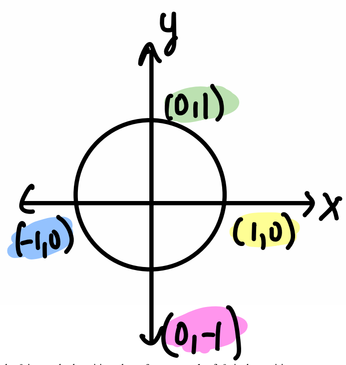
$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \cos\theta \neq 0 \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}, \sin\theta \neq 0$$





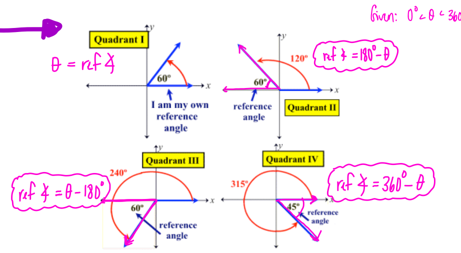
θ	30°	45°	60°
Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
Sin θ	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
Cos θ	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
Tan θ	$\frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

θ	0°	90°	180°	270°	360°
Radians	0	$\pi/2$	π	$\frac{3\pi}{2}$	2π
Sin θ	0	1	0	−1	0
Cos θ	1	0	−1	0	1
Tan θ	0	undefined	0	undefined	0



Given an angle θ in standard position, the ***reference angle*** of θ, is the positive acute angle formed by the terminal side of θ and the positive or negative portion of the x-axis.

Given: $0° < \theta < 360°$



Express as a function of a positive acute angle (reference angles)

θ uadrant?

Reference ∢

Sign

Evaluate/find the value of

Q

R

S

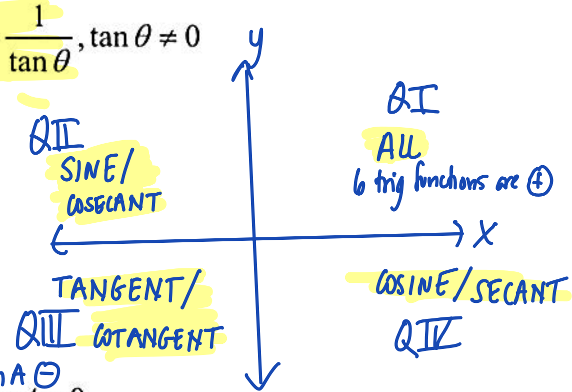
Table

Name:______________________________ Date:___________

PC: Reciprocal Trig Functions Ms. Loughran

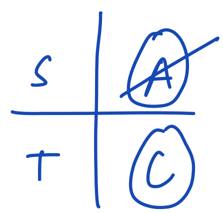
Each of the basic trigonometric functions has a corresponding reciprocal function. The **secant** function (sec) is the reciprocal of the cosine function, the **cosecant** function (csc) is the reciprocal of the sine function, and the **cotangent** function (cot) is the reciprocal of the tangent function.

$\sec\theta = \dfrac{1}{\cos\theta}, \cos\theta \neq 0$ $\csc\theta = \dfrac{1}{\sin\theta}, \sin\theta \neq 0$ $\cot\theta = \dfrac{1}{\tan\theta}, \tan\theta \neq 0$



Also since $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, then $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$.

1. Name the quadrant in which $\angle A$ must lie if $\overset{\cos A \oplus}{\sec A > 0}$ and $\overset{\sin A \ominus}{\csc A < 0}$.



QIV

2. Find the exact value of: (a) $\sec 120°$
(b) $\cot 210°$

	30°	45°	60°
$\sin\theta$	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
$\tan\theta$	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

(a) $\sec 120°$ ← reciprocal of $\cos 120°$

Q II
R $180° - 120° = 60°$
S −
T

$-\cos 60°$

$-\frac{1}{2}$

-2

(b) $\cot 210°$ ← reciprocal of $\tan 210°$

Q III
R $210° - 180° = 30°$
S ⊕
T

$+\tan 30°$

$\frac{\sqrt{3}}{3}$

$\frac{3}{\sqrt{3}}$

Exercises 4–10: Find the exact value of each expression.

5 csc 225° ← reciprocal of sin 225°

Q III

R $225° - 180° = 45°$

S −

T

$-\sin 45°$

$-\frac{\sqrt{2}}{2}$

$\frac{-2}{\sqrt{2}}$

7 cot 420° $\xrightarrow{\text{reciprocal}}$ of tan 420°

$420° - 360°$

tan 60°

Q I

R 60°

S ⊕

T

$\sqrt{3}$

$\frac{1}{\sqrt{3}}$

9 (sec 150°)(cos 150°)

$\frac{1}{\cancel{\cos 150°}} \cdot \cancel{\cos 150°} = 1$

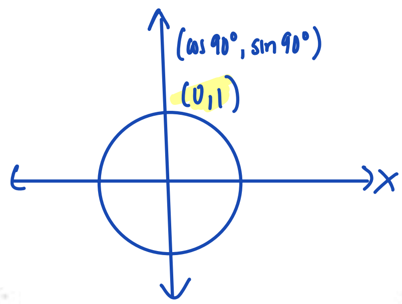
17 Which expression is equivalent to csc 45°?

(1) $\frac{1}{\sin 45°}$

(2) $\frac{1}{\sec 45°}$

(3) $\frac{1}{\tan 45°}$

(4) sin (−45°)



19 If $g(x) = \sin x + \csc x$, find $g(90°)$.

(1) 1

(2) 2

(3) 0

(4) −2

reciprocal of 90°
the reciprocal of 1
is $\frac{1}{1} = 1$

$g(90°) = \sin 90° + \csc 90°$

$= 1 + 1 = 2$

Homework 03-18

15. $\dfrac{\cos\theta\sin^2\theta}{1-\cos\theta} = \cos\theta + \cos^2\theta$

$\dfrac{\cos\theta(1-\cos^2\theta)}{1-\cos\theta}$

$\dfrac{\cos\theta(\cancel{1-\cos\theta})(1+\cos\theta)}{\cancel{1-\cos\theta}}$

$\cos\theta(1+\cos\theta)$

$\cos\theta+\cos^2\theta = \cos\theta+\cos^2\theta$

16. $\dfrac{\tan\theta-\cot\theta}{\tan\theta+\cot\theta} = 2\sin^2\theta - 1$

$\dfrac{(\cancel{\cos\theta}\sin\theta)\dfrac{\sin\theta}{\cancel{\cos\theta}} - \dfrac{\cos\theta}{\cancel{\sin\theta}}(\cos\theta\cancel{\sin\theta})}{(\cancel{\cos\theta}\sin\theta)\dfrac{\sin\theta}{\cancel{\cos\theta}} + \dfrac{\cos\theta}{\cancel{\sin\theta}}(\cos\theta\cancel{\sin\theta})}$

$\dfrac{\sin^2\theta-\cos^2\theta}{\sin^2\theta+\cos^2\theta}$

$\dfrac{\sin^2\theta-\cos^2\theta}{1}$

$\sin^2\theta-(1-\sin^2\theta)$

$\sin^2\theta-1+\sin^2\theta$

$2\sin^2\theta-1 = 2\sin^2\theta-1$

17. $\csc x - \sin x = \dfrac{\cot x}{\sec x}$

$\dfrac{1}{\sin x} - \sin x \quad\Big|\quad \dfrac{\frac{\cos x}{\sin x}}{\frac{1}{\cos x}}$

$\dfrac{1-\sin^2 x}{\sin x} \quad\Big|\quad \dfrac{\cos x}{\sin x}\cdot\dfrac{\cos x}{1}$

$\dfrac{\cos^2 x}{\sin x} = \dfrac{\cos^2 x}{\sin x}$

18. $\dfrac{\tan x\csc^2 x}{1+\tan^2 x} = \cot x$

$\dfrac{\tan x\csc^2 x}{\sec^2 x}$

$\dfrac{\dfrac{\cancel{\sin x}}{\cos x}\cdot\dfrac{1}{\sin^{\cancel{2}} x}}{\dfrac{1}{\cos^2 x}}$

$\dfrac{\dfrac{1}{\cos x\sin x}}{\dfrac{1}{\cos^2 x}}$

$\dfrac{1}{\cancel{\cos x}\sin x}\cdot\cos^{\cancel{2}} x$

$\dfrac{\cos x}{\sin x} = \dfrac{\cos x}{\sin x}$

(4) $\frac{\sin x \cdot \cos x}{\tan x} = \frac{\sin x \cdot \cos x}{\frac{\sin x}{\cos x}} = \cancel{\sin x} \cdot \cos x \cdot \frac{\cos x}{\cancel{\sin x}} = \cos^2 x$ (B)

(8) $\frac{\cot\theta}{\csc\theta} = \frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} = \frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1} = \cos\theta$ (C)

(12) $\csc^2\theta - \cot^2\theta - \sin^2\theta$

$\underbrace{\frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}} - \sin^2\theta$

these 2 have the same denominator

$\frac{1-\cos^2\theta}{\sin^2\theta}$ — Pythagorean Identity

$\frac{\sin^2\theta}{\sin^2\theta} - \sin^2\theta$

$1-\sin^2\theta$

$\cos^2\theta$

or since we have an identity that says $\cot^2\theta + 1 = \csc^2\theta$

$-\cot^2\theta \quad -\cot^2\theta$

$\underbrace{1 = \csc^2\theta - \cot^2\theta}$

$\underbrace{\csc^2\theta - \cot^2\theta} - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta$

(16) $\underbrace{\sin^2\theta + \cos^2 x} - b^2$

$1 - b^2$

$(1-b)(1+b)$ (A)

(20) $\frac{\sin^2 x + \cos^2 x}{\cos x} = \frac{1}{\cos x} = \sec x$ (A)

(24) $\sin\theta(\cot\theta - \csc\theta)$

$\sin\theta\left(\frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$

$\cos\theta - 1$ (D)

(28) $(1+\sin\theta)(1-\sin\theta)$ *multiply out

$1 - \sin\theta + \sin\theta - \sin^2\theta$

$1 - \sin^2\theta$

$\cos^2\theta$

22 $\dfrac{\cot A}{\csc A} = \dfrac{\frac{\cos A}{\sin A}}{\frac{1}{\sin A}} = \dfrac{\cos A}{\cancel{\sin A}} \cdot \dfrac{\cancel{\sin A}}{1} = \cos A$ (C)

36 $f(x) = \dfrac{\sin x}{\cos x} = \tan x$ tan x is undefined when $x = \pi/2,\ 3\pi/2, \ldots$ (A)

40 $\dfrac{\sin \theta}{\tan \theta} = \dfrac{\sin \theta}{\frac{\sin \theta}{\cos \theta}} = \cancel{\sin \theta} \cdot \dfrac{\cos \theta}{\cancel{\sin \theta}} = \cos \theta$ (B)

44 $\csc \theta \cdot \tan \theta \cdot \cos \theta$

$\dfrac{1}{\cancel{\sin \theta}} \cdot \dfrac{\cancel{\sin \theta}}{\cancel{\cos \theta}} \cdot \cancel{\cos \theta} = 1$ (C)