# Do Now: Try #1

Name:_____     Date:_____
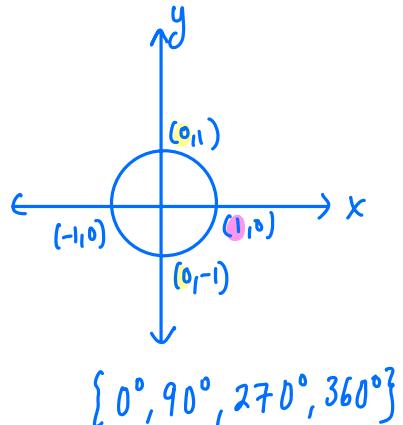PC: Solving Quadratic Trig Equations        Ms. Loughran

Examples:

1. For $0° \leq \theta \leq 360°$, solve: $\cos^2 \theta = \cos \theta$

let $x = \cos \theta$

$$x^2 = x$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$

| $x = 0$ | $x = 1$ |
|---|---|
| $\cos \theta = 0$ | $\cos \theta = 1$ |
| $\theta = 90°, 270°$ | $\theta = 0°, 360°$ |

$\{0°, 90°, 270°, 360°\}$

(0,1) (−1,0) (1,0) (0,−1)

2. For $0° \leq \theta \leq 360°$, solve: $2\cos^2 \theta + 7\cos \theta - 4 = 0$

let $x = \cos \theta$

$$2x^2 + 7x - 4 = 0$$

$\{60°, 300°\}$

$$2x^2 + 8x - x - 4 = 0$$
$$2x(x+4) - 1(x+4) = 0$$
$$(2x-1)(x+4) = 0$$

| $x = \frac{1}{2}$ | $x = -4$ |
|---|---|
| $\cos \theta = \frac{1}{2}$ | $\cos \theta = -4$ |
| ref $\angle : 60°$ | ✗ |

| s | A |
|---|---|
| t | C |

QI: 60°
QIV 360°−60° = 300°

$-1 \leq \cos \theta \leq 1$
same for sine
$-1 \leq \sin \theta \leq$

3. For $0° \leq \theta \leq 360°$, solve: $\sin^2 \theta - 1 = 0$

$$(\sin \theta - 1)(\sin \theta + 1) = 0$$

| $\sin \theta = 1$ | $\sin \theta = -1$ |
|---|---|
| $\theta = 90°$ | $\theta = 270°$ |

(0,1) (−1,0) (1,0) (0,−1)

$\{90°, 270°\}$

*↓ in radians*

4. For $0 \leq x \leq 2\pi$, solve: $\csc^2 x - \csc x + 3 = 5$

let $y = \csc x$

$$y^2 - y + 3 = 5$$
$$y^2 - y - 2 = 0$$
$$(y - 2)(y + 1) = 0$$

$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}$

| S | A |
|---|---|
| T | C |

ref ∡: 30°

QI: 30°
QII: 180° - 30° = 150°

$y = 2$ | $y = -1$

$\csc x = 2$ | $\csc x = -1$

$\sin x = \frac{1}{2}$ | $\sin x = -1$    $x = 270°$

5. To the nearest degree, solve for $\theta$ in the interval $0° \leq \theta \leq 360°$:
$$\sin^2 \theta - 4\sin \theta + 2 = 0$$
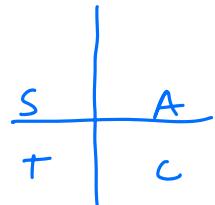
let $x = \sin \theta$

$\theta = 36°, 144°$

$$x^2 - 4x + 2 = 0$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{8}}{2} = \frac{4 \pm 2\sqrt{2}}{2} = 2 \pm \sqrt{2}$$

$\sqrt{4}\sqrt{2}$

| S | A |
|---|---|
| T | C |

QI: 35.85...°
QII: 144.14..°

$\sin \theta = 2 + \sqrt{2}$ ✗

$\sin \theta = 2 - \sqrt{2}$   ✔ + ✗

ref ∡ $\sin^{-1}(2 - \sqrt{2}) = 35.85...°$

6. To the nearest degree, solve for $x$ in the interval $0° < x < 360°$:

$\sin x$    $\sin x - 3 = \dfrac{-1}{\sin x}$  $\sin x$  (Note: $\sin x \neq 0$).

let $y = \sin x$

$y - 3 = \dfrac{-1}{y}$

$$\sin^2 x - 3\sin x = -1$$

let $y = \sin x$

$$y^2 - 3y = -1$$
$$y^2 - 3y + 1 = 0$$

$\{22°, 158°\}$

$$y = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$y = \frac{3 \pm \sqrt{5}}{2}$$

$\sin x = \dfrac{3 + \sqrt{5}}{2}$

$\sin x = 2.61...$ ✗

$\sin x = \dfrac{3 - \sqrt{5}}{2}$

$\sin x = .38...$

ref ∡ 22.455...

QI  22.455...°
QII  157.544...°

# Homework 03-26

**13**  $9\sin \beta - 2 = 4\sin \beta - 1$

$$5\sin \beta = 1$$
$$\sin \beta = \tfrac{1}{5}$$

$$\sin^{-1}\left(\tfrac{1}{5}\right) = 11.536\ldots°$$

QI  $\beta = 11.53\ldots°$
QII  $\beta = 180° - 11.536\ldots° = 168.46\ldots°$

$$\{11.5°, 168.5°\}$$

**14**  $-2(\tan \beta - 4) = 3(4 - \tan \beta)$

$$-2\tan \beta + 8 = 12 - 3\tan \beta$$
$$\tan \beta = 4$$
$$\tan^{-1}(4) = 75.963\ldots°$$

QI  $\beta = 75.963\ldots°$
QIII  $\beta = 180° + 75.963\ldots° = 255.963\ldots°$

$$\{76.0°, 256.0°\}$$

**15**  $3\sec \beta + 12 = \dfrac{3}{4}(8\sec \beta - 4)$

$$3\sec\beta + 12 = 6\sec \beta - 3$$
$$15 = 3\sec\beta$$
$$5 = \sec\beta$$
$$\cos \beta = \tfrac{1}{5}$$
$$\cos^{-1}\left(\tfrac{1}{5}\right) = 78.46\ldots°$$

QI  $\beta = 78.46\ldots°$
QIV  $\beta = 360° - 78.46\ldots° = 281.53\ldots$

$$\{78.5°, 281.5°\}$$

**16**  $\dfrac{1}{2}\csc \beta + 1 = \dfrac{1}{4}(\csc \beta + 8)$

$$\tfrac{1}{2}\csc \beta + 1 = \tfrac{1}{4}\csc\beta + 2$$

$$4\left(\tfrac{1}{4}\csc \beta = 1\right)$$
$$\csc \beta = 4$$
$$\sin \beta = \tfrac{1}{4}$$
$$\sin^{-1}\left(\tfrac{1}{4}\right) = 14.477\ldots$$

QI  $\beta = 14.477\ldots°$
QII  $\beta = 180 - 14.477\ldots° = 165.522\ldots$

$$\{14.5°, 165.5°\}$$

**17**  $2\tan \beta - \sqrt{3} = 2\sqrt{3} - \tan \beta$

$$3\tan \beta = 3\sqrt{3}$$
$$\tan \beta = \sqrt{3} \quad \text{don't need calculator}$$

QI :  $\beta = 60.0°$
QIII  $\beta = 180° + 60° = 240.0°$

$$\{60.0°, 240.0°\}$$