Name:_______________________________ Date:____________

PC: Ellipse Do Now Ms. Loughran

$c^2 = a^2 + b^2$

$c^2 = 4 + 9 = 13$

$c = \sqrt{13}$ ⇄

1. Given $-4y^2 + 9x^2 - 36x + 8y - 4 = 0$. Find each of the following.

Center: $(2,1)$

$(4,1)$

Vertices: $(2 \pm 2, 1)$ $(0,1)$

$9x^2 - 36x \quad -4y^2 + 8y \quad = 4$

$9(x^2 - 4x + 4) - 4(y^2 - 2y + 1) = \cancel{4}4 + 9(4) - 4(1)$

Asymptotes: $y - 1 = \pm \frac{3}{2}(x - 2)$

$\frac{9(x-2)^2}{36} - \frac{4(y-1)^2}{36} = \frac{36}{36}$

HTA

Foci: $(2 \pm \sqrt{13}, 1)$

$\frac{(x-2)^2}{4} - \frac{(y-1)^2}{9} = 1$

$a^2 = 4, a = 2$ ⇄

$b^2 = 9, b = 3$

2. Given $\frac{\cancel{5}(y+1)^2}{50} + \frac{\cancel{10}(x-2)^2}{50} = \frac{50}{50}$. Find each of the following.

Center: $(2,-1)$

VMA

$\frac{(x-2)^2}{5} + \frac{(y+1)^2}{10} = 1$

$a^2 = 10, a = \sqrt{10}$ ↑↓

$b^2 = 5, b = \sqrt{5}$ ⇄

Major axis length:

$2\sqrt{10}$

Minor axis length:

$2\sqrt{5}$

Vertices: $(2, -1 \pm \sqrt{10})$

$c^2 = a^2 - b^2$

$c^2 = 10 - 5 = 5$

$c = \sqrt{5}$ ↑↓

Covertices: $(2 \pm \sqrt{5}, -1)$

Foci: $(2, -1 \pm \sqrt{5})$

Limits are used to describe how a function behaves as the independent variable moves towards a certain value of as it moves towards $\pm\infty$.

Pre-Calculus

FINDING LIMITS GRAPHICALLY
AN INTRODUCTION

L. Lewis

Limit is not concerned with what happens at a but rather as we are approaching a.

Consider a function $y = f(x)$ graphed below:

$\lim\limits_{x\to a^-}$ → "limit as x approaches a from the left"

$\lim\limits_{x\to a^+}$ → "limit as x approaches a from the right"

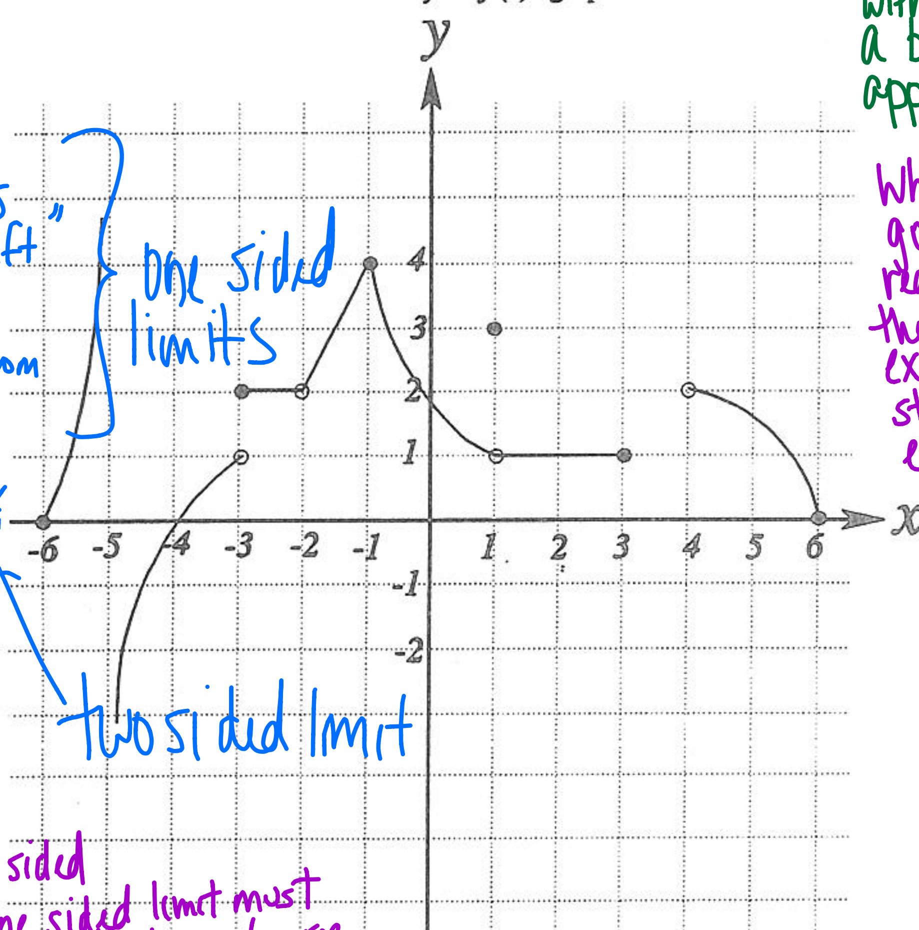
one sided limits

$\lim\limits_{x\to a}$ "the limit as x approaches a"

two sided limit

↑ this limit only exists if $\lim\limits_{x\to a^-} = \lim\limits_{x\to a^+}$

That means for a two sided limit to exist each one sided limit must exist and the one sided limits must agree.



When the limit is going off to $\pm\infty$ that really means that the limit does not exist but we can state "how it doesn't exist." We know it is increasing or decreasing without bound.

does not exist

Find each limit if it exists, and find each function value, if possible.

1. a) $\lim\limits_{x\to -1^+} f(x) =$ 4
   b) $\lim\limits_{x\to -1^-} f(x) =$ 4
   c) $\lim\limits_{x\to -1} f(x) =$ 4
   d) $f(-1) =$ 4

2. a) $\lim\limits_{x\to 1^+} f(x) =$ 1
   b) $\lim\limits_{x\to 1^-} f(x) =$ 1
   c) $\lim\limits_{x\to 1} f(x) =$ 1
   d) $f(1) =$ 3

3. a) $\lim\limits_{x\to -2^+} f(x) =$ 2
   b) $\lim\limits_{x\to -2^-} f(x) =$ 2
   c) $\lim\limits_{x\to -2} f(x) =$ 2
   d) $f(-2) =$ not defined

4. a) $\lim\limits_{x\to -3^+} f(x) =$ 2
   b) $\lim\limits_{x\to -3^-} f(x) =$ 1
   c) $\lim\limits_{x\to -3} f(x) =$ dne
   d) $f(-3) =$ 2

5. a) $\lim\limits_{x\to 3^+} f(x) =$ dne
   b) $\lim\limits_{x\to 3^-} f(x) =$ 1
   c) $\lim\limits_{x\to 3} f(x) =$ dne
   d) $f(3) =$ 1

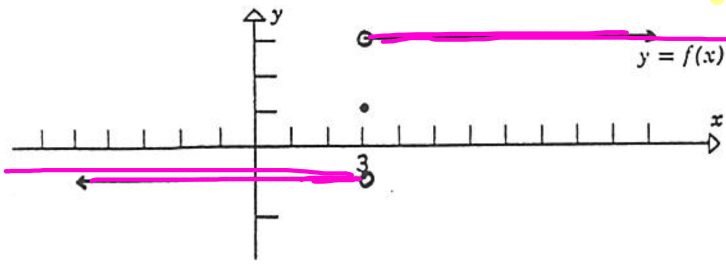
6. a) $\lim\limits_{x\to -5^+} f(x) =$ $-\infty$
   b) $\lim\limits_{x\to -5^-} f(x) =$ ∞
   c) $\lim\limits_{x\to -5} f(x) =$ dne
   d) $f(-5) =$ undefined

1. For the function f graphed below, find

(a) $\lim_{x\to 3^-} f(x)$ -1 (b) $\lim_{x\to 3^+} f(x)$ 3

(c) $\lim_{x\to 3} f(x)$ dne look for closed (d) $f(3) = 1$

(e) $\lim_{x\to -\infty} f(x)$ -1 (f) $\lim_{x\to +\infty} f(x)$. 3

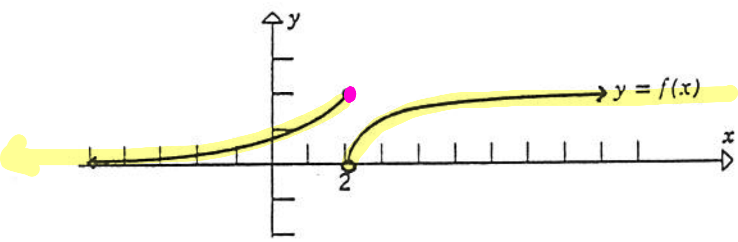


2. For the function f graphed below, find

(a) $\lim_{x\to 2^-} f(x)$ 2 (b) $\lim_{x\to 2^+} f(x)$ 0

(c) $\lim_{x\to 2} f(x)$ dne (d) $f(2)$ 2

(e) $\lim_{x\to -\infty} f(x)$ 0 (f) $\lim_{x\to +\infty} f(x)$. 2

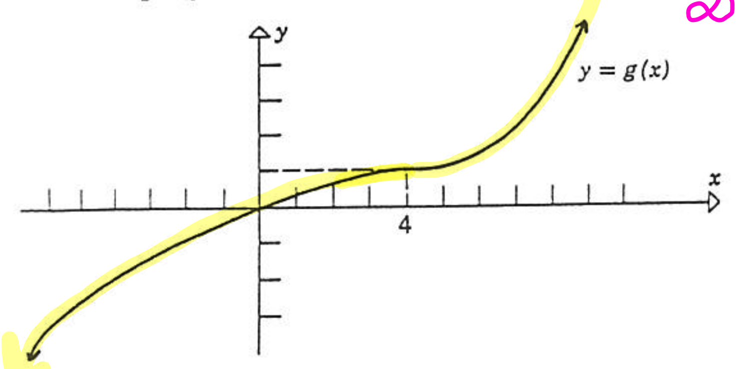


3. For the function g graphed below, find

(a) $\lim_{x\to 4^-} g(x)$ 1 (b) $\lim_{x\to 4^+} g(x)$ 1

(c) $\lim_{x\to 4} g(x)$ 1 (d) $g(4)$ 1

(e) $\lim_{x\to -\infty} g(x)$ $-\infty$ (f) $\lim_{x\to +\infty} g(x)$. ∞

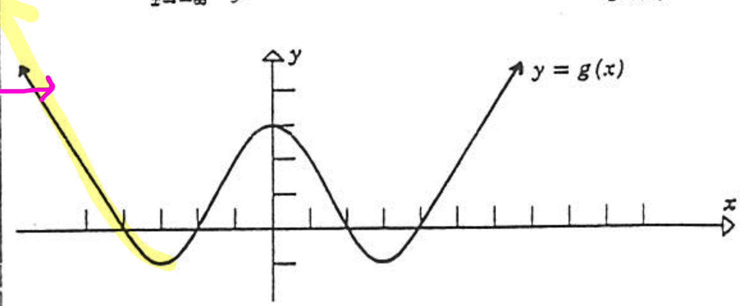


4. For the function g graphed below, find

(a) $\lim_{x\to 0^-} g(x)$ 3 (b) $\lim_{x\to 0^+} g(x)$ 3

(c) $\lim_{x\to 0} g(x)$ 3 (d) $g(0)$ 3

(e) $\lim_{x\to -\infty} g(x)$ ∞ (f) $\lim_{x\to +\infty} g(x)$. ∞

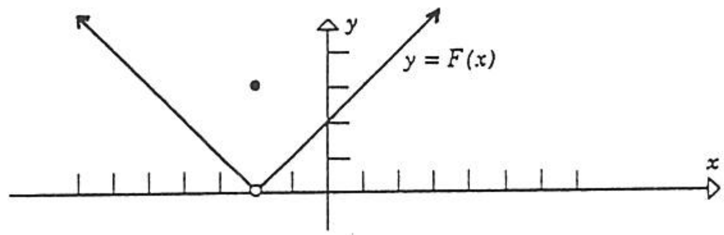


5. For the function F graphed below, find

(a) $\lim_{x\to -2^-} F(x)$ (b) $\lim_{x\to -2^+} F(x)$

(c) $\lim_{x\to -2} F(x)$ (d) $F(-2)$

(e) $\lim_{x\to -\infty} F(x)$ (f) $\lim_{x\to +\infty} F(x)$.

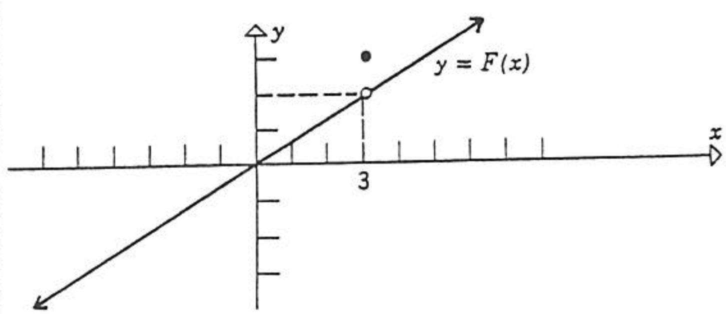


6. For the function F graphed below, find

(a) $\lim_{x\to 3^-} F(x)$ (b) $\lim_{x\to 3^+} F(x)$

(c) $\lim_{x\to 3} F(x)$ (d) $F(3)$

(e) $\lim_{x\to -\infty} F(x)$ (f) $\lim_{x\to +\infty} F(x)$.

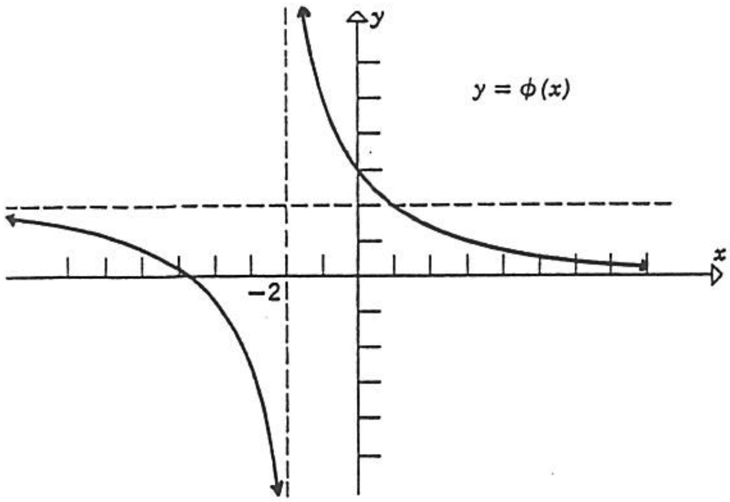


7. For the function ϕ graphed below, find

(a) $\lim_{x\to -2^-} \phi(x)$ (b) $\lim_{x\to -2^+} \phi(x)$

(c) $\lim_{x\to -2} \phi(x)$ (d) $\phi(-2)$

(e) $\lim_{x\to -\infty} \phi(x)$ (f) $\lim_{x\to +\infty} \phi(x)$.

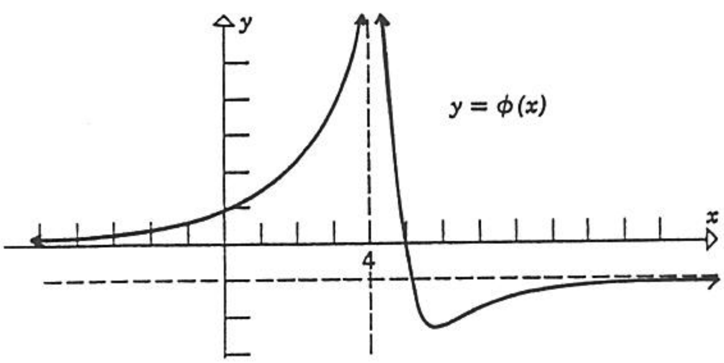


8. For the function ϕ graphed below, find

(a) $\lim_{x\to 4^+} \phi(x)$ (b) $\lim_{x\to 4^-} \phi(x)$

(c) $\lim_{x\to 4} \phi(x)$ (d) $\phi(4)$

(e) $\lim_{x\to -\infty} \phi(x)$ (f) $\lim_{x\to +\infty} \phi(x)$.

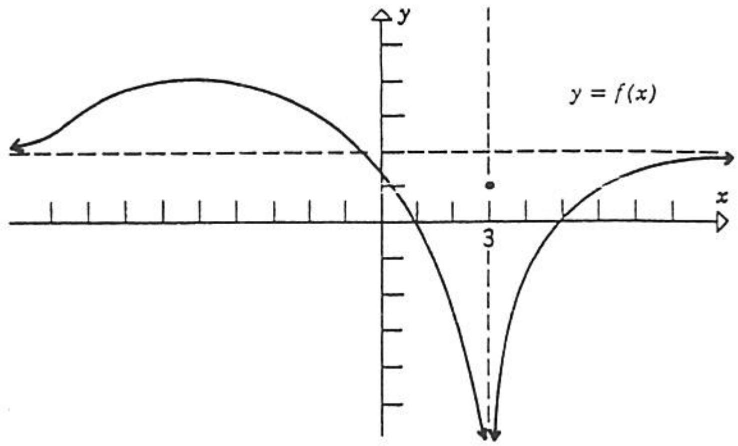


9. For the function f graphed below find

(a) $\lim_{x\to 3^-} f(x)$ (b) $\lim_{x\to 3^+} f(x)$

(c) $\lim_{x\to 3} f(x)$ (d) $f(3)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to +\infty} f(x)$.

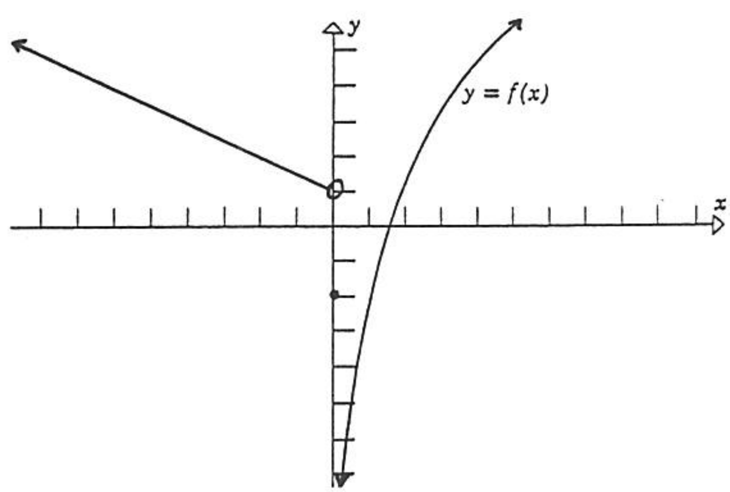


10. For the function f graphed below, find

(a) $\lim_{x\to 0^-} f(x)$ (b) $\lim_{x\to 0^+} f(x)$

(c) $\lim_{x\to 0} f(x)$ (d) $f(0)$

(e) $\lim_{x\to -\infty} f(x)$ (f) $\lim_{x\to +\infty} f(x)$.

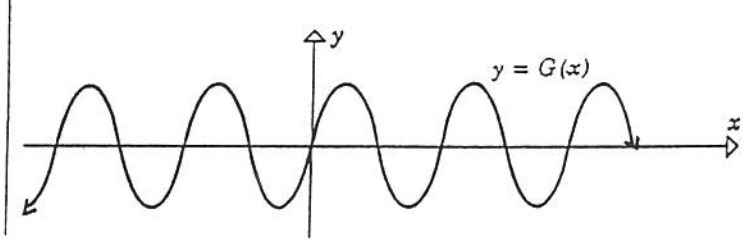


11. For the function G graphed below, find

(a) $\lim_{x\to 0^-} G(x)$ (b) $\lim_{x\to 0^+} G(x)$

(c) $\lim_{x\to 0} G(x)$ (d) $G(0)$

(e) $\lim_{x\to -\infty} G(x)$ (f) $\lim_{x\to +\infty} G(x)$.



Key to Review of Conics sheet

① $y^2+2x^2-3=0$ ellipse

② $y+2x^2-3=0$ parabola

③ $y+2x-3=0$ none (line)

④ $2y^2+2x^2-3=0$ circle

⑤ $y^2-2x^2-3=0$ hyperbola

⑥ $y^2+x^2+2x-3=0$

$y^2+x^2+2x+1=3+1$

$y^2+(x+1)^2=4$ circle

⑦ $y^2+4y+2x^2-4x=-2$

$y^2+4y+4+2(x^2-2x+1)=-2+4+2(1)$

$(y+2)^2+2(x-1)^2=4$

$\frac{(y+2)^2}{4}+\frac{(x-1)^2}{2}=1$

ellipse

⑧ $4y^2+32y-x^2-16x-16=0$

$4(y^2+8y+16)-1(x^2+16x+64)=16+4(16)-1(64)$

$4(y+4)^2-(x+8)^2=16$

$\frac{(y+4)^2}{4}-\frac{(x+8)^2}{16}=1$ hyperbola

⑨ $y^2+x^2-2x-3=0$

$y^2+x^2-2x=3$

$y^2+x^2-2x+1=3+1$

$y^2+(x-1)^2=4$

$c=(1,0)$ radius $=\sqrt{4}=2$

10 $\frac{(y-3)^2}{4} + \frac{(x+1)^2}{5} = 1$

$c^2 = a^2 - b^2$
$c^2 = 5 - 4 = 1$
$c = 1$

$c: (-1,3)$

ellipse $a^2 = 5 \rightarrow a = \sqrt{5}$
$b^2 = 4 \quad b = 2$

foci: $(-1 \pm 1, 3)$ → $(0,3)$, $(-2,3)$

horizontal major axis

vertices: $(-1+\sqrt{5}, 3), (-1-\sqrt{5}, 3)$
covertices: $(-1, 5), (-1, 1)$

11 $\frac{(y-1)^2}{4} - \frac{(x+4)^2}{25} = 1$

vertices: $(-4, 3), (-4, -1)$

hyperbola $c(-4, 1)$
$y+$ vertical transverse axis
$a^2 = 4 \rightarrow a = 2$
$b^2 = 25 \rightarrow b = 5$

$c^2 = a^2 + b^2$
$c^2 = 4 + 25 = 29$
$c = \sqrt{29}$

foci: $(-4, 1 \pm \sqrt{29})$

12 $\frac{x^2}{100} + \frac{(y+1)^2}{144} = 1$

ellipse vertical major axis
$a^2 = 144 \rightarrow a = 12$
$b^2 = 100 \rightarrow b = 10$
$c: (0, -1)$

$c: (0, -1)$
vertices: $(0, 11), (0, -13)$
covertices: $(10, -1), (-10, -1)$
$c^2 = a^2 - b^2 = 144 - 100 = 44$
$c = \sqrt{44} = \sqrt{4}\sqrt{11} = 2\sqrt{11}$
foci: $(0, -1 \pm 2\sqrt{11})$

13 $\frac{(x+6)^2}{} - \frac{(y-1)^2}{9} = 1$

$c^2 = a^2 + b^2$
$c^2 = 1 + 9 = 10$
$c = \sqrt{10}$

hyperbola horizontal transverse axis
$c: (-6, 1)$
$a = 1$
$b = 3$

vertices: $(-5, 1)$, $(-7, 1)$
foci: $(-6 \pm \sqrt{10}, 1)$

14 same equation as #8

$$\frac{(y+4)^2}{4} - \frac{(x+8)^2}{16} = 1$$

asym: $y-k = \pm \frac{a}{b}(x-h)$

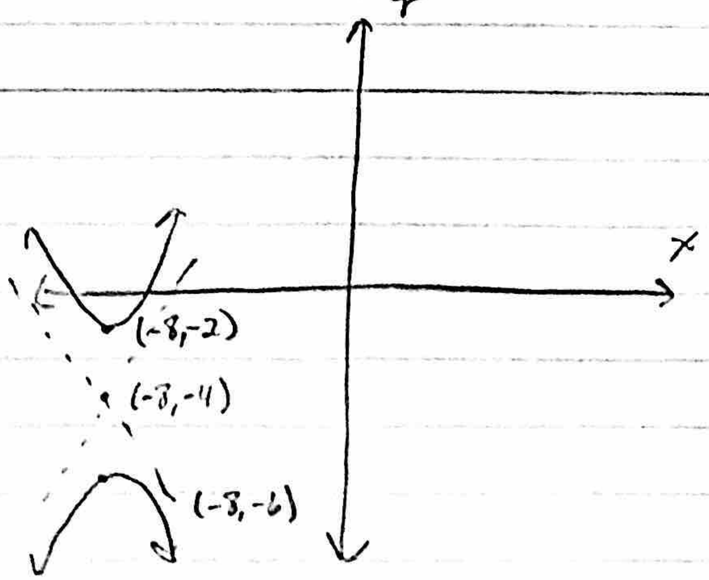
$y+4 = \pm \frac{1}{2}(x+8)$

hyperbola $c: (-8, -4)$
y ⊕ vertical transverse axis
$a = 2$, $b = 4$

vertices: $(-8, -2)$ $(-8, -6)$

$c^2 = a^2 + b^2$
$c^2 = 4 + 16 = 20$
$c = \sqrt{20} = 2\sqrt{5}$

foci: $(-8, -4 \pm 2\sqrt{5})$



(15) Same equation as #7

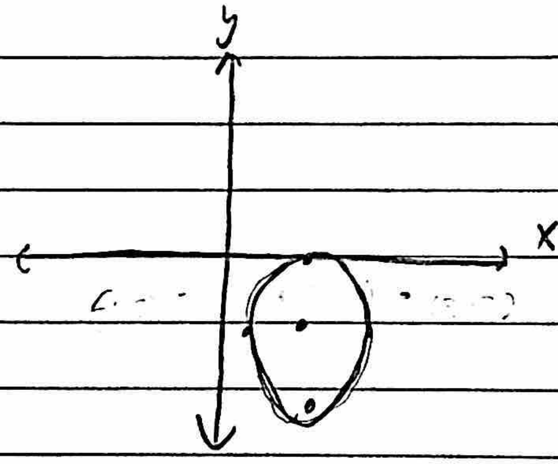
$$\frac{(y+2)^2}{4} + \frac{(x-1)^2}{2} = 1$$

$c^2 = a^2 - b^2 = 4-2$
$c^2 = 2 \quad c = \sqrt{2}$

ellipse c: $(1,-2)$ $a=2$, $b=\sqrt{2}$ vertical major axis

vertices: $(1,0)$, $(1,-4)$
covertices: $(1+\sqrt{2},-2)$, $(1-\sqrt{2},-2)$

foci: $(1, -2\pm\sqrt{2})$



(16) $x^2 - \frac{(y-1)^2}{9} = 1$

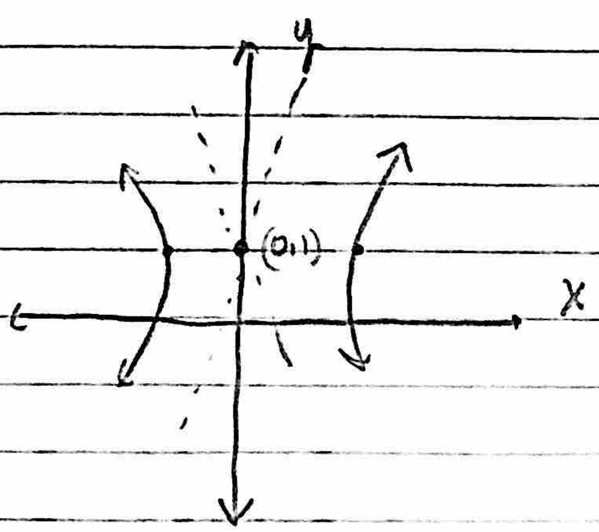
$c^2 = a^2 + b^2$
$c^2 = 1+9 = 10$
$c = \sqrt{10}$

hyperbola c: $(0,1)$

x ⊕ Horizontal tranverse axis foci: $(0\pm\sqrt{10}, 1)$
$a=1$
$b=3$

vertices: $(1,1)$, $(-1,1)$

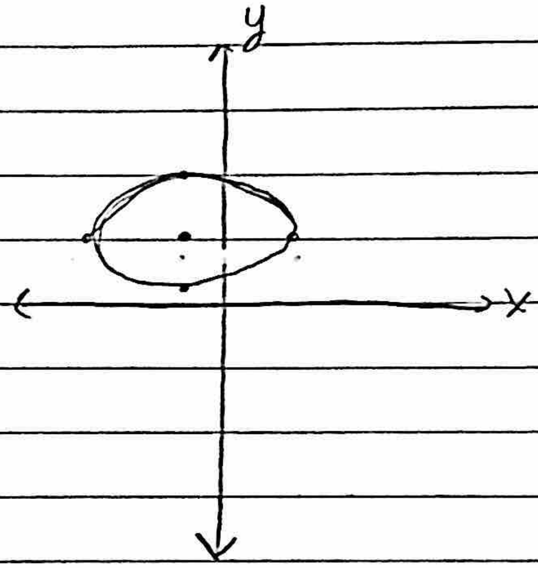
asym: $y-k = \pm\frac{b}{a}(x-h)$
$y-1 = \pm\frac{3}{1}(x-0)$



(17) $\dfrac{(y-3)^2}{4} + \dfrac{(x+1)^2}{9} = 1$

ellipse - horizontal major axis

$a=3$, $b=2$



$C: (-1,3)$

vertices: $(-4,3)$, $(2,3)$

covertices: $(-1,5)$, $(-1,1)$

$c^2 = a^2 - b^2$

$c^2 = 9-4 = 5$

$c = \sqrt{5}$

foci: $(-1 \pm \sqrt{5}, 3)$

(18) $(-1,4)$, $(3,-4)$

center $\Rightarrow$ midpoint $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right) = \left(\dfrac{-1+3}{2}, \dfrac{4-(-4)}{2}\right)$

$(1, 0)$

$$\text{distance} = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$
$$= \sqrt{(3-(-1))^2 + (-4-4)^2}$$
$$= \sqrt{16+64}$$
$$= \sqrt{80} = \sqrt{16}\sqrt{5}$$
$$= 4\sqrt{5} \leftarrow \text{diameter}$$

radius $= 2\sqrt{5}$

Eq: $(x-1)^2 + (y-0)^2 = (2\sqrt{5})^2$

$(x-1)^2 + y^2 = 20$