Do Now: From the Exercises section of the Sum and Difference of Angles Formulas packet (yesterday's packet) # 43

QI

43. If x and y are positive acute angles such that $\cos x = \frac{12}{13}$ A H and $\cos y = \frac{4}{5}$ A H, find the value of $\cos(x + y)$.

O:5 O:3

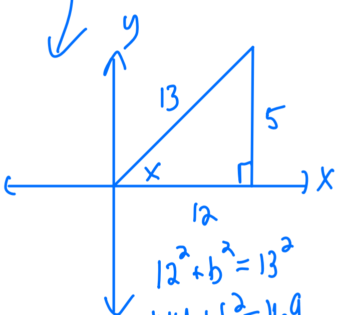
$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\left(\frac{12}{13}\right)\left(\frac{4}{5}\right) - \left(\frac{+5}{13}\right)\left(\frac{+3}{5}\right) = \frac{33}{65}$$

Or use the Pythagorean Triples:

3, 4, 5

5, 12, 13

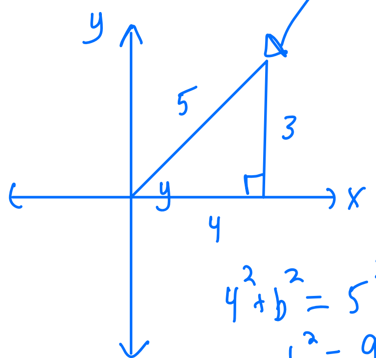


$12^2 + b^2 = 13^2$

$144 + b^2 = 169$

$b^2 = 25$

$b = \pm 5$



$4^2 + b^2 = 5^2$

$b^2 = 9$

$b = \pm 3$

Name:______________________________________ Date:__________

PC: Double Angle Formulas Ms. Loughran

Double Angles

Double-Angle Formulas

$\sin 2x = 2\sin x \cos x$	$\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$	$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

One application of these formulas is in evaluating trigonometric functions.

A second application of these formulas is in solving trigonometric equations.

Examples:

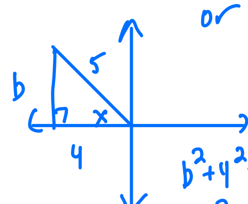
A: 4
H: 5 O: 3

1. If $\angle x$ is in Quadrant II and $\cos x = -\frac{4}{5}$. Find the value of $\sin 2x$.

need $\sin x \rightarrow$ use Pythagorean triple 3,4,5

$$\sin 2x = 2\sin x \cos x$$
$$= 2\left(+\frac{3}{5}\right)\left(\frac{-4}{5}\right) = \frac{-24}{25}$$

or



$b^2 + 4^2 = 5^2$

$b^2 = 9$

$b = \pm 3$

2. Prove the identity: $\sin 2x = \dfrac{2\tan x}{1+\tan^2 x}$

$$\frac{2\cdot\frac{\sin x}{\cos x}\cos^2 x}{\cos^2 x\left(1+\frac{\sin^2 x}{\cos^2 x}\right)}$$

$$\frac{2\sin x\cos x}{\cos^2 x + \sin^2 x}$$

$$\frac{2\sin x\cos x}{1}$$

$2\sin x \cos x = 2\sin x\cos x$

QI

1. If A is a positive acute angle and $\cos A = \frac{4}{5}$, what is the value of $\cos 2A$?

(1) 1 (2) $\frac{7}{25}$ (3) $\frac{9}{25}$ (4) $\frac{24}{25}$

$$\cos 2x = 2\cos^2 x - 1$$
$$= 2\left(\frac{4}{5}\right)^2 - 1 = \frac{7}{25}$$

Double Angles

$\sin 2x = 2 \sin x \cos x$	*Double-Angle Formulas* $\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$	$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

O:3 H:5 A:4 QII

2. If $\sin x = \frac{3}{5}$ and angle x is obtuse, then the value of $\sin 2x$ is

(1) $\frac{6}{5}$ (2) $-\frac{6}{5}$ (3) $\frac{24}{25}$ (4) $-\frac{24}{25}$

$$\sin 2x = 2\sin x \cos x$$
$$= 2\left(\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{-24}{25}$$

O:3 H:5 A:4 QI

3. If $\sin x = \frac{3}{5}$ and x is an acute angle, what is the numerical value of $\sin 2x$?

$$\sin 2x = 2\sin x \cos x$$
$$= 2\left(\frac{3}{5}\right)\left(+\frac{4}{5}\right) = \frac{24}{25}$$

4. If $\sin A = \frac{2}{5}$, find the value of $\cos 2A$.

$\cos 2A = 1 - 2\sin^2 A$

$= 1 - 2\left(\frac{2}{5}\right)^2$

$\frac{17}{25}$

Double Angles

	Double-Angle Formulas	
$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$	$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

31. $(\sin x - \cos x)^2$ is equivalent to

(1) 1 (3) $1 - \sin 2x$

(2) $-\cos 2x$ (4) $1 - \cos 2x$

$-\cos x \sin x$

$(\sin x - \cos x)(\sin x - \cos x)$

$-\cos x \sin x$

Double Angles

	Double-Angle Formulas	
$\sin 2x = 2\sin x\cos x$	$\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$	$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$

$\sin^2 x - \underbrace{2\cos x \sin x}_{} + \cos^2 x$

$1 \quad - \sin 2x$

Sum and Difference of Angles Formulas HW

(27) $\cos 70^\circ \cos 40^\circ - \sin 70^\circ \sin 40^\circ$

$\cos(70^\circ + 40^\circ) = \cos 110^\circ$ (3)

(29) $\sin 42^\circ \cos 48^\circ + \cos 42^\circ \sin 48^\circ$

$\sin(42^\circ + 48^\circ)$

$\sin 90^\circ = 1$ (1)

(30) $\sin 95^\circ \cos 24^\circ + \cos 96^\circ \sin 24^\circ$

$\sin(95^\circ + 24^\circ)$

$\sin 120^\circ$ Q2 S+

$\sin 60^\circ = \sqrt{3}/2$ (1)

(31) $\sin 210^\circ \cos 30^\circ - \cos 210^\circ \sin 30^\circ$

$\sin(210^\circ - 30^\circ)$

$\sin 180^\circ = 0$ (3)

(32) $\sin 90^\circ \cos 30^\circ - \cos 90^\circ \sin 30^\circ$

$\sin(90^\circ - 30^\circ)$

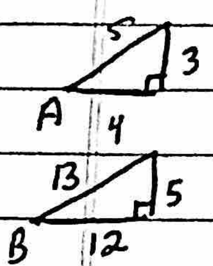
$\sin 60^\circ$

$\frac{\sqrt{3}}{2}$

(39) $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$= \left(\frac{4}{5}\right)\left(\frac{12}{13}\right) \quad \left(\frac{3}{5}\right)\left(\frac{5}{13}\right)$

If ∡A and ∡B are acute ∡s, they are in QI where all trig functions are positive



$13^2 = 5^2 + x^2$

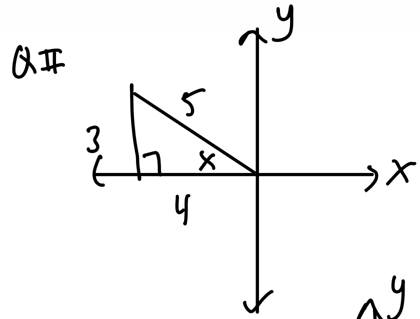
$x = 12$

$\frac{48}{65} + \frac{15}{65} = \frac{63}{65}$ (4)

(40) $\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$= \frac{\frac{1}{2}+\frac{1}{3}}{1-\left(\frac{1}{2}\right)\left(\frac{1}{3}\right)} = \frac{\frac{5}{6}}{1-\frac{1}{6}} = \frac{\frac{5}{6}}{\frac{5}{6}} = 1$ (1)

(41) $\tan(x+y) = \frac{\tan x + \tan y}{1-\tan x \tan y} = \frac{1+2}{1-(1)(2)} = \frac{3}{1-2} = \frac{3}{-1} = -3$

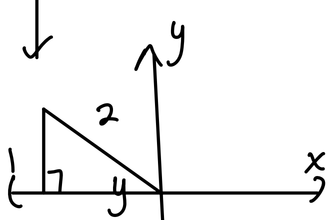


(42) Obtuse ∡s → QII

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$= \left(\frac{3}{5}\right)\left(-\frac{\sqrt{3}}{2}\right) + \left(\frac{-4}{5}\right)\left(\frac{1}{2}\right)$

$\frac{-3\sqrt{3}}{10} + -\frac{4}{10} = \frac{-3\sqrt{3}-4}{10}$



$1^2 + b^2 = 2^2$

$b^2 = 3$

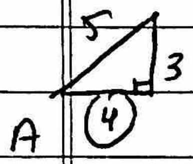
$b = \pm\sqrt{3}$

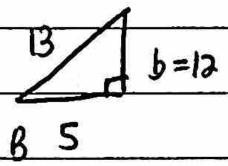
Positive acute ∡s means they are in QI

(44) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$= \left(\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{3}{5}\right)\left(\frac{12}{13}\right)$$

$$\frac{20}{65} - \frac{36}{65} = \frac{-16}{65}$$





$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = \pm 12$