Do Now: From the Exercises section of the Sum and Difference packet # 37

QI

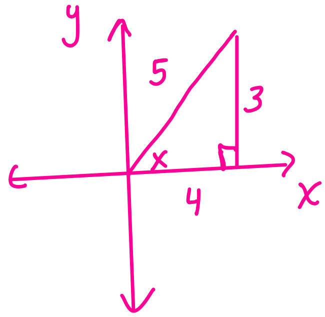
37. If x and y are positive acute angles, and $\sin x = \frac{3}{5}$ and $\sin y = \frac{1}{2}$, then $\sin(x+y)$ is equal to

(1) $\frac{3\sqrt{3}-4}{10}$

(2) $\frac{3\sqrt{3}+4}{10}$

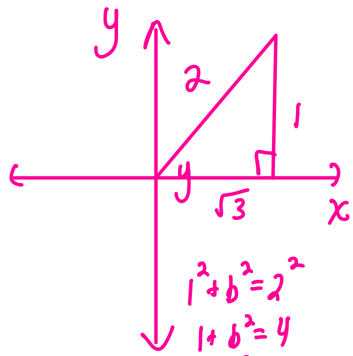
(3) $\frac{12}{25}+\frac{\sqrt{3}}{4}$

(4) $\frac{12}{25}-\frac{\sqrt{3}}{4}$



$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$
$$= \left(\frac{3}{5}\right)\left(\frac{+\sqrt{3}}{2}\right) + \left(\frac{+4}{5}\right)\left(\frac{1}{2}\right)$$

$$\frac{3\sqrt{3}}{10} + \frac{4}{10} = \frac{3\sqrt{3}+4}{10}$$



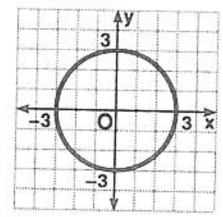
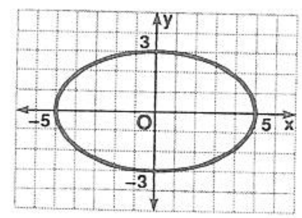
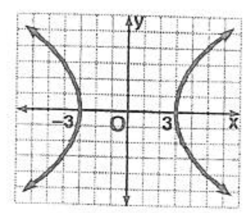
$1^2 + b^2 = 2^2$
$1 + b^2 = 4$
$b^2 = 3$
$b = \pm\sqrt{3}$

Name:______________________________________ Date:____________

PC: Special Relations Ms. Loughran

The General Equation $ax^2 + by^2 = c$

Depending on the values of the coefficients, the general equation $ax^2 + by^2 = c$, where $a, b, c \neq 0$, describes the graph of a *circle*, *ellipse*, or *hyperbola*.

Values of Coefficients	Name of Graph	Example	
$a = b$ and have the same sign as c	*circle*	$2x^2 + 2y^2 = 18$ or $x^2 + y^2 = 9$ circle with center at origin and radius = 3	
$a \neq b$ and have the same sign as c	*ellipse*	$9x^2 + 25y^2 = 225$ ellipse with center at origin and x-intercepts = ± 5 y-intercepts = ± 3	
a, b have different signs	*hyperbola*	$x^2 - y^2 = 9$ hyperbola with center at origin and x-intercepts = ± 3 no y-intercepts	

Recall: The equation of a parabola contains only one square term: either $y = ax^2 + bx + c$ or $x = ay^2 + by + c$
The equation of a straight line contains no square terms: $ax + by = c$

EXERCISES

In 1-14, identify the graph of the given relation as

(1) a circle (3) a hyperbola
(2) an ellipse (4) a parabola

1. $4y^2 = 25 - 4x^2$
   $4x^2 + 4y^2 = 25$ circle (1)
2. $2x^2 + 3y^2 = 24$
3. $x^2 = y^2 + 9$ hyperbola (3)
   $x^2 - y^2 = 9$
4. $x^2 = 6 - y$
5. $4x^2 - 100 = 25y^2$
   $4x^2 - 25y^2 = 100$ hyperbola (3)
6. $3y^2 = 6 - x^2$
7. $3x^2 + 2y^2 = 6$ ellipse (2)
8. $4x^2 + 16y^2 = 25$
9. $x^2 + y = 9$ parabola (4)
   (only one square term)
10. $2x^2 = 5 - 2y^2$
11. $y^2 = 6 - 3x^2$ ellipse (2)
    $3x^2 + y^2 = 6$
12. $2x^2 - 9 = 2y^2$
13. $4x^2 - 4y^2 = 9$ hyperbola (3)
14. $x^2 - \frac{y^2}{16} = 1$

15. Which of the following is the equation of a hyperbola?
    (1) $x^2 = 10 - y^2$ $x^2 + y^2 = 10$ circle
    (2) $x = y^2 - 9$ parabola
    (3) $y^2 = x^2 - 1$ $-x^2 + y^2 = -1$
    (4) $4x^2 + y^2 = 9$ ellipse

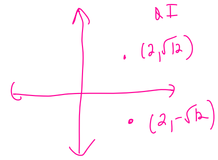
16. The graph of which equation is an ellipse?
    (1) $3x^2 - 4y^2 = 7$
    (2) $\frac{y + 6}{x - 1} = 3$
    (3) $y = 2x^2 + 3x - 5$
    (4) $x^2 + 5y^2 = 2$

17. Which is an equation of a circle?
    (1) $2x^2 - 2y^2 = 18$ hyperbola
    (2) $2x^2 + 3y^2 = 36$ ellipse
    (3) $3x^2 + 3y^2 = 21$
    (4) $x^2 = y^2 + 16$ hyperbola

18. Which equation has a hyperbola as its graph?
    (1) $x^2 = 10 + y$
    (2) $x^2 = 10 - y^2$
    (3) $3x^2 = 10 - 2y^2$
    (4) $3x^2 = 10 + 2y^2$

19. Which equation has an ellipse as its graph?
    (1) $2x^2 = 8 - 3y$ parabola
    (2) $2x^2 = 8 + 3y^2$ hyperbola
    (3) $2x^2 = 8 - 3y^2$ $2x^2 + 3y^2 = 8$
    (4) $2x = 8 - 3y$ line (no square terms)

20. Which is an equation of a circle?
    (1) $2x^2 + y^2 = 7$
    (2) $x = \frac{y}{8}$
    (3) $x^2 - y^2 = 10$
    (4) $5(x^2 + y^2) = 12$



$2^2+y^2=16$
$4+y^2=16$
$y^2=12$
$y=\pm\sqrt{12}$

pts of intersection
$(2, \sqrt{12})$
$(2, -\sqrt{12})$

21. Which is an equation of a parabola?

(1) $x^2 = 3 + y^2$ (3) $x = 3 + y$
(2) $x = 3 + y^2$ (4) $y^2 = 3x^2 + 3$

one square term

22. The graph of the relation $ay = bx^2 + c$ in which neither a nor b is 0 is

(1) a parabola (3) an ellipse
(2) a straight line (4) a hyperbola

23. If a, b, and c are positive unequal numbers, the graph of $ax^2 + by^2 = c$ is

(1) a circle (3) an ellipse
(2) a parabola (4) a hyperbola

24. The graph of $ax^2 + by^2 = c$, in which a, b, and c are real numbers, is an ellipse if

(1) $a = b, a > 0, b < 0, c > 0$
(2) $a = b, a > 0, b > 0, c < 0$
(3) $a \neq b, a > 0, b > 0, c > 0$
(4) $a \neq b, a > 0, b < 0, c > 0$

then are 2 square terms

25. If $a \neq 0$, $b \neq 0$, and $c \neq 0$, the graph of $ax^2 + by^2 = c$ can not be

(1) an ellipse (3) a parabola
(2) a circle (4) a hyperbola

26. The graph of the equation $\frac{x^2}{4} + \frac{y^2}{16} = 1$ passes through the point whose coordinates are

(1) (0, 0) (2) (0, 2) (3) (0, 4) (4) (4, 0)

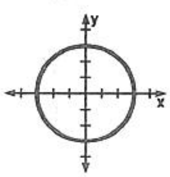
27. Which relation is a function? (has to pass the VLT)

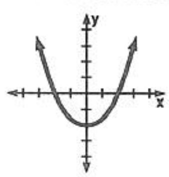
(1) $\{(x, y)| x^2 + y = 4\}$ $y=-x^2+4$ (3) $\{(x, y)| x^2 - y^2 = 4\}$ hyperbola
(2) $\{(x, y)| x^2 + y^2 = 4\}$ circle (4) $\{(x, y)| x^2 + 4y^2 = 4\}$ ellipse

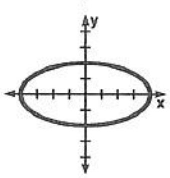
28. If the replacement set is the set of real numbers, what is the domain of the relation represented by $\{(x, y)| x^2 + 4y^2 = 16\}$?

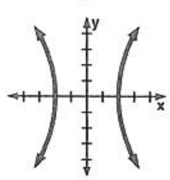
(1) $\{y| -2 \leq y \leq 2\}$ (3) $\{x| -4 \leq x \leq 4\}$
(2) $\{y| -2 < y < 2\}$ (4) $\{x| -4 < x < 4\}$

29. Which is the graph of a quadratic relation for which the domain consists of all the real numbers?

(1)


(3)


(2)


(4)


30. If the graphs of the equations $x^2 + y^2 = 9$ and $y = 3$ are drawn on the same set of axes, what is the total number of points common to both graphs?

(1) 1 (2) 2 (3) 3 (4) 0

31. When drawn on the same set of axes, the points of intersection of the graphs of $x^2 + y^2 = 16$ and $x = 2$ are located in quadrants

(1) I and III ~~(3)~~ II and III
(2) I and IV ~~(4)~~ II and IV

32. The graphs of the equations $x^2 + y^2 = 25$ and $y = x^2$ are drawn on the same set of axes. The total number of points common to these graphs is

(1) 1 (2) 2 (3) 3 (4) 4

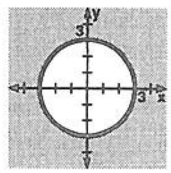
33. The graph of $x^2 + y^2 = 25$ and the graph of $x - 4 = 0$ are drawn on the same set of axes. A point of intersection of the graphs is

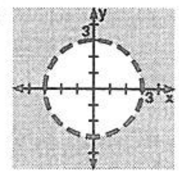
$x=4$

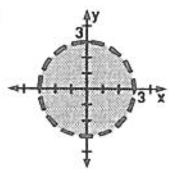
(1) (5, 0) (2) (−4, −3) (3) (4, −3) (4) (−3, 4)

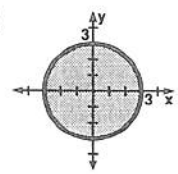
$x^2+y^2=25$
$4^2+y^2=25$
$16+y^2=25$
$y^2=9$
$y=\pm 3$ (4,3) (4,-3)

~~**34.**~~ What is the graph of the solution set of $x^2 + y^2 > 9$?

(1)


(3)


(2)


(4)


35. Each equation in column A has one of the geometric figures in column B as its graph. List the numbers 1-5 on your answer paper and after each number write the letter that indicates the corresponding graph.

Column A	Column B
(1) $x^2 + y^2 - 4 = 0$	**a.** The point (0, 0)
(2) $4x^2 + y^2 - 1 = 0$	**b.** Two straight lines parallel to the y-axis
(3) $x^2 - y - 4 = 0$	**c.** Two straight lines intersecting at the origin
(4) $x^2 + 4y^2 = 0$	**d.** A parabola that crosses the y-axis at (0, −4)
(5) $x^2 - 4y^2 = 0$	**e.** A circle whose center is the origin and whose radius is 2
	f. An ellipse that crosses the y-axis at (0, 1) and (0, −1)
	g. A hyperbola that crosses the y-axis at (0, 2) and (0, −2)

Homework 04-11

6) $\sin 2x = 2\sin x \cos x$ x is a post. acute $\measuredangle \rightarrow$ QI

$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right)$ if adj $= 5$, hyp $= 13$

$= \frac{120}{69}$ opp $= 12$

7) $\cos 2x = 1 - 2\sin^2 x$ $\rightarrow$ I chose this formula for $\cos 2x$ b/c I had $\sin x = \frac{5}{6}$

$1 - 2\left(\frac{5}{6}\right)^2$

$= \frac{-7}{18}$

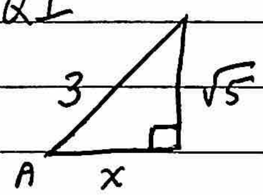
8) $\tan 2A = \dfrac{2\tan A}{1-\tan^2 A} = \dfrac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2} = \dfrac{3}{4}$

9) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta} = \dfrac{2(1)}{1-(1)^2} = \dfrac{2}{0}$ (4) undefined

10) a) $\sin 2A = 2\sin A\cos A$ $\measuredangle A$ in QI

$2\left(\frac{\sqrt{5}}{3}\right)\left(\frac{2}{3}\right)$

$\frac{4\sqrt{5}}{9}$



$x^2 + (\sqrt{5})^2 = 3^2$

$x^2 + 5 = 9$

$x^2 = 4$

$x = \pm 2$

b) $\cos 2A = 1 - 2\sin^2 A$

$1 - 2\left(\frac{\sqrt{5}}{3}\right)^2 = 1 - 2\left(\frac{5}{9}\right) = 1 - \frac{10}{9} = \frac{-1}{9}$

(23) $\sin 2\theta \sec\theta$

$2\sin\theta\cancel{\cos\theta}\cdot\frac{1}{\cancel{\cos\theta}} = 2\sin\theta$ (4)

(24) $\frac{1}{2}\sec x \sin 2x$

$\frac{1}{\cancel{2}}\cdot\frac{1}{\cancel{\cos x}}\cdot\cancel{2}\sin x\cancel{\cos x} = \sin x$ (1)

(25) $2\sin^2 A + \underbrace{\cos^2 A}$ → using 1st formula for $\cos 2A$

$\underline{2\sin^2 A} + \cos^2 A \underline{- \sin^2 A}$ can combine these

$\sin^2 A + \cos^2 A$

1 (1)

(26) $\frac{\sin 2A}{\sin^2 A} = \frac{2\cancel{\sin}A\cos A}{\sin^{\cancel{2}} A} = \frac{2\cos A}{\sin A} = 2\cot A$ (4)

(27) $\frac{\sin 2A}{2\sin A} = \frac{\cancel{2\sin A}\cos A}{\cancel{2\sin A}} = \cos A$ (2)

(28) $\frac{\sin 2A}{\cos A} - \sin A$

using formula

$\frac{2\sin A\cancel{\cos A}}{\cancel{\cos A}} - \sin A$

$2\sin A - 1\sin A$

$\sin A$ (3)

(29) $\frac{2\cos x}{\sin 2x}$

using formula

$\frac{\cancel{2\cos x}}{\cancel{2}\sin x\cancel{\cos x}}$

$\frac{1}{\sin x}$

$\csc x$ (4)

① $\sin(22^\circ + 18^\circ) = \sin 40$ (A)

② $\sin 2(30^\circ) = \sin 60^\circ$ (C)

③ $\cos 2(40) = \cos 80^\circ$ (C)

④ $\cos(70 + 40) = \cos 110^\circ$ (B)

⑤ $\sin A \cos B - \cos A \sin B$

$\left(\frac{4}{5}\right)\left(\frac{8}{17}\right) - \left(\frac{3}{5}\right)\left(\frac{15}{17}\right) = \frac{32}{85} - \frac{45}{85} = \frac{-13}{85}$

⑥ $\cos A \cos B - \sin A \sin B$

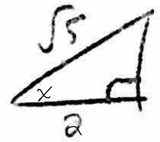
$\left(\frac{3}{5}\right)\left(\frac{8}{17}\right) - \left(\frac{4}{5}\right)\left(\frac{15}{17}\right) = \frac{24}{85} - \frac{60}{85} = -\frac{36}{85}$

⑦ $\cos A \cos B + \sin A \sin B$

$\left(\frac{3}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{4}{5}\right)\left(\frac{15}{17}\right) = \frac{24}{85} + \frac{60}{85} = \frac{84}{85}$

$5 = 2^2 + b^2$

$5 =$



$2^2 + b^2 = (\sqrt5)^2$

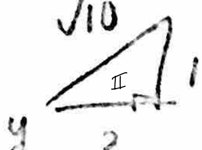
$4 + b^2 = 5$

$b^2 = 1$

$b = \pm 1$

⑧ $\sin x \cos y - \cos x \sin y$

$\left(\frac{1}{\sqrt5}\right)\left(-\frac{3}{\sqrt{10}}\right) - \left(\frac{2}{\sqrt5}\right)\left(\frac{1}{\sqrt{10}}\right)$



$3^2 + 1^2 = c^2$

$9 + 1 = c^2$

$10 = c^2$

$\pm\sqrt{10} = c$

$\frac{3}{\sqrt{50}} - \frac{2}{\sqrt{50}} = \frac{1}{\sqrt{50}}$

⑨ $\sin A \cos B + \cos A \sin B$

$\left(\frac{4}{5}\right)\left(\frac{8}{17}\right) + \left(\frac{3}{5}\right)\left(\frac{15}{17}\right) = \frac{32 + 45}{85} = \frac{77}{85}$

(10) $\cos 2A = 1 - 2\sin^2 A = 1 - 2\left(\frac{3}{5}\right)^2 = 1 - 2\left(\frac{9}{25}\right)$

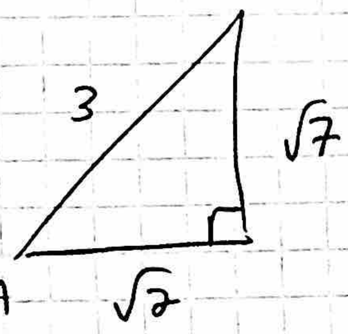
$1 - \frac{18}{25} = \frac{7}{25}$

(11) $\cos 2\theta = 2\cos^2\theta - 1$

$2\left(-\frac{3}{5}\right)^2 - 1$

$2\left(\frac{9}{25}\right) - 1$

$\frac{18}{25} - 1 = \frac{-7}{25}$



(12) $\sin 2A = 2\sin A\cos A$

$2\left(\frac{\sqrt{7}}{3}\right)\left(\frac{\sqrt{2}}{3}\right) = \frac{2\sqrt{14}}{9}$